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April 29th. [illegible]

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A

TREATISE ON OPTICS.

BY

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MEMBER OF THE IMPERIAL ACADEMY OF PETERSBURG, AND OF
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A NEW EDITION.

WITH AN APPENDIX, CONTAINING AN ELEMENTARY VIEW OF THE APPLICATION OF ANALYSIS TO REFLEXION AND REFRACTION,

BY A. D. BACHE, A. M.

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NOTE BY THE AMERICAN EDITOR.

My object in undertaking the revision of the Treatise on Optics by Dr. Brewster was, principally, to introduce an Appendix, containing such a discussion of the subjects of Reflexion and Refraction, as might adapt the work to use in those of our colleges in which considerable extension is given to the course of Natural Philosophy. In this revision, I have thought it best, without specially calling the attention of the reader to them, to correct such errors as my comparatively limited knowledge of the subject assured me, would not have been passed over by the author in a second Edition.

A. D. BACHE.

Philadelphia, Jan., 1833.

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June 13 - 1848 Review -

A TREATISE ON OPTICS.

INTRODUCTION.

(1). Optics, from a Greek word which signifies *to see*, is that branch of knowledge which treats of the properties of *light* and of *vision*, as performed by the human eye.

(2). Light is an emanation, or something which proceeds from bodies, and by means of which we are enabled to see them by the eye. All visible bodies may be divided into two classes—*self-luminous* and *non-luminous*.

Self-luminous bodies, such as the stars, flames of all kinds, and bodies which shine by being heated or rubbed, are those which possess in themselves the property of discharging light. *Non-luminous* bodies are those which have not the power of discharging light of themselves, but which throw back the light which falls upon them from self-luminous bodies. One non-luminous body may receive light from another non-luminous body, and discharge it upon a third; but in every case the light must originally come from a self-luminous body. When a lighted candle is brought into a dark room, the form of the flame is seen by the light which proceeds from the flame itself; but the objects in the room are seen by the light which they receive from the candle, and again throw back; while other objects, on which the light of the candle does not fall, receive light from the white ceiling and walls, and thus become visible to the eye.

(3). All bodies, whether self-luminous or non-luminous, discharge light of the same color with themselves. A *red* flame or a red-hot body discharges *red* light; and a piece of red cloth discharges *red* light, though it is illuminated by the *white* light of the sun.

(4). Light is emitted from every visible point of a luminous or of an illuminated body, and in every direction in which the point is visible. If we look at the flame of a candle, or at a sheet of white paper, and magnify them ever so much, we shall not observe any points destitute of light.

(5.) Light moves in straight lines, and consists of separate and independent parts, called *rays* of light. If we admit the light of the sun into a dark room through a small hole, it will illuminate a spot on the wall exactly opposite to the sun,—the middle of the spot, the middle of the hole, and the middle of the sun, being all in the same straight line. If there is dust or smoke in the room, the progress of the light in straight lines will be distinctly seen. If we stop a very small portion of the admitted light, and allow the rest to pass, or if we stop nearly the whole light, and allow only the smallest portion to pass, the part which passes is not in the slightest degree affected by its separation from the rest. The smallest portion of light which we can either stop or allow to pass is called a *ray of light.*

(6). Light moves with a velocity of 192,500 miles in a second of time. It travels from the sun to the earth in seven minutes and a half. It moves through a space equal to the circumference of our globe in the 8th part of a second, a flight which the swiftest bird could not perform in less than three weeks.

(7). When light falls upon any body whatever, part of it is reflected or driven back, and part of it enters the body, and is either lost within it or transmitted through it. When the body is bright and well polished like *silver*, a great part of the light is reflected, and the remainder lost within the silver, which can transmit light only when hammered out into the thinnest film. When the body is transparent, like *glass* or *water*, almost all the light is transmitted, and only a small part of it reflected. The light which is driven back from bodies is reflected according to particular laws, the consideration of which forms that branch of optics called *catoptrics;* and the light which is transmitted through transparent bodies is transmitted according to particular laws, the consideration of which constitutes the subject of *dioptrics.*

PART I.

ON THE REFLEXION AND REFRACTION OF LIGHT.

CATOPTRICS.

(8.) Catoptrics is that branch of optics which treats of the progress of rays of light after they are reflected from surfaces either plane or curved, and of the formation of images from objects placed before such surfaces.

CHAP. I.

REFLEXION BY SPECULA AND MIRRORS.

(9.) Any substance of a regular form employed for the purpose of reflecting light, or of forming images of objects, is called a *speculum* or *mirror*. It is generally made of metal or glass, having a highly polished surface. The name of mirror is commonly given to reflectors that are made of glass; and the glass is always quicksilvered on the back, to make it reflect more light. The word *speculum* is used to describe a reflector which is metallic, such as those made of silver, steel, or of grain tin mixed with copper.

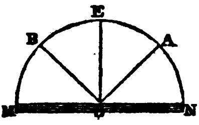
(10.) Specula or mirrors are either *plane*, *concave*, or *convex*.

A *plane speculum* is one which is perfectly flat, like a looking-glass; a *concave speculum* is one which is hollow like the inside of a watch-glass; and a *convex speculum* is one which is round like the outside of a watch-glass.

As the light which falls upon glass mirrors is intercepted by the glass before it is reflected from the quick-silvered surface, we shall suppose all our mirrors to be formed of polished metal, as they are in almost all optical instruments.

(11.) When a ray of light, A D, *fig*. 1., falls upon a plane speculum, M N, at the point D, it will be reflected or driven back in a direction D B, which is as much inclined to E D a line perpendicular to M N, as the ray A D was; that is, the angle B D E is equal to A D E, or the circular arc B E is equal to E A.

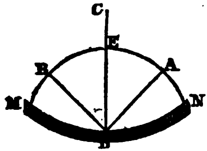
Fig. 1.



The ray A D is called the *incident ray*, and D B the reflected ray, A D E the *angle of incidence*, and B D E the *angle of reflexion;* and a plane passing through A D and D B, or the plane in which these two lines lie, is called the *plane of incidence*, or the *plane of reflexion.*

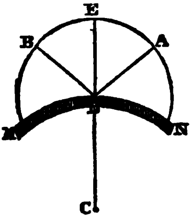
(12.) When the speculum is *concave*, as M N, *fig*. 2., then if C be the centre of the circle of which M N is a part, the incident ray A D and the reflected ray D B will form equal angles with the line C D, which is perpendicular to the small portion of the speculum on which the ray falls at D. Hence in this case also the angle of incidence A D E is equal to the angle of reflexion B D E.

Fig. 2.



(13.) When the speculum is convex, as M N, *fig*. 3., let C be the centre of the circle of which M N forms a part, and C E a line drawn through D; then the angle of incidence A D E will be equal to the angle of reflexion B D E.

Fig. 3.



These results are found to be true by experiment; and they may be easily proved by admitting a ray of the sun's light through a hole in the window-shutter, and making it fall on the mirrors M N in the direction A D, when it will be seen reflected in the direction D B. If the incident ray A D is made to approach the perpendicular D E, the reflected ray D B will also approach the perpendicular D E; and when the ray A D falls in the direction E D, it will be reflected in the direction D E. In like manner when the ray A D approaches to D N, the ray D B will approach to D M.

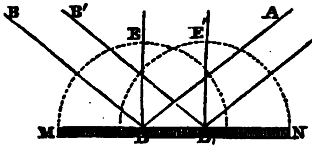
(14.) As these results are true under all circumstances, we may consider it as a general law, that *when light falls upon any surface, whether plane or curved, the angle of its reflexion is equal to the angle of its incidence.*

Hence we have a method of universal application for finding the direction of a reflected ray when we know the direction of the incident ray. If A D, for example, *figs*. 1, 2, 3., is the direction in which the incident ray falls upon the mirror at D, draw the perpendicular D E in *fig*. 1., and in *fig*. 2 or *fig*. 3. draw a line from D to C, the centre of the curved surface M N; and, having described a circle M B E A N round D as a centre, take the distance A E in the compasses and carry it from E to B, and having drawn a line from D to B, D B will be the direction of the reflected ray.

Reflexion of Rays from Plane Mirrors.

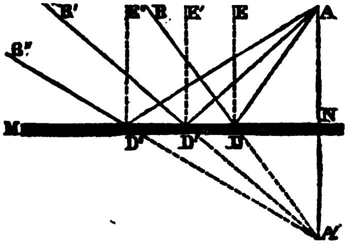
(15.) *Reflexion of parallel rays.* When parallel or equidistant rays, A D, A′ D′, *fig.* 4., are incident upon a plane mirror, M N, they will continue to be parallel after reflexion. By the method already explained, describe arches of circles round D, D′ as centres, and make the arch from E towards B equal to that between A D and D E, and also the arch from E′ towards B′ equal to that between A′ D′ and D′ E′; then drawing the lines D B, D′ B′, it will be found that these lines are parallel. If the space between A D and A′ D′ is filled with other rays parallel to A D, so as to constitute a parallel beam or mass of light, A A′ D′ D, the reflected rays will be all parallel to B D, and will constitute a parallel reflected beam. The reflected beam, however, will be inverted; for the side A D, which was uppermost before reflexion, will be undermost, as at D B, after reflexion.

Fig. 4.



(16.) *Reflexion of diverging rays.* Diverging rays are those which proceed from a point, A, and separate as they advance, like A D, A D′, A D″. When such rays fall upon a

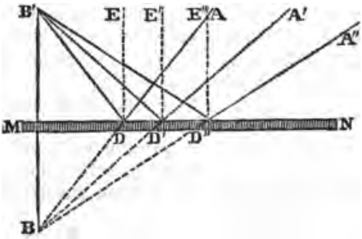
Fig. 5.



plane mirror M N, *fig.* 5., they will be reflected in directions D B, D′ B′, D″ B″, making the angles B D E, B′ D′ E′, B″ D″ E″ respectively, equal to A D E, A D′ E′, A D″ E″; the lines D E, D′ E′, D″ E″ being drawn from the points D, D′, D″, where the rays are incident, perpendicular to M N; and by continuing the reflected rays backwards, they will be found to meet at a point A′ as far *behind* the mirror M N as A is *before* it; that is, if A N A′ be drawn perpendicular to M N, A′ N will be equal to A N. Hence the rays will have the same divergency after reflexion as they had before it. If we consider A D″ D as a divergent beam of light included between A D and A D″, then the reflected beam included between D B and D″ B″ will diverge from A′, and will be inverted after reflexion.

(17.) *Reflexion of converging rays.* Converging rays are those which proceed from several points A A′ A″, *fig.* 6., towards one point B. When such rays fall upon a plane

Fig. 6.



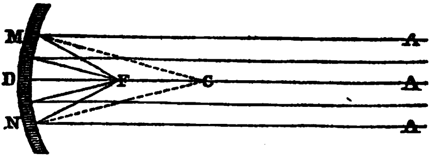
mirror, M N, they will be reflected in directions D B′, D′ B′, D″ B′, forming the same angles with the perpendiculars D E, D′ E′, D″ E″, as the incident rays did, and converging to a point B′ as far before the mirror as the point B is behind it. If we consider A D D″ A″ as a converging beam of light, D″ B′ D will be its form after reflexion.

In all these cases the reflexion does nothing more than invert the incident beam of light, and shift its point of divergence or convergence to the opposite side of the mirror.

Reflexion of Rays from Concave Mirrors.

(18.) *Reflexion of parallel rays.* Let M N, *fig.* 7., be a concave mirror whose centre of concavity is C; and let A D, A M, A N be parallel rays, or a parallel beam of light falling upon it, at and near to the vertex D. Then, since C M, C N are perpendicular to the surface of the mirror at the points M and N, C M A, C N A will be the angles of incidence of the rays A M, A N. Make the angles of reflexion C M F, C N F equal to C M A, C N A, and it will be found that the lines M F, N F meet at F in the line A D, and these lines M F, N F will be the reflected rays. The ray A C D being perpendicular to the mirror at D, because it passes through the centre C, will be reflected in an opposite direction D F; so that all the three rays, A M, A D, and A N, will meet at one point, F. In like manner it will be found that all other rays between A M and A N, falling upon other points of the mirror between M and N, will be reflected to the same point F. The point F, in which a concave mirror collects the rays which fall upon it, is called the *focus*, or *fire-place*, because the rays thus collected have the power of burning any inflammable body placed there. When the rays which the mirror collects are parallel, as in the present case, the point F is called its *principal focus*, or its *focus for parallel rays*. When we consider that the rays which form the beam A M N A occupy a large space before they fall upon the mirror M N, and by reflexion are condensed upon a small space at F, it is easy to understand how they have the power of burning bodies placed at F.

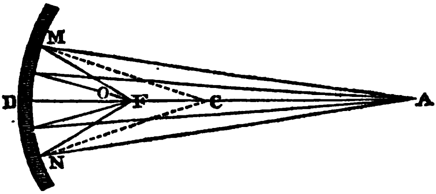
Fig. 7.



Rule.—The distance of the focus F from the nearest point or vertex D of the mirror M N is in spherical mirrors, whatever be their substance, equal to one half of C D, the radius of the mirror's concavity. The distance F D is called the *principal focal distance* of the mirror. The truth of this rule may be found by projecting *fig*. 7. upon a large scale, and by taking the points M N near to D.

(19.) *Reflexion of diverging rays.* Let M N, *fig*. 8., be a concave mirror, whose centre of concavity is C; and let rays A M, A D, A N, diverging or radiating from the point A, fall upon the mirror at the points, M, D, N, and be reflected from these points; M and N being near to D. The lines C M, C D, and C N being perpendicular to the mirror at the points M, D, and N, we shall find

Fig. 8.



the reflected rays M F, N F, by making the angle F M C equal to A M C, and F N C equal to A N C; and the point F where these rays meet will be the focus where the diverging rays A M, A N are collected. By comparing *fig.* 7. with *fig.* 8. it is obvious that, as the incident ray A M in *fig.* 8. is nearer the perpendicular C M than the same ray is in *fig.* 7., the reflected ray M F will also be nearer the perpendicular C M than the same ray in *fig.* 7.; and as the same is true of the reflected ray N F, it follows that the point F must be nearer C in *fig.* 8. than in *fig.* 7.; that is, in the reflexion of diverging rays the focal distance D F of the mirror is greater than its focal distance for parallel rays.

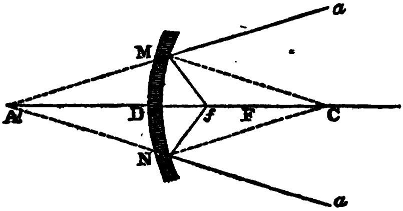
If we suppose the point of divergence A, *fig.* 8., or the *radiant point*, as it is called, to approach to C, the incident rays A M, A N will approach to the perpendiculars C M, C N, and consequently the reflected rays M F, N F will also approach to C M, C N; that is, as the radiant point A approaches to the centre of concavity C, the focus F also approaches to it, so that when A reaches C, F will also reach C; that is, when rays diverge from the centre, C, of a concave mirror, they will all be reflected to the same point.

If the radiant point A passes C towards D, then the focus F will pass C towards A; so that if the light now diverges from F it will be collected in A, the points that were formerly the radiant points being now the foci. From this relation, or interchange, between the radiant points and the foci, the points A and F have been called *conjugate foci*, because if either of them be the *radiant* point the other will be the *focal* point.

If in *fig.* 7. we suppose F to be the radiant point, then the focal point A will be at an infinite distance; that is, the rays will never meet in a focus, but will be parallel, like M A, N A in *fig.* 7.

In like manner it is obvious, that if the point F is at *f*, as in *fig.* 9., the reflected rays will be M *a*, N *a*; that is, they will diverge from some point, A′, behind the mirror M N; and as

Fig. 9.



f approaches to D, they will diverge more and more, as if the point A′, from which they seem to diverge, approached to D. The point A′ behind the mirror, from which the rays M *a*, N *a* seem to proceed, or at which they would meet if they moved backwards in the directions *a* M, *a* N is called their *virtual focus*, because they only *tend* to meet in that focus.

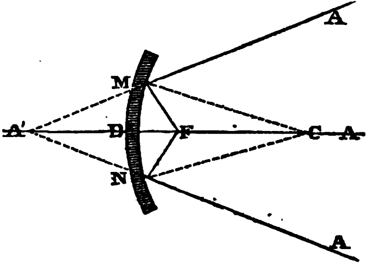
In all these cases the distance of the focus F may be determined either by projection or by the following rule, the radius of the concavity of the mirror, C D, and the distance, A D, of the radiant point, being given.

Rule. Multiply the distance, A D, of the radiant point from the mirror by the radius, C D, of the mirror, and divide this product by the difference between twice the distance of the radiant point and the radius of the mirror, and the quotient will be F D, the conjugate focal distance required.

In applying this rule we must observe, what will be readily seen from the figures, that if twice A D is less than C D (as at *f*, *fig.* 9.), the rays will not meet before the mirror, but will have a virtual focus behind it, the distance of which from D will be given by the rule.

(20.) *Reflexion of converging rays.* Let M N, *fig.* 10., be a concave mirror whose centre of concavity is C, and let rays A M, A D, A N, converging to a point A′ behind the mirror, fall upon the mirror at the points M, D, and N, and suffer reflexion at these points; M and N being near to D. The lines C M, C D, and C N being perpendicular to the mirror at the points M, D, and N, we shall find the reflected rays M F and N F by making the angle F M C equal to A M C, and F N C equal to A N C; and the point F, where these rays meet, will be the *focus* where the converging rays A M, A N are collected. By comparing *fig.* 10. with *fig.* 7. it will be manifest, that, as the incident ray A M in *fig.* 10. is farther from the perpendicular C M than the same ray A M in *fig.* 7., the reflected ray M F in *fig.* 10

Fig. 10.



will also be farther from the perpendicular C M than the same ray in *fig.* 7.; and as the same is true of the reflected ray N F, it follows that the point F must be farther from C in *fig.* 10. than in *fig.* 7.; that is, in the reflexion of converging rays, the conjugate focal distance D F of the mirror is less than its distance for parallel rays.

If we suppose the point of convergence A', *fig.* 10., to approach to D, or the rays A M, A N to become more convergent, then the incident rays A M, A N will recede from the perpendiculars C M, C N; and as the reflected rays M F, N F will also recede from C M, C N, the focus F will likewise approach to D; and when A' reaches D, F will also reach D.

If the rays A M, A N become less convergent, that is, if their point of convergence A' recedes farther from D to the left, the focus F will recede from D to the right; and when A' is infinitely distant, or when A M, A N are parallel, as in *fig.* 7., F will be half-way between D and C.

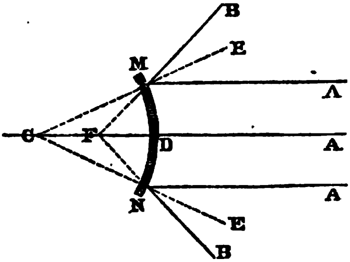
In these cases the place of the focus F will be found by the following rule.

Rule. Multiply the distance of the point of convergence from the mirror by the radius of the mirror, and divide this product by the sum of twice the distance of the radiant point and the radius C D, and the quotient will be the distance of the focus, or F D, the focus F being always in front of the mirror.

Reflexion of Rays from Convex Mirrors.

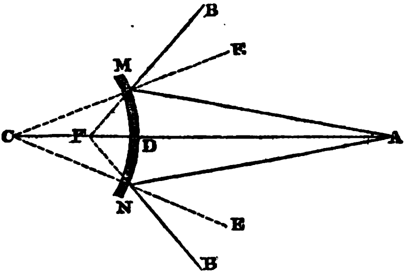
(21.) *Reflexion of parallel rays.* Let M N, *fig.* 11., be a convex mirror whose centre is C, and let A M, A D, A N be parallel rays falling upon it. Continue the lines C M and C N to E, and M E, N E will be perpendicular to the surface of the mirror at the points M and N. The rays A M, A N will therefore be reflected in directions M B, N B, the angles of reflexion E M B, E N B being equal to the angles of incidence E M A, E N A. By continuing the reflected rays B M, B N backwards, they will be found to meet at F, their virtual focus behind the mirror; and the focal distance D F for parallel rays will be almost exactly one half of the radius of convexity C D, provided the points M and N are taken near D.

Fig. 11.



(22.) *Reflexion of diverging rays.* Let M N, *fig.* 12., be a convex mirror, C its centre of convexity, and A M, A N rays diverging from A, which fall upon the mirror at the points M, N. The lines C M E and C N E will be, as before, perpendicular to the mirror at M and N; and consequently, if we make the angles of reflexion E M B, E N B equal to the angles of incidence E M A, E N A, M B, N B will be the reflected rays which, when continued backwards, will meet at

Fig. 12.



F, their virtual focus behind the mirror. By comparing *fig.* 12. with *fig.* 11., it is obvious that the ray A M, *fig.* 12., is farther from M E than in *fig.* 11., and consequently the reflected ray M B must also be farther from it. Hence, as the same is true of the ray N B, the point F, where these rays meet, must be nearer to D in *fig.* 12. than in *fig.* 11.; that is, in the reflexion of diverging rays, the virtual focal distance D F is less than for parallel rays.

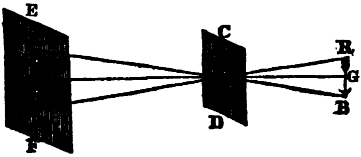
For the same reason, if we suppose the point of divergence A to approach the mirror, the virtual focus F will also approach it; and when A arrives at D, F will also arrive at D. In like manner, if A recedes from the mirror, F will recede from it; and when A is infinitely distant, or when the rays become parallel, as in *fig.* 11., F will be half-way between D and C. In all these cases, the focus is a virtual one behind the mirror.*

CHAP. II.

IMAGES FORMED BY MIRRORS.

(23.) THE image of any object is a picture of it formed either in the air, or in the bottom of the eye, or upon a white ground, such as a sheet of paper. Images are generally formed by mirrors or lenses; though they may be formed also by placing a screen, with a small aperture, between the object and the sheet of paper which is to receive the image. In order to understand this, let C D be a screen or window-shutter with a small aperture, A, and E F a sheet of white paper placed in a dark room. Then, if an illuminated object, R G B, is placed on the outside of the shutter, we shall observe an inverted image of this object painted on the paper at *r g b*. In order to understand how this takes place, let us suppose the object R B to have three distinct colors, *red* at R, *green* at G, and *blue* at B; then it is plain that the *red* light from R will

Fig. 13.



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* For a discussion of the subjects in this chapter, see (in the College Edition) the Appendix of American Editor, Chapter I.

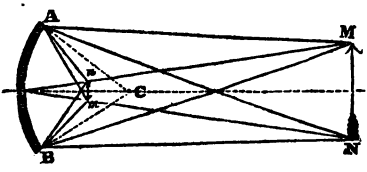
pass in straight lines through the aperture A, and fall upon the paper E F at *r*. In like manner the *green* light from G will fall upon the paper at *g*, and the *blue* light from B will fall upon the paper at *b*; thus painting upon the paper an *inverted* image, *r b*, of the object, R B. As every colored point in the object R B has a colored point corresponding to it, and opposite to it on the paper E F, the image *b r* will be an accurate picture of the object R B, provided the aperture A is very small. But if we increase the aperture, the image will become less distinct; and it will be nearly obliterated when the aperture is large. The reason of this is, that, with a large aperture, two adjacent points of the object will throw their light on the same point of the paper, and thus create confusion in the image.

It is obvious from *fig.* 13., that the size of the image *b r* will increase with the distance of the paper E F behind the hole A. If A *g* is equal to A G, the image will be equal to the object; if A *g* is less than A G, the image will be less than the object; and if A *g* is greater than A G, the image will be greater than the object.

As each point of an object throws out rays in all directions, it is manifest that those only which fall upon the small aperture at A concur in forming the image *b r*; and as the number of these rays is very small, the image *b r* must have very little light, and therefore cannot be used for any optical purposes. This evil is completely remedied in the formation of images by mirrors and lenses.

(24.) *Formation of images by concave mirrors.* Let A B, *fig.* 14., be a concave mirror whose centre is C, and let M N be an object placed at some distance before it. Of all the

Fig. 14.



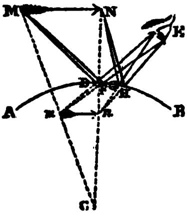
rays emitted in every direction by the point M, the mirror receives only those which lie between M A and M B, or a cone of rays M A B whose base is the spherical mirror, the section of which is A B. If we draw the reflected rays A *m*, B *m*, for all the incident rays M A, M B, by the methods already described, we shall find that they will all meet at the point *m*, and will there paint the extremity M of the object. In like manner, the cone of rays N A B flowing from the other extremity N of the object will be reflected to a focus at *n*, and will there paint that point of the object. For the same reason, cones of rays flowing from intermediate points between M and N will be reflected to intermediate points in the image between *m* and *n*, and *m n* will be an exact inverted picture of the object M N. It will also be very bright, because a great number of rays concur in forming each point of the image. The distance of the image from the mirror is found by the same rule which we have given for finding the focus of diverging rays, the points M, *m* in *fig.* 14. corresponding with A and F in *fig.* 8.

If we measure the relative sizes of the object M N and its image *m n*, we shall find that in every case the size of the image is to the size of the object as the distance of the image from the mirror is to the distance of the object from it.

If the concave mirror A B is large, and if the object M N is very bright, such as a plaster of Paris statue strongly illuminated, the image *m n* will appear suspended in the air; and a series of instructive experiments may be made by varying the distance of the object, and observing the variation in the size and place of the image. When the object is placed at *m n*, a magnified representation of it will be formed at M N.

(25.) *Formation of images by convex mirrors.* In concave mirrors there is, in all cases, a real image of the object formed in front of the mirror, excepting when the object is placed between the principal focus and the mirror, in which case it gives a virtual image formed behind it; whereas in convex mirrors the image is always a virtual one formed behind the mirror.

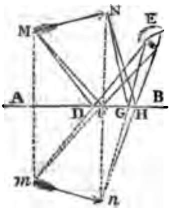
Fig. 15.



Let A B, *fig.* 15., be a convex mirror whose centre is C, and M N an object placed before it; and let the eye of the observer be situated anywhere in front of the mirror, as at E. Out of the great number of rays which are emitted in every direction from the points M, N of the object, and are subsequently reflected from the mirror, a few only can enter the eye at E. Those which do enter the eye, such as D E, F E and G E, H E, will be reflected from the portions D F, G H of the mirror so situated with respect to the eye and the points M, N that the angles of incidence and reflexion will be equal. The ray M D will be reflected in a direction

D E, forming the same angle that M D does with the perpendicular C N, and the ray N G in the direction G E. In like manner, F E, H E will be the reflected rays corresponding to the incident ones M F, N H. Now, if we continue backwards the rays D E, F E, they will meet at *m*; and they will therefore appear to the eye to have come from the point *m* as their focus. For the same reason the rays G E, H E will appear to come from the point *n* as their focus, and *m n* will be the virtual image of the object M N. It is called virtual because it is not formed by the actual union of rays in a focus, and cannot be received upon paper. If the eye E is placed in any other position before the mirror, and if rays are drawn from M and N, which after reflexion enter the eye, it will be found that these rays continued backwards will have their virtual foci at *m* and *n*. Hence, in every position of the eye before the mirror, the image will be seen in the same spot *m n*. If we draw the lines C M, C N from the centre of the mirror, we shall find that the points *m*, *n* are always in these lines. Hence it is obvious that the image *m n* is always *erect*, and less than the object. It will approach to the mirror as the object M N approaches to it, and it will recede from it as M N recedes; and when M N is infinitely distant, and the rays which it emits become parallel, the image *m n* will be half-way between C and the mirror. In other positions of the object the distance of the image will be found by the rule already given for diverging rays falling upon convex mirrors. The size of the image is to the size of the object, as C *m*, the distance of the image from the centre of the mirror, is to C M, the distance of the object. In approaching the mirror, the image and object approach to equality; and when they touch it, they are both of the same size. Hence it follows that objects are always seen diminished in convex mirrors, unless when they actually touch the mirror.

Fig. 16.



(26.) *Formation of images by plane mirrors.* Let A B, *fig.* 16., be a plane mirror or looking-glass, M N an object situated before it, and E the place of the eye; then, upon the very same principles which we have explained for a convex mirror, it will be found that an image of M N will be formed at *m n*, the virtual foci *m*, *n* being determined by continuing back the reflected rays D E, F E till they meet at *m*, and G E, H E till they meet at *n*. If we join the points M, *m* and N, *n*, the lines M *m*, N *n* will be perpendicular to the mirror A B, and consequently parallel; and the image will be at the same distance, and have the same position behind the mirror that the object has before it. Hence we see the reason why the images of all objects seen in a looking-glass have the same form and distance as the objects themselves.*

DIOPTRICS.

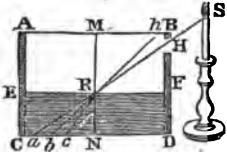
(27.) **Dioptrics** is that branch of optics which treats of the progress of those rays of light which enter transparent bodies and are transmitted through their substance.

CHAP. III.

REFRACTION.

(28.) When light passes through a drop of water or a piece of glass, it obviously suffers some change in its direction, because it does not illuminate a piece of paper placed behind these bodies in the same manner as it did before they were placed in its way. These bodies have therefore exercised some action, or produced some change upon the light, during its progress through them.

Fig. 17.



In order to discover the nature of this change, let A B C D be an empty vessel, having a hole H in one of its sides B D, and let a lighted candle S be placed within a few feet of it, so that a ray of its light S H may fall upon the bottom C D of the vessel, and form a round spot of light at *a*. The beam of light S H R *a* will be a straight line. Having marked the point *a* which the ray from S strikes, pour water into the vessel till it rises to the level E F. As soon as the surface of the water has become smooth, it will be seen that the round spot which was formerly at *a* is now at *b*, and that the ray S H R *b* is bent at R; H R and R *b* being two straight lines meeting at R, a point in the surface of the water. Hence it follows, that all objects seen under water are not seen in their true direction by a person whose eye is not immersed in the water. If a fish, for example, is lying at *b*, *fig.* 17., it will be seen by an eye at S in the direction S *a*, the direction of the refracted ray R S; so that, in order to shoot it with a ball, we must direct the gun to a point nearer us than the point *a*. For the same reason, every point of an object under water appears in a place different from its true place; and the difference between the real and apparent place of any point of an object increases with its depth beneath the surface, and with the obliquity of the ray R S by which it is seen. A straight stick, one half of which is immersed in water, will therefore appear crooked or bent into an angle at the point where it enters the water. A straight rod S R *a*, for example, will appear bent like S R *b*; and a rod bent will, for a like reason, appear straight. This effect must have been often observed in the case of an oar dipping into transparent water.

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* For the formation of images by mirrors, see (in the College edition) the Appendix of Am. ed. chap. ii.

If in place of water we use *alcohol*, *oil*, or *glass*, the surfaces of all these bodies coinciding with the line E F, we shall find that they all have the power of bending the ray of light S R at the point R; the alcohol bending it more than the water, the oil more than the alcohol, and the glass more than the oil. In the case of glass, the ray would be bent into the direction R *c*. The power which thus bends or changes the direction of a ray of light is called *refraction*,—a name derived from a Latin word, signifying *breaking back*,—because the ray S R *a* is broken at R, and the water is said to refract, or break the ray, at R. Hence we may conclude that if a ray of light, passing through air, falls in an oblique or slanting direction on the surface of solid or fluid bodies that are transparent, it will be refracted *towards* a line, M N, perpendicular to the surface E F at the point R, where the ray enters it; and that the quantity of this refraction, or the angle *a* R *b*, varies with the nature of the body. The power by which bodies produce this effect is called their *refractive power*, and bodies that produce it in different degrees are said to have different refractive powers.

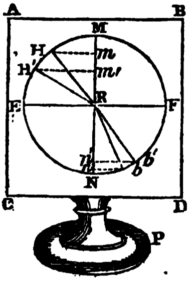
Let the vessel A B C D be now emptied, and let a bright object, such as a sixpence, be cemented on the bottom of it at *a*. If the observer places himself a few feet from the vessel, he will find a position where he will see the sixpence at *a* through the hole H. If water be now poured into the vessel up to E F, the observer will no longer see the sixpence; but if another sixpence is placed at *a*, and is moved towards *b*, it will become visible when it reaches *b*. Now, as the ray from the sixpence at *b* reaches the eye, it must come out of the water at a point, R, in the surface, found by drawing a straight line, S H R, through the eye and the hole H; and consequently *b* R must be the direction of the ray, which makes the sixpence visible, before its refraction at R. But if this ray had moved onwards in a straight line, without being refracted at R, its path would have been *b h*; whereas, in consequence of the refraction, its path is R H. Hence it follows, that when a ray of light, passing through any dense medium, such as water, &c., in a direction oblique or slanting to its surface, quits the medium at any point, and enters a rarer medium, such as air, it is refracted from the line perpendicular to the surface at the point where it quits it.

When the ray S H R from the candle falls, or is incident upon the surface E F of the water, and is refracted in the direction R *b*, towards the perpendicular M N, the angle M R H which it makes with the perpendicular, is called *the angle of incidence;* and the angle N R *b*, which the ray R *b* bent or refracted at R makes with the same perpendicular, is called *the angle of refraction.* The ray H R is called the *incident ray*, and R *b* the *refracted ray.* But when the light comes out of the water from the sixpence at *b*, and is refracted at R in the direction R H, *b* R is the incident ray and R H the refracted ray. The angle N R *b* is the angle of incidence, and M R H the angle of refraction.

Hence it follows, that *when light passes out of a rarer into a denser medium, as from air to water, the angle of incidence is greater than the angle of refraction;* and when light passes *out of a denser into a rarer medium, as out of water into air, the angle of incidence is less than the angle of refraction:* and these angles are so related to one another, that when the ray which was refracted in the one case becomes the incident ray, what was formerly the incident ray becomes the refracted ray.

(29.) In order to discover the law, or rule, according to which the rays of light enter or quit water, or other refracting media, so that we may be able to determine the refracted ray when we know the direction of the incident ray, describe a circle M N upon a square board A B C D, *fig.* 18. standing upon a heavy pedestal P, and draw the two diameters M N, E F perpendicular to one another, and also to the sides, A B, A C of the piece of wood. Let a small tube, H R, be so made that it may be attached to the board along any radius H R, H′ R, or, what would be still better, that it may move freely round R as a centre. Let the board with its pedestal be

Fig. 18.



placed in a pool or tub of water, or in a glass vessel of water, so that the surface of the water may coincide with the line E F without touching the end R of the tube H R. When the tube is in the position M R, perpendicular to the surface E F of the water, admit a ray of light down the tube, and it will be seen that it enters the water at R, and passes straight on to N, without suffering any change in its direction. Hence it follows, *that a ray of light incident perpendicularly on a refracting surface experiences no refraction or change in its direction.* If we now place a sixpence at N, we shall see it through the tube M R; so that the rays from the sixpence quit the water at R, and proceed in the same straight line N R M. Hence *a ray of light quitting a refracting surface perpendicularly undergoes no refraction or change of direction.* If we now bring the tube into the position H R, and make a ray of light pass along it, the ray will be refracted at R in some direction R *b*, the angle of refraction N R *b* being less than the angle of incidence M R H. If we now with a pair of compasses, take the shortest distance *b n* of the point *b* from the perpendicular M N, and make a scale of equal parts, of which *b n* is one part, the scale being divided into tenths and hundredths, and if we set the distance H *m* upon this scale, we shall find it to be 1·336 of these parts, or 1⅓ nearly. If this experiment is repeated at any other position, H′ R, of the tube where R *b′* is the refracted ray, we shall find that on a new scale, in which *b′ n′* is one part, H′ *m′* will also be 1·336 parts. But the lines H *m*, H′ *m′* are called the *sines of the angles of incidence* H R M, H′ R M, and *b n*, *b′ n′* the *sines of the angles of refraction* *b* R N, *b′* R N. Hence it follows, that in water the sine of the angle of incidence is to the sine of the angle of refraction as 1·336 to 1, whatever be the position of the ray with respect to the surface E F of the water. This truth is called by optical writers *the constant ratio of the sines.* By placing a sixpence at *b*, we shall find that it will be seen through the tube when it has the position H R; and placing it at *b′*, it will be seen through it in the position H′ R. Hence, when light quits the surface of water, the sine of its angle of incidence *b* R N will be to the sine of its angle of refraction H R M as 1 to 1·366, as these are the measures of the sines *b n*, H *m*; and since these are also the measures of *b′ n′* H′ *m′* upon another scale, in which *b′ n′* is unity, we may conclude that, when light emerges from water into air, the sines of the angles of incidence and refraction are in the constant ratio of 1 to 1·336.

If we make the same experiment with other bodies, we shall obtain different degrees of refraction at the same angles;

but in every case the sines of the angles of incidence and refraction will be found to have a constant ratio to each other.

The number 1·336, which expresses this ratio for *water*, is called the *index of refraction* for water, and sometimes its *refractive power*.

(30.) As philosophers have determined the index of refraction for a great variety of bodies, we are able, from those determinations, to ascertain the direction of any ray when refracted at any angle of incidence from the surface of a given body, either in entering or quitting it. Thus, in the case of water, let it be required to find the direction of a ray, H R, *fig.* 18., after it is refracted at the surface E F of water: draw R M perpendicular to E F at the point R, where the ray H R enters the water, and from H draw H *m* perpendicular to M R. Take H *m* in the compasses, and make a scale in which this distance occupies 1·336 parts, or $1\frac{1}{3}$ nearly. Then, taking 1 on the same scale, place one foot of the compasses in the quadrant N F, and move that foot towards or from N till the other foot falls upon some one point *n* in the perpendicular R N, and in no other point of it. Let *b* be the point on which the first foot of the compasses is placed when the second falls upon *n*, then the line R *b* passing through this point will be the refracted ray corresponding to the incident ray H R.

(31.) Table I. (Appendix) contains the index of refraction for some of the substances most interesting in optics.

(32.) As the bodies contained in these tables have all different densities, the indices of refraction annexed to their names cannot be considered as showing the relation of their absolute refractive powers, or the refractive powers of their ultimate particles. The small refractive index of hydrogen, for example, arises from its particles being at so great a distance from one another; and, if we take its specific gravity into account, we shall find that, instead of having a less refractive power than all other bodies, its ultimate particles exceed all other bodies in their absolute action upon light.

Sir Isaac Newton has shown, upon the supposition that the ultimate particles of bodies are equally heavy, and that the law of the forces which different media exert is of the same *form* in all, that the absolute refractive power is equal to the excess of the square of the index of refraction above unity, divided by the specific gravity of the body.

In this way Table II. (Appendix) has been calculated.

Mr. Herschel has justly remarked, that if, according to the doctrines of modern chemistry, material bodies consist of a finite number of atoms, differing in their actual weight for every differently compounded substance, the intrinsic refractive power of the atoms of any given medium will be the product arising from multiplying the number for the medium, in Table II. by the weight of its atom.

(33.) In examining Table II., it appears that the substances which contain fluoric acid have the least absolute refractive power, while all inflammable bodies have the greatest. The high absolute refractive power of oil of cassia, which is placed above all other fluids, and even above *diamond*, indicates the great inflammability of its ingredients.†

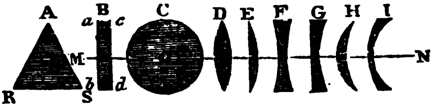
CHAP. IV.*

REFRACTION THROUGH PRISMS AND LENSES.

(34). By means of the law of refraction explained in the preceding pages, we are enabled to trace a ray of light in its passage through any medium or body of any figure, or through any number of bodies, provided we can always find the inclination of the incident ray to that small portion of the surface where the ray either enters or quits the body.

The bodies generally used in optical experiments, and in the construction of optical instruments, where the effect is produced by refraction, are *prisms*, *plane glasses*, *spheres*, and *lenses*, a section of each of which is shown in the annexed figure.

Fig. 19.



1. The most common *optical prism*, shown at A, is a solid having two plane surfaces A R, A S, which are called its *refracting surfaces*. The face R S, equally inclined to A R and A S, is called the *base* of the prism.

2. A *plane glass*, shown at B, is a plate of glass with two plane surfaces, *a b*, *c d*, parallel to each other.

3. A *spherical lens*, shown at C, is a sphere, all the points in its surface being equally distant from the centre O.

4. A *double convex lens*, shown at D, is a solid formed by *two* convex spherical surfaces, having their centres on opposite sides of the lens. When the radii of its two surfaces are equal, it is said to be *equally convex;* and when the radii are unequal, it is said to be an *unequally convex lens.*

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* For the subjects treated in this and in the preceding chapter, see (in the College edition) the Appendix of Am. ed. chap. iii.

† See Note No. I. at the close of author's Appendix.

5. A *plano-convex lens*, shown at E, is a lens having one of its surfaces *convex* and the other *plane*.

6. A *double concave lens*, shown at F, is a solid bounded by *two concave* spherical surfaces, and may be either equally or unequally concave.

7. A *plano-concave lens*, represented at G, is a lens one of whose surfaces is *concave* and the other *plane*.

8. A *meniscus*, shown at H, is a lens one of whose surfaces is *convex* and the other *concave*, and in which the two surfaces meet if continued. As the convexity *exceeds* the concavity, it may be regarded as a convex lens.

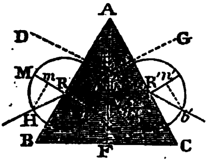
9. A *concavo-convex lens*, shown at I, is a lens one of whose surfaces is *concave* and the other *convex*, and in which the two surfaces will not meet though continued. As the concavity exceeds the convexity, it may be regarded as a concave lens.

In all these lenses a line, M N, passing through the centres of their curved surfaces, and perpendicular to their plane surfaces, is called the *axis*. The figures represent only the sections of the lenses, as if they were cut by a plane passing through their axis; but the reader will understand that the convex surface of a lens is like the outside of a watch-glass, and the concave surface like the inside of a watch-glass.

In showing the progress of light through such lenses, and in explaining their properties, we shall still use the sections shown in the above figure; for since every section of the same lens passing through its axis has exactly the same form, what is true of the rays passing through one section must be true of the rays passing through every section, and consequently through the whole surface.

(35.) *Refraction of light through prisms.* As prisms are introduced into several optical instruments, and are essential parts of the apparatus used for decomposing light and examining the properties of its component parts, it is necessary that the reader should be able to trace the progress of light through their two refracting surfaces. Let A B C be a prism of plate glass whose index of refraction is 1·500, and let H R be a ray of light falling obliquely upon its first surface A B at the point R. Round R as a centre, and with any radius H R, describe the circle H M *b*. Through R draw M R N perpendicular to A B, and H *m* perpendicular to M R. The angle H R M will be the angle of incidence of the ray H R, and H *m* its sine, which in the present

Fig. 20.



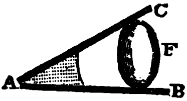
case is 1·500. Then having made a scale in which the distance H *m* is 1·500, or 1½ parts, take 1 part or unity from the same scale, and having set one foot of the compasses on the circle somewhere about *b*, move it to different points of the circle till the other foot strikes only one point *n* of the line R N; the point *b* thus found will be that through which the refracted ray passes, R *b* will be the refracted ray, and *n* R *b* the angle of refraction, because the sine *b n* of this angle has been made such that its ratio to H *m*, the sine of the angle of incidence, is as 1 to 1·500. The ray R *b* thus refracted will go on in a straight line till it meets the second surface of the prism at R′, where it will again suffer refraction in the direction R′ *b*′. In order to determine this direction, make R′ H′ equal to R H, and, with this distance as radius, describe the circle H′ *b*′. Draw R′ N perpendicular to A C, and H′ *m*′ perpendicular to R′ N, and form a scale on which H′ *m*′ shall be 1 part, or 1·000, and divide it into tenths and hundredths. From this scale take in the compasses the index of refraction 1·500, or 1½ of these parts; and having set one foot somewhere in the line R′ *n*′, move it to different parts of it till the other foot falls upon some part of the circle about *b*′, taking care that the point *b*′ is such, that when one foot of the compasses is placed there, the other foot will touch the line R′ *n*′, continued, only in one place. Join R′ *b*′. Then, since H′ R′ *m*′ is the angle of incidence on the second surface A C, and H′ *m*′ its sine, and since *n*′ *b*′, the sine of the angle *b*′ R′ *n*′, has been made to have to H′ *m*′ the ratio of 1·500 to 1, *b*′ R′ *n*′ will be the angle of refraction, and R′ *b*′ the refracted ray.

If we suppose the original ray H R to proceed from a candle, and if we place our eye at *b*′ behind the prism so as to receive the refracted ray *b*′ R′, it will appear as if it came in the direction D R′ *b*′, and the candle will be seen in that direction; the angle H E D representing its angular change of direction, or the *angle of deviation*, as it is called.

In the construction of *fig.* 20., the ray H R has been made to fall upon the prism at such an angle that the refracted ray R R′ is equally inclined to the faces A B, A C, or is parallel to the base B C of the prism; and it will be found that the angle of incidence on the face of the prism, H R B is equal to the angle of emergence *b*′ R′ C. Under these circumstances we shall find, by making the angle H R B either greater or less than it is in the figure, that the angle of deviation H E D is less than at any other angle of incidence. If we, therefore, place the eye behind the prism at *b*′, and turn the prism round in the plane B A C, sometimes bringing A towards the eye and sometimes pushing it from it, we shall easily discover the position where the image of the candle seen in the direction b' D has the least deviation. When this position is found, the angles H R B and b' R$'$ C are equal, and R R$'$ is parallel to B C, and perpendicular to A F, a line bisecting the refracting angle B A C of the prism. Hence it may be shown by the similarity of triangles, or proved by projection, that the angle of refraction b R n at the first surface is equal to B A F, half the refracting angle of the prism. But since B A F is known, the angle of refraction b R n is also known; and the angle of incidence H R M being found by the preceding methods, we may determine the index of refraction for any prism by the following analogy. As the sine of the angle of refraction is to the sine of the angle of incidence, so is unity to the index of refraction; or the index of refraction is equal to the sine of the angle of incidence divided by the sine of the angle of refraction.

(36.) By this method, which is very simple in practice, we may readily measure the refractive powers of all bodies. If the body be solid, it must be shaped into a prism; and if it is soft or fluid, it must be placed in the angle B A C of a hollow prism A B C, *fig.* 21., made by cementing together three pieces of plate glass, A B, A C, B C. A very simple hollow prism for this purpose may be made by fastening together at any angle two pieces of plate glass, A B, A C, with a bit of wax, F. A drop of the fluid may then be placed in the angle at A, where it will be retained by the force of capillary attraction.

Fig. 21.

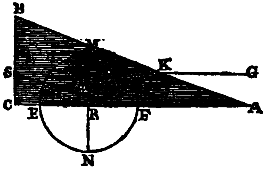


When light is incident upon the second surface of a prism, it may fall so obliquely that the surface is incapable of refracting it, and therefore the incident light is *totally reflected* from the second surface. As this is a curious property of light, we must explain it at some length.

On the total Reflexion of Light.

(37.) We have already stated, that when light falls upon the first or second surfaces of transparent bodies, a certain portion of it is reflected, and another and much greater portion transmitted. The light is in this case said to be *partially reflected.* When the light, however, falls very obliquely upon the *second* surface of a transparent body, it is wholly reflected, and not a single ray suffers refraction, or is transmitted by the surface. Let A B C be a prism of glass, whose index of refraction is 1·500: let a ray of light G K, *fig.* 22., be refracted

Fig. 22.

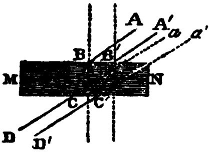


at K by the first surface A B, so as to fall on the point R of the second surface very obliquely, and in the direction K R. Upon R as a centre, and with any radius, R H, describe the circle H M E N F; then, in order to find the refracted ray corresponding to H R, make a scale on which H *m* is equal to 1, and take in the compasses 1·500 or $1\frac{1}{2}$ from that scale, and setting one foot in the quadrant E N, try to find some point in it, so that the other foot may fall only in one point of the radius R N. It will soon be seen that there is no such point, and that 1·500 is greater even than E R, the sine of an angle E R N of 90°. If the distance 1·500 in the compasses had been less than E R, the ray would have been refracted at R; but as there is no angle of refraction whose sine is 1·500, the ray does not emerge from the prism, but suffers total reflexion at R in the direction R S, so that the angle of reflexion M R S is equal to the angle of incidence M R H. If we construct *fig.* 22. so as to make the incident ray H R take different positions between M R and F R, we shall find that the refracted ray will take different positions between R N and R E. There will be some position of the incident ray about H R, where the refracted ray will just coincide with R E; and that will happen when the quantity 1·500, taken from the scale on which H *m* is equal to 1, is exactly equal to R E, or radius. At all positions of the incident ray between this line and F R, refraction will be impossible, and the ray incident at R will be totally reflected. It will also be found that the sine of the angle of incidence at R, at which the light begins to be totally reflected, is equal to $\frac{1}{1\cdot500}$, or ·666, or $\frac{2}{3}$, which is the sine of 41° 48′, the angle of total reflexion for plate glass.

The passage from partial to total reflexion may be finely seen, by exposing one side, A C, of a prism A B C, *fig.* 20., to the light of the sky, or at night to the light reflected from a large sheet of white paper. When the eye is placed behind the other side, A B, of the prism, and looks at the image of the sky, or the paper, as reflected from the base, B C, of the prism, it will see when the angle of incidence upon B C is less than 41° 48′, the faint light produced by partial reflexion; but by turning the prism round, so as to render the incidence gradually more oblique, it will see the faint light pass suddenly into a bright light, and separated from the faint light by a colored fringe, which marks the boundary of the two reflexions at an angle of 41° 48′. But, at all angles of incidence above this, the light will suffer total reflexion.

Refraction of Light through Plane Glasses.

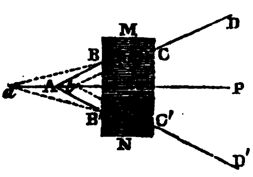
Fig. 23.



(38.) Let M N, *fig.* 23., be the section of a plane glass with parallel faces; and let a ray of light, A B, fall upon the first surface at B, and be refracted into the direction B C: it will again be refracted at its emergence from the second surface at C, in a direction, C D, parallel to A B; and to an eye at D it will appear to have proceeded in a direction *a* C, which will be found by continuing D C backwards. It will thus appear to come from a point *a* below A, the point from which it was really emitted. This may be proved by projecting the figure by the method already described; though it will be obvious also from the consideration, that if we suppose the refracted ray to become the incident ray, and to move backwards, the incident ray will become the refracted ray. Thus the refracted ray B C, falling at equal angles upon the two surfaces of the plane glass, will suffer equal refractions at B and C, if we suppose it to move in opposite directions; and consequently the angles which the refracted rays B A, C D form with the two refracting surfaces will be equal, and the rays parallel.

If we suppose another ray, A′ B′, parallel to A B, to fall upon the point B′, it will suffer the same refraction at B′ and C′, and will emerge in the direction C′ D′, parallel to C D, as if it came from a point *a*′. Hence *parallel rays falling upon a plane glass will retain their parallelism after passing through it.*

Fig. 24.



(39). If rays diverging from any point, A, *fig.* 24., such as A B, A B′, are incident upon a plane glass, M N, they will be refracted into the directions B C, B′ C′ by the first surface, and C D, C′ D′ by the second. By continuing C B, C′ B′ backwards, they will be found to meet at *a*, a point farther from the glass than A. Hence, if we suppose the surface B B′ to be that of standing water, placed horizontally, an eye within it would see the point A removed to *a*, the divergency of the rays B C, B′ C′ having been diminished by refraction at the surface B B′. But when the rays B C, B′ C′ suffer a second refraction, as in the case of a plane glass, we shall find, by continuing D C, D′ C′ backwards, that they will meet at *b*, and the object at A will seem to be brought nearer to the glass; the rays C D, C′ D′, by which it is seen, having been rendered more divergent by the two refractions. A plane glass, therefore, diminishes the distance of the divergent point of diverging rays.

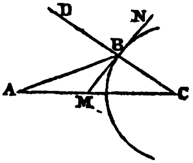
If we suppose D C, D′ C′ to be rays converging to *b*, they will be made to converge to A by the refraction of the two surfaces; and consequently a plane glass causes to recede from it the convergent point of converging rays.

If the two surfaces B B′, C C′ are equally curved, the one being convex and the other concave, like a watch-glass, they will act upon light nearly like a plane glass; and accurately like a plane glass, if the convex and concave sides are so related that the rays B A, C D are incident at equal angles on each surface: but this is not the case when the surfaces have the same centre, unless when the radiant point A is in their common centre. For these reasons, glasses with parallel surfaces are used in windows and for watch-glasses, as they produce very little change upon the form and position of objects seen through them.

Refraction of Light through Curved Surfaces.

(40). When we consider the inconceivable minuteness of the particles of light, and that a single ray consists of a succession of those particles, it is obvious that the small part of any curved surface on which it falls, and which is concerned in refracting it, may be regarded as a plane. The surface of a lake, perfectly still, is known to be a curved surface of the same radius as that of the earth, or about 4000 miles; but a square yard of it, in which it is impossible to discover any curvature, is larger in proportion to the radius of the earth than the small space on the surface of a lens occupied by a ray of light is in relation to the radius of that surface. Now, mathematicians have demonstrated that a line touching a curve at any point may be safely regarded as coinciding with an infinitely small part of the curve; so that when a ray of light, A B, *fig.* 25., falls upon a curved refracting surface at B, its

Fig. 25.

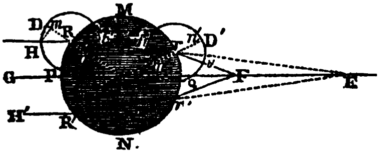


angle of incidence must be considered as A B D, the angle which the ray A B forms with a line D C, perpendicular to a line M N, which touches, or is a tangent to, the curved surface at B. In all spherical surfaces, such as those of lenses, the tangent M N is perpendicular to the radius C B of the surface. Hence, in spherical surfaces the consideration of the tangent M N is unnecessary; because the radius C D, drawn through the point of incidence B, is the perpendicular from which the angle of incidence is to be reckoned.

Refraction of Light through Spheres.

(41.) Let M N be the section of a sphere of glass whose centre is C, and whose index of refraction is 1·500; and let parallel rays, *fig.* 26., H R, H′ R′, fall upon it at equal distances on each side of the axis G C F. If the ray H R is incident at R, describe the circle H D *b* round R; through C and R draw the line C R D, which will be perpendicular to the surface at R, and draw H *m* perpendicular to R D. Draw the ray R *b r* through a point *b* found by the method already explained, and so that the sine *b n* of the angle of refraction *b* R C may be 1 on the same scale on which H *m* is 1·500, or 1½; then R *b* will be the ray as refracted by the first surface of the sphere. In like manner draw R′ *r*′ for the refracted ray corresponding to H′ R′.

Fig. 26.



If we continue the rays R *r*, R′ *r*′, they will meet the axis at E, which will be the focus of parallel rays for a single convex surface R P R′; and the focal distance P E may be found by the following rule.

Rule *for finding the principal focus of a single convex surface.* Divide the index of refraction by its excess above unity, and the quotient will be the principal focal distance, P E; the radius of the surface, or C R, being 1. If C R is given in inches, then we have to multiply the result by that number of inches. When the surface is that of glass, of which the index of refraction is 1·5, then the focal distance, P E, will always be equal to thrice the radius, C R.

Round *r* as a centre, with a radius equal to R H, describe the circle D′ *b′ h*, and, by the method formerly explained, find a point *b′* in the circle, such that *b′ n′*, the sine of the angle of refraction *b′ r n′*, is 1·500 or 1½ on the same scale on which *h m′*, the sine of the angle of incidence, is 1 part, and *r b′* F will be the ray refracted at the second surface. In the same manner we shall find *r′* F to be the refracted ray corresponding to the incident ray R′ *r′*, F being the point where *r b′* cuts the axis G E. Hence the point F will be the *focus of parallel rays* for the sphere of glass M N.

If *diverging rays* fall upon the points R, R′, it is quite clear, from the inspection of the figure, that their focus will be on some point of the axis G F more remote from the sphere than F, the distance of their focus increasing as the radiant point from which they diverge approaches to the sphere. When the radiant point is as far before the sphere as F is behind it, then the rays will be refracted into parallel directions, and the focus will be infinitely distant. Thus, if we suppose the rays F *r*, F *r′* to diverge from F, then they will emerge after refraction in the parallel directions R H, R′ H′.

If *converging rays* fall upon the points R R′, it is equally manifest that their focus will be at some point of the axis, G F, nearer the sphere than its principal focus F; and their convergency may be so great that their focus will fall within the sphere. All these truths may be rendered more obvious, and would be more deeply impressed upon the mind, by tracing rays of different degrees of divergency and convergency through the sphere, by the methods already so fully explained.

(42.) In order to form an idea of the effect of a sphere made of substances of different refractive powers, in bringing parallel rays to a focus, let us suppose the sphere to have a radius of one inch, and let the focus F be determined as in *fig.* 26., when the substances are,

	Index of Refraction.	Distance, F Q, of the Focus from the Sphere.
Tabasheer	1·11145	4 inches.
Water	1·3358	1 —
Glass	1·500	½ —
Zircon	2·000	0 —

Hence we find that in tabasheer the distance F Q is 4 inches; in water, 1 inch; in glass, half an inch; and in zircon, nothing; that is, *r* and F coincide with Q, after a single refraction at R.

When the index of refraction is greater than 2·000, as in diamond and several other substances, the ray of light R *r* will cross the axis at a point somewhere between C and Q. Under certain circumstances the ray R *r* will suffer total reflexion at *r*, towards another part of the sphere, where it will again suffer total reflexion, being carried round the circumference of the sphere, without the power of making its escape, till the ray is lost by absorption. Now, as this is true of every possible section of the sphere, every such ray, R *r*, incident upon it in a circle equidistant from the axis, G F, will suffer similar reflexions.

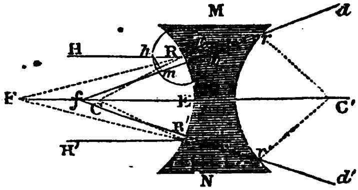
Rule *for finding the focus* F *of a sphere.* The distance of the focus, F, from the centre, C, of any sphere may be thus found. Divide the index of refraction by twice its excess above 1, and the quotient is the distance, C F, in radii of the sphere. If the radius of the sphere is 1 inch, and its refractive power 1·500, we shall have C F equal to 1½ inches, and Q F equal to half an inch.

Refraction of Light through Convex and Concave Surfaces.

(43.) The method of tracing the progress of a ray which enters a convex surface, is shown in *fig.* 26. for the ray H R, and of tracing one entering a concave surface of a rare medium, or quitting a convex surface of a dense one, is shown for the ray R *r*, in the same figure.

When the ray enters the concave surface of a dense medium, or quits a similar surface, and enters the convex surface of a rare medium, the method of tracing its progress is shown in *fig.* 27., where M N is a dense medium (suppose glass) with two concave surfaces, or a thick concave lens. Let C, C be the centres of the two surfaces lying in the axis C C′, and H R, H′ R′ parallel rays incident on the first surface. As C R is perpendicular to the surface at R, H R C will be the angle of incidence; and if a circle is described with a radius

Fig. 27.



R h, $h\,m$ will be the sine of that angle. From a scale on which $h\,m$ is 1·500, take in the compasses 1, and find some point, b, in the circle where, when one foot of the compasses is placed, the other will fall only on one point, n, of the perpendicular C R: the line R b drawn through this point will be the refracted ray. By continuing this ray b R backwards, it will be found that it meets the axis at F. In like manner it will be seen that the ray H′ R′ will be refracted in the direction R′ r', as if it also diverged from F. Hence F will be the virtual focus of parallel rays refracted by a single concave surface, and may be found by the following rule.

Rule *for finding the principal focus of a single concave surface.* Divide the index of refraction by its excess above unity, and the quotient will be the principal focal distance F E, the radius of the surface, or C E, being 1. If the radius C E is given in inches, we have only to multiply E F, thus obtained by that number of inches, to have the value of F E in inches.

If, by a similar method, we find the refracted ray $r\,d$ at the emergence of the ray $r\,b$ from the second surface $r\,r'$ of the lens, and continue it backwards, it will be found to meet the axis at f; so that the divergent rays R r, R′ r' are rendered still more divergent by the second surface, and f will be the focus of the lens M N.

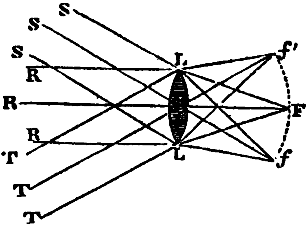
Refraction of Light through Convex Lenses.

(44). *Parallel rays.* Rays of light falling upon a convex lens parallel to its axis are refracted in precisely the same manner as those which fall upon a sphere; and the refracted ray may be found by the very same methods. But as a sphere has an axis in every possible direction, every incident ray must be parallel to an axis of it; whereas, in a lens which has only one axis, many of the incident rays must be oblique to that axis. In every case, whether of spheres or lenses, all the rays that pass along the axis suffer no refraction, because the axis is always perpendicular to the refracting surface.

When parallel rays, R L, R C, R L, *fig.* 28., fall upon a double convex lens, L L, parallel to its axis R F, the ray R C which coincides with the axis will pass through without suffering any refraction, but the other rays, R L, R L, will be refracted at each of the surfaces of the lens; and the refracted rays corresponding to them, viz. L F, L F, will be found, by the method already given, to meet at some point, F, in the axis.

When the rays are oblique to the axis, as S L, S L, T L,

Fig. 28.



T L, the rays S C, T C, which pass through the centre, C, of the lens, will suffer refraction at each surface; but as the two refractions are equal, and in opposite directions, the finally refracted rays C *f*, C *f'* will be parallel to S C, T C. Hence, in considering oblique rays, such as S L, T L, we may regard lines S *f*, T *f'* passing through the centre, C, of the lens as the directions of the refracted rays corresponding to S C, T C. By the methods already explained, it will be found that S L, S L will be refracted to a common point, *f*, in the direction of the central ray S *f*, and T L, T L, to the point *f'*. The focal distance F C, or *f* C, may be found numerically by the following rule, when the thickness of the lens is so small that it may be neglected.

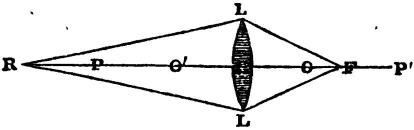
Rule *for finding the principal focus, or the focus of parallel rays, for a glass lens unequally convex.* Multiply the radius of the one surface by the radius of the other, and divide twice this product by the sum of the same radii.

When the lens is of glass and equally convex, the focal distance will be equal to the radius.

Rule *for the principal focus of a plano-convex lens of glass.* With either side of the lens turned to parallel rays, the distance of the focus, when the lens is thin, will be equal to twice the radius of the convex surface.

(45.) *Diverging rays.* When diverging rays, R L, R L, *fig.* 29., radiating from the point R, fall upon the double convex lens L L, whose principal focus is at O and O′, their focus will be at some point F more remote than O. If R approaches to L L, the focus F will recede from L L. When R comes to P, so that P C is equal to twice the principal focal distance

Fig. 29.



C O, the focus F will be at P′ as far behind the lens as the radiant point P is before it. When R comes to O′, the focus F will be infinitely distant, or the rays L F, L F will be parallel; and when R is between O′ and C, the refracted rays will diverge and have a virtual focus before the lens. The focus F of a glass lens, when the thickness is small, will be found by the following rule.

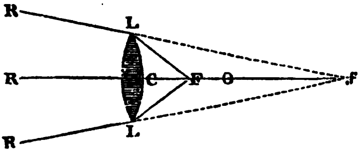
Rule *for finding the focus of a convex lens for diverging rays.* Multiply twice the product of the radii of the two surfaces of the lens by the distance, R C, of the radiant point, for a dividend. Multiply the sum of the two radii by the same distance R C, and from this product subtract twice the product of the radii, for a divisor. Divide the above dividend by the divisor, and the quotient will be the focal distance, C F, required.

If the lens is *equally convex*, the rule will be this. Multiply the distance of the radiant point, or R C, by the radius of the surfaces, and divide that product by the difference between the same distance and the radius, and the quotient will be the focal length, C F, required.

When the lens is *plano-convex*, divide twice the product of the distance of the radiant point multiplied by the radius by the difference between that distance and twice the radius, and the quotient will be the distance of the focus from the centre of the lens.

(46.) For *converging rays.* When rays, R L, R L, converging to a point *f*, *fig.* 30., fall upon a convex lens L L, they

Fig. 30.



will be so refracted as to converge to a point or focus F nearer the lens than its principal focus O. As the point of convergence *f* recedes from the lens, the point F will also recede from it towards O, which it just reaches when the point *f* becomes infinitely distant. When *f* approaches the lens, F also approaches it. The focus F of a glass lens may be found when the thickness is small, by the following rule:—

Rule *for finding the focus of converging rays.* Multiply twice the product of the radii of the two surfaces of the lens by the distance *f* C of the point of convergence, for a dividend. Multiply the sum of the two radii by the same distance *f* C, and to this product add twice the product of the radii, for a divisor. Divide the above dividend by the divisor, and the quotient will be the focal distance C F required.

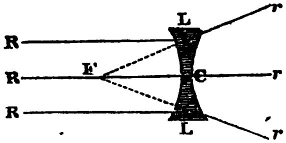
If the lens is *equally convex,* multiply the distance *f* C by the radius of the surface, and divide that product by the sum of the same distance and the radius, and the quotient will be the focal length F C required.

When the lens is *plano-convex,* divide twice the product of the distance *f* C multiplied by the radius by the sum of that distance and twice the radius, and the quotient will be the focal distance F C required.

Refraction of Light through Concave Lenses.

(47.) *Parallel rays.* Let L L be a double concave lens, and

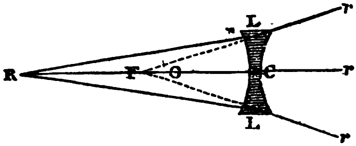
g. 31.



R L, R L parallel rays incident upon it; these rays will diverge after refraction in the directions L *r*, L *r*, as if they radiated from a point F, which is the virtual focus of the lens. The rule for finding F C is the same as for a convex lens.

(48.) *Diverging rays.* When the lens L L receives the

Fig. 32.



rays R L, R L diverging from R, they will be refracted into lines, L *r*, L *r*, diverging from a focus F, more remote from the lens than the principal focus O, and the focal distance, F C, will be found by the following rule:—

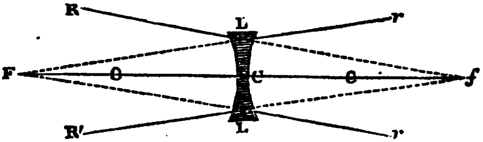
RULE *for finding the focus of a concave lens of glass, for diverging rays.* Multiply twice the product of the radii by the distance, R C, of the radiant point for a dividend. Multiply the sum of the radii by the distance R C, and add to this twice the product of the radii, for a divisor. Divide the dividend by the divisor, and the quotient will be the focal distance.

If the lens is equally concave, the rule will be this. Multiply the distance of the radiant point by the radius, and divide the product by the sum of the same distance and the radius, and the quotient will be the focal distance.

When the lens is plano-concave, multiply twice the radius by the distance of the radiant point, and divide this product by the sum of the same distance and twice the radius; the quotient will be the focal distance.

(49). *Converging rays.* When rays, R L, R L, *fig.* 33.,

Fig. 33.



converging to a point *f*, fall upon a concave lens, L L, they will be refracted so as to have their virtual focus at F, and the distance F C will be found by the rule given for convex lenses. The rule for finding the focus of converging rays is exactly the same as that for diverging rays in a double convex lens.

When the lens is plano-concave, the rule for finding the focus of converging rays is the same as for diverging rays on a plano-convex lens.

Refraction of Light through Meniscuses and Concavo-convex Lenses of Glass.

(50.) The general effect of a *meniscus* in refracting parallel, diverging, and converging rays, is the same as that of a *convex* lens of the same focal length; and the general effect of a *concavo-convex* lens is the same as that of a concave lens of the same focal length.

RULE *for a meniscus with parallel rays.* Divide twice the product of the two radii by their difference, and the quotient will be the focal distance required.

Rule *for a meniscus with diverging rays.* Multiply twice the distance of the radiant point by the product of the two radii for a dividend. Multiply the difference between the two radii by the same distance of the radiant point, and from this product take twice the product of the radii for a divisor. Divide the above dividend by this divisor, and the quotient will be the focal distance required.

The truth of the preceding rules and observations is capable of being demonstrated mathematically; but the reader who has not studied mathematics may obtain an ocular demonstration of them, by projecting the rays and lenses in large diagrams, and determining the course of the rays after refraction by the methods already described. We would recommend to him also to submit the rules and observations to the test of direct experiment with the lenses themselves.

CHAP. V.

ON THE FORMATION OF IMAGES BY LENSES, AND ON THEIR MAGNIFYING POWER.*

(51.) We have already described, in Chapter II., the principle of the formation of images by small apertures, and by the convergency of rays to foci by reflexion from mirrors. Images are formed, by refraction, by lenses in the very same manner as they are formed, by reflexion, in mirrors; and it is a universal rule, that when an image is formed by a convex lens, it is inverted in position relatively to the position of the object, and its magnitude is to that of the object as its distance from the lens is to the distance of the object from the lens.

If M N is an object placed before a convex lens, L L, *fig.* 34., every point of it will send forth rays in every direction. Those rays which fall upon the lens L L will be refracted to foci behind the lens, and at such distances from it as may be determined by the Rules in the last chapter. Since the focus where any point of the object is represented in its image is in the straight line drawn from that point of the object through the middle point C of the lens, the upper end M of the object will be represented somewhere in the line M C *m*, and the lower end N somewhere in the line N C *n*, that is, at the points *m*, *n*, where the rays L *m*, L *m*, L *n*, L *n* cross the lines M C *m*, N C *n*. Hence *m* will represent the upper, and *n* the lower end of the object M N. It is also evident, that in the two triangles M C N, *m* C *n*, *m n*, the length of the image must be to M N the length of the object as C *m*, the distance of the image, is to C M, the distance of the object from the lens.

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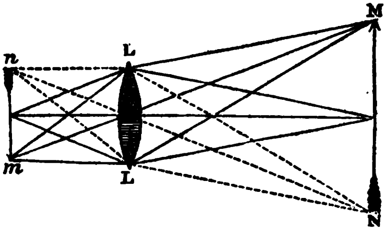
* See, in the College edition, Appendix of Am. ed. chap. iv.

Fig. 34.



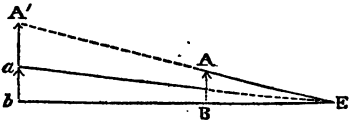
We are enabled, therefore, by a lens, to form an image of an object at any distance behind the lens we please, greater than its principal focus, and to make this image as large as we please, and in any proportion to the object. In order to have the image large, we must bring the object near the lens, and in order to have it small, we must remove the object from the lens; and these effects we can vary still farther, by using lenses of different focal lengths or distances.

When the lenses have the same focus, we may increase the brightness of the image by increasing the size of the lens or the area of its surface. If a lens has an area of 12 square inches, it will obviously intercept twice as many rays proceeding from every point of the object as if its area were only 6 square inches; so that, when it is out of our power to increase the brightness of the object by illuminating it, we may always increase the brightness of the image by using a larger lens.

(52.) Hitherto we have supposed the image *m n* to be received upon white paper, or stucco, or some smooth and white surface on which a picture of it is distinctly formed; but if we receive it upon ground glass, or transparent paper, or upon a plate of glass one of whose sides is covered with a dried film of skimmed milk, and if we place our eye 6 or 8 inches or more behind this semi-transparent ground interposed at *m n*, we shall see the inverted image *m n* as distinctly as before. If we keep the eye in this position, and remove the semi-transparent ground, we shall see an image in the air distinctly and more bright than before. The cause of this will be readily understood, if we consider that all the rays which form by their convergence the points *m*, *n* of the image *m n*, cross one another at *m*, *n*, and diverge from these points exactly in the same manner as they would do from a real object of the same size and brightness placed in *m n*. The image *m n* therefore of any object may be regarded as a new object; and by placing another lens behind it, another image of the image *m n* would be formed, exactly of the same size and in the same place as it would have been had *m n* been a real object. But since the new image of *m n* must be inverted, this new image will now be an erect image of the object M N, obtained by the aid of two lenses; so that, by using one or more lenses, we can obtain direct or inverted images of any object at pleasure. If the object M N is a movable one, and within our reach, it is unnecessary to use two lenses to obtain an erect image of it: we have only to turn it upside down, and we shall obtain, by means of one lens, an image erect in reality, though still inverted in relation to the object.

(53.) In order to explain the power of lenses in magnifying objects and bringing them near us, or rather in giving magnified images of objects, and bringing the images near us, we must examine the different circumstances under which the same object appears when placed at different distances from the eye. If an eye placed at E looks at a man *a b*, *fig.* 35., placed at a distance, his general outline only will be seen, and neither his age, nor his features, nor his dress will be recognized. When he is brought gradually nearer to us, we discover the separate parts of his dress, till at the distance of a few feet we perceive his features; and when brought still nearer, we can count his very eye-lashes, and observe the minutest lines upon his skin. At the distance E *b* the man is seen under the angle *b* E *a*, and at the distance E B he is seen under the greater angle B E A or *b* E A′, and his *apparent* magnitudes, *a b*, A′ *b*, are measured in those different positions by the angles *b* E *a*, B E A, or *b* E A′. The apparent magnitude of the smallest object may, therefore, be equal

Fig. 35.



to the apparent magnitude of the greatest. The head of a pin, for example, may be brought so near the eye that it will appear to cover a whole mountain, or even the whole visible surface of the earth, and in this case the apparent magnitude of the pin's head is said to be equal to the apparent magnitude of the mountain, &c.

Let us now suppose the man *a b* to be placed at the distance of 100 feet from the eye at E, and that we place a convex glass of 25 feet focal distance, half-way between the object *a b* and the eye, that is 50 feet from each; then, as we have previously shown, an inverted image of the man will be formed 50 feet behind the lens, and of the very same size as the object, that is, six feet high. If this object is looked at by the eye, placed 6 or 8 inches behind it, it will be seen exceedingly distinct, and nearly as well as if the man had been brought nearer from the distance of 100 feet to the distance of 6 inches, at which we can examine minutely the details of his personal appearance, Now, in this case, the man, though not actually magnified, has been apparently magnified, because his apparent magnitude is greatly increased, in the proportion nearly of 6 inches to 100 feet, or of 200 to 1.

But if, instead of a lens of 25 feet focal length, we make use of a lens of a shorter focus, and place it in such a position between the eye and the man, that its conjugate foci may be at the distance of 20 and 80 feet from the lens, that is, that the man is 20 feet before the lens, and his image 80 feet behind it, then the size of the image is *four* times that of the object, and the eye placed 6 inches behind this magnified image will see it with the greatest distinctness. Now in this case the image is magnified 4 times directly by the lens, and 200 times by being brought 200 times nearer the eye; so that its apparent magnitude will be 800 times as large as before.

If, on the other hand, we use a lens of a still smaller focal length, and place it in such a position between the eye and the man, that its conjugate foci may be at the distance of 75 and 25 feet from the lens, that is, that the man is 75 feet before the lens, and his image 25 feet behind it, then the size of the image will be only one third of the size of the object; but though the image is thus diminished *three* times in size, yet its apparent magnitude is increased 200 times by being brought within 6 inches of the eye, so that it is still magnified, or its apparent magnitude is increased $\frac{200}{3}$, or 67 times, nearly.

At distances less than the preceding, where the focal length of the lens forms a considerable part of the whole distance, the rule for finding the magnifying power of a lens, when the eye views, at the distance of 6 inches, the image formed by the lens, is as follows. From the distance between the image and the object in feet, subtract the focal distance of the lens in feet, and divide the remainder by the same focal distance. By this quotient divide twice the distance of the object in feet, and the new quotient will be the magnifying power, or the number of times that the apparent magnitude of the object is increased.

When the focal length of the lens is quite inconsiderable, compared with the distance of the object, as it is in most cases, the rule becomes this. Divide the focal length of the lens by the distance at which the eye looks at the image; or, as the eye will generally look at it at the distance of 6 inches, in order to see it most distinctly, divide the focal length by 6 inches; or, what is the same thing, double the focal length in feet, and the result will be the magnifying power.

(54.) Here, then, we have the principle of the simplest *telescope;* which consists of a lens, whose focal length exceeds *six* inches, placed at one end of a tube whose length must always be six inches greater than the focal length of the lens. When the eye is placed at the other end of the tube, it will see an inverted image of distant objects, magnified in proportion to the focal length of the lens. If the lens has a focal length of 10 or 12 feet, the magnifying power will be from 20 to 24 times, and the satellites of Jupiter will be distinctly seen through this single lens telescope. To a very short-sighted person, who sees objects distinctly at a distance of *three* inches, the magnifying power would be from 40 to 48.

A single concave mirror is, upon the same principle, a *reflecting telescope*, for it is of no consequence whether the image of the object is formed by refraction or reflexion. In this case, however, the image *m n*, *fig.* 14., cannot be looked at without standing in the way of the object; but if the reflection is made a little obliquely, or if the mirror is sufficiently large, so as not to intercept all the light from the object, it may be employed as a telescope. By using his great mirror, 4 feet in diameter and 40 feet in focal length, in the way now mentioned, Dr. Herschel discovered one of the satellites of Saturn.

But there is still another way of increasing the apparent magnitude of objects, particularly of those which are within our reach, which is of great importance in optics. It will be proved, when we come to treat of vision, that a good eye sees the visible outline of an object very distinctly when it is placed at a great distance, and that, by a particular power in the eye, we can accommodate it to perceive objects at different distances. Hence, in order to obtain distinct vision of any object, we have only to cause the rays which proceed from it to enter the eye in parallel lines, as if the object itself was very distant. Now, if we bring an object, or the image of an object, very near to the eye, so as to give it great apparent magnitude, it becomes indistinct; but if we can, by any contrivance, make the rays which proceed from it enter the eye nearly parallel, we shall necessarily see it distinctly. But we have already shown that when rays diverge from the focus of any lens, they will emerge from it parallel. If we, therefore, place an object, or an image of one, in the focus of a lens held close to the eye, and having a small focal distance, the rays will enter the eye parallel, and we shall see the object very distinctly, as it will be magnified in the proportion of its present short distance from the eye to the distance of six inches, at which we see small objects most distinctly. But this short distance is equal to the focal length of the lens, so that the magnifying power produced by the lens will be equal to six inches divided by the focal length of the lens. A lens thus used to look at or magnify any object is a *single microscope;* and when such a lens is used to magnify the magnified image produced by another lens, the two lenses together constitute a *compound microscope.*

When such a lens is used to magnify the image produced in the single lens telescope from a distant object, the two lenses together constitute what is called the *astronomical refracting telescope;* and when it is used to magnify the image produced by a concave mirror from a distant object, the two constitute a *reflecting telescope,* such as that used by Le Maire and Herschel: and when it is used to magnify an enlarged image, M N, *fig.* 14., produced from an object *m n,* placed before a concave mirror, the two constitute a *reflecting microscope.* All these instruments will be more fully described in a future chapter.

CHAP. VI.

SPHERICAL ABERRATION OF LENSES AND MIRRORS.*

(55.) In the preceding chapters we have supposed that the rays refracted at spherical surfaces meet exactly in a focus; but this is by no means strictly true: and if the reader has in any one case projected the rays by the methods described, he must have seen that the rays nearest the axis of a spherical surface, or of a lens, are refracted to a focus *more remote* from the lens than those which are incident at a distance from the axis of the lens. The rules which we have given for the foci of lenses and surfaces are true for rays very near the axis.

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* For a discussion of these subjects, see (in the college edition) the Appendix of Am. ed. chap. v.

In order to understand the cause of spherical aberration, let L L be a plano-convex lens one of whose surfaces is spherical, and let its plane surface L *m* L be turned towards parallel rays R L, R L. Let R′ L′, R′ L′ be rays very near the axis A F of the lens, and let F be their focus after refraction. Let R L, R L be parallel rays incident at the very margin of the lens, and it will be found by the method of projection that the corresponding refracted rays L *f*, L *f* will meet at a point *f* much nearer the lens than F. In like manner intermediate rays between R L and R′ L′ will have their foci intermediate between *f* and F. Continue the rays L *f*, L *f*, till they meet at G and H a plane passing through F, and perpendicular to F A. The distance *f* F is called the *longitudinal spherical aberration*, and G H the *lateral spherical aberration of the lens*. In a plano-convex lens placed like that in the figure, the longitudinal spherical aberration *f* F is no less than $4\frac{1}{2}$ times *m n* the thickness of the lens. It is obvious that such a lens cannot form a distinct picture of any object in its focus F. If it is exposed to the sun, the central part of the lens L′ *m* L′ whose focus is at F, will form a pretty bright image of the sun at F; but as the rays of the sun which pass through the outer part L L of the lens have their foci at points between *f* and F, the rays will, after arriving at those points, pass on to the plane G H, and occupy a circle whose diameter is G H; hence the image of the sun in the focus F will be a bright disc surrounded and rendered indistinct by a broad halo of light growing fainter and fainter from F to G and H. In like manner, every object seen through such a lens, and every image formed by it, will be rendered confused and indistinct by spherical aberration.

Fig. 36.

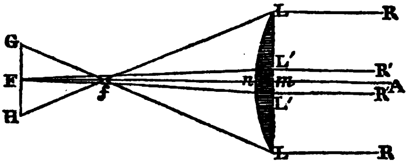
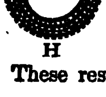


Fig. 37.



These results may be illustrated experimentally by taking a ring of black paper, and covering up the outer parts of the face L L of the lens. This will diminish the halo G H, and the indistinctness of the image, and if we cover up all the lens excepting a small part in the centre, the image will become perfectly distinct, though less bright than before, and the focus will be at F. If, on the contrary, we cover up all the central part, and leave only a narrow ring at the circumference of the lens, we shall have a very distinct image of the sun formed about *f*.

(56.) If the reader will draw a very large diagram of a plano-convex and of a double convex lens, and determine the refracted rays at different distances from the axis where parallel rays fall on each of the surfaces of the lens, he will be able to verify the following results for glass lenses.

1. In a *plano-convex* lens, with its plane side turned to parallel rays as in *fig*. 36., that is, turned to distant objects if it is to form an image behind it, or turned to the eye if it is to be used in magnifying a near object, the spherical aberration will be 4½ times the thickness, or 4½ times $m\,n$.

2. In a *plano-convex lens*, with its convex side turned towards parallel rays, the aberration is only $1\frac{17}{100}$ths of its thickness. In using a plano-convex lens, therefore, it should always be so placed that parallel rays either enter the convex surface or emerge from it.

3. In a *double convex lens* with equal convexities, the aberration is $1\frac{67}{100}$ths of its thickness.

4. In a *double convex lens* having its radii as 2 to 5, the aberration will be the same as in a plano-convex lens in Rule 1, if the side whose radius is 5 is turned towards parallel rays; and the same as the plano-convex lens in Rule 2, if the side whose radius is 2 is turned to parallel rays.

5. The lens which has *the least spherical aberration* is a double convex one, whose radii are as 1 to 6. When the face whose radius is 1 is turned towards parallel rays, the aberration is only $1\frac{7}{100}$ths of its thickness; but when the side with the radius 6 is turned towards parallel rays, the aberration is $3\frac{45}{100}$ths of its thickness.

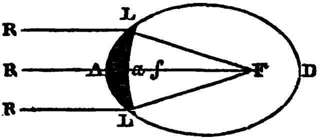
These results are equally true of plano-concave and double concave lenses.

If we suppose the lens of least spherical aberration to have its aberration equal to 1, the aberrations of the other lenses will be as follows:—

Best form, as in Rule 5.	1·000
Double convex or concave, with equal curvatures .	1·561
Plano-convex or concave in best position, as in Rule 2.	1·093
Plano-convex or concave in worst position, as in Rule 1.	4·206

(57.) As the central parts of the lens L L, *fig.* 36., refract the rays too little, and the marginal parts too much, it is evident that if we could increase the convexity at *n* and diminish it gradually towards L, we should remove the spherical aberration. But the ellipse and the hyperbola are curves of this kind, in which the curvature diminishes from *n* to L; and mathematicians have shown how spherical aberration may be entirely removed, by lenses whose sections are ellipses or hyperbolas. This curious discovery we owe to Descartes.

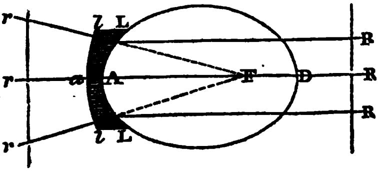
Fig. 38.



If A L D L, for example, is an ellipse whose greater axis A D is to the distance between its foci F, *f*, as the index of refraction is to unity, then parallel rays R L, R L incident upon the elliptical surface L A L will be refracted by the single action of that surface into lines, which would meet exactly in the focus F, if there were no second surface intervening between L A L and F. But as every useful lens must have two surfaces, we have only to describe a circle L *a* L round F as a centre, for the second surface of the lens L L. As all the rays refracted at the surface L A L converge accurately to F, and as the circular surface L *a* L is perpendicular to every one of the refracted rays, all these rays will go on to F without suffering any refraction at the circular surface. Hence it follows that a meniscus whose convex surface is part of an ellipsoid, and whose concave surface is part of any spherical surface whose centre is in the farther focus, will have no spherical aberration, and will refract parallel rays incident on its convex surface to the farther focus.

In like manner a concavo-convex lens, L L, whose concave

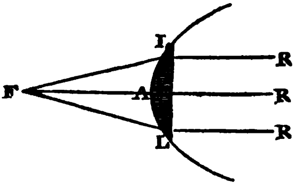
Fig. 39.



surface L A L is part of the ellipsoid A L D L, and whose convex surface *l a l* is a circle described round the farther focus of the ellipse, will cause parallel rays R L, R L to diverge in directions L *r*, L *r*, which when continued backwards will meet exactly in the focus F, which will be its virtual focus.

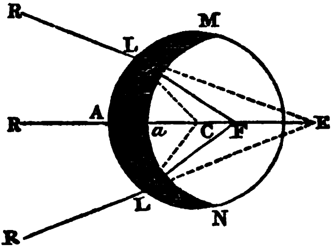
If a plano-convex lens has its convex surface, L A L, part of a hyperboloid formed by the revolution of a hyperbola whose greater axis is to the distance between the foci as unity is to the index of refraction; then parallel rays, R L, R L, falling perpendicularly on the plane surface will be refracted without aberration to the farther focus of the hyperboloid. The same property belongs to a plano-concave lens, having a similar hyperbolic surface, and receiving parallel rays on its plane surface.

Fig. 40.



A meniscus with spherical surfaces has the property of refracting all converging rays to its focus, if its first surface is convex, provided the distance of the point of convergence or divergence from the centre of the first surface is to the radius of the first surface as the index of refraction is to unity.

Fig. 41.



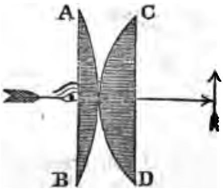
Thus, if M L L N is a meniscus, and R L, R L rays converging to the point E, whose distance E C from the centre of the first surface L A L of the meniscus is to the radius C A or C L as the index of refraction is to unity, that is as 1·500 to 1, in glass; then if F is the focus of the first surface, describe with any radius less than F A a circle M *a* N for the second surface of the lens. Now it will be found by projection that the rays R L, R L, whether near the axis A E or remote from it, will be refracted accurately to the focus F, and as all these rays fall perpendicularly on the second surface, they will still pass on without refraction to the focus F. In like manner it is obvious that rays F L, F L diverging from F will be refracted into R L, R L, which diverge accurately from the virtual focus.

When these properties of the ellipse and hyperbola, and of the solids generated by their revolution, were first discovered, philosophers exerted all their ingenuity in grinding and polishing lenses with elliptical and hyperbolical surfaces, and various ingenious mechanical contrivances were proposed for this purpose. These, however, have not succeeded, and the practical difficulties which yet require to be overcome are so great, that lenses with spherical surfaces are the only ones now in use for optical instruments.

But though we cannot remove or diminish the spherical aberration of single lenses beyond 1$\frac{7}{100}$ths of their thickness, yet by combining two or more lenses, and making opposite aberrations correct each other, we can remedy this defect to a very considerable extent in some cases, and in other cases remove it altogether.

(58.) Mr. Herschel has shown, that if two plano-convex lenses A B, C D, whose focal lengths are as 2·3 to 1, are placed with their convex sides together, A B the least convex being next the eye when the combination is to be used as a microscope, the aberration will be only 0·248, or one fourth of that of a single lens in its best form. When this lens is used to form an image, A B must be turned to the object. If the focal lengths of the two lenses are equal, the spherical aberration will be 0·603, or a little more than one-half of a single lens in its best form.

Fig. 42.



Mr. Herschel has also shown that the spherical aberration may be *wholly removed* by combining a meniscus C D with a double convex lens A B, as in *figs.* 43. and 44., the lens A B being turned to the eye when it is used for a microscope, and to the object when it is to be used for forming images, or as a burning-glass.

Fig. 43.

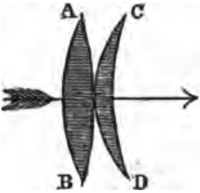
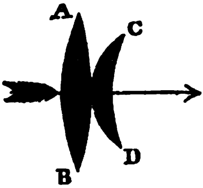


Fig. 44.



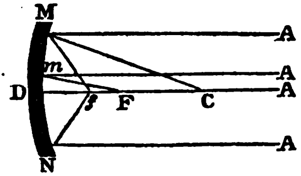
The following are the radii of curvature for these lenses, as computed by Mr. Herschel; the first supposes, as a condition, that the focal length of the compound lens shall be as near 10·000 as is consistent with correcting the aberration; and the second, that the same focal length shall be the least possible:—

	Fig. 43.	Fig. 44.
Focal length of the *double convex* lens A B	+ 10·000	+ 10·000
Radius of its first or outer surface	+ 5·833	+ 5·833
Radius of its second surface . .	— 35·000	— 35·000
Focal length of the meniscus C D	+ 17·829	+ 5·497
Radius of its first surface	+ 3·688	+ 2·054
Radius of its second surface . .	+ 6·291	+ 8·128
Focal length of the compound lens	+ 6·407	+ 3·474

Spherical Aberration of Mirrors.

(59.) We have already stated, that when parallel rays, A M, A N, are incident on a spherical mirror, M N, they are refracted to the same focus, F, only when they are incident very near the axis, A D. If F is the focus of those very near the axis, such as A *m*, then the focus of those more remote, such as A M, will be at *f* between F and D, and *f* F will be the

Fig. 45.



longitudinal spherical aberration, which will obviously increase with the diameter of the mirror when its curvature remains the same, and with the curvature when its diameter is constant. The images, therefore, formed by mirrors will be indistinct, like those formed by spherical lenses, and the indistinctness will arise from the same cause.

It is manifest that if M N were a curve of such a nature that a line, A M, parallel to its axis, A D, and another line, *f* M drawn from M to a fixed point, *f*, should always form equal angles with a line, C M, perpendicular to the curve M N, we should in this case have a surface which would reflect parallel rays exactly to a focus *f*, and form perfectly distinct images of objects. Such a curve is the *parabola;* and, therefore, if we could construct mirrors of such a form that their section M N is a parabola, they would have the invaluable property of reflecting parallel rays to a single focus. When the curvature of the mirror is very small, opticians have devised methods of communicating to it a parabolic figure; but when the curvature is great, it has not yet been found practicable to give it this figure.

In the same manner it may be shown, that when diverging rays fall upon a concave mirror of a spherical form, they will be reflected to different points of the axis; and that if a surface could be formed so that the incident and reflected rays should form equal angles with a line perpendicular to the surface at the point of incidence, the reflected rays would all meet in a single point as their focus. A surface whose section is an ellipse has this property; and it may be proved that rays diverging from one focus of an ellipse will be reflected accurately to the other focus. Hence in reflecting microscopes the mirror should be a portion of an ellipsoid; the axis of the mirror being the axis of the ellipsoid, and the object being placed in the focus nearest the mirror.

On Caustic Curves formed by Reflexion and Refraction.[*]

(60.) *Caustics formed by reflexion.*—As the rays incident on different points of a reflecting surface at different distances from its axis are reflected to different foci in that axis, it is evident that the rays thus reflected must cross one another at particular points, and wherever the rays cross they will illuminate the white ground on which they are received with twice as much light as falls on other parts of the ground. These luminous intersections form curve lines, called *caustic lines* or *caustic curves;* and their nature and form will, of course, vary with the aperture of the mirror, and the distance of the radiant point.

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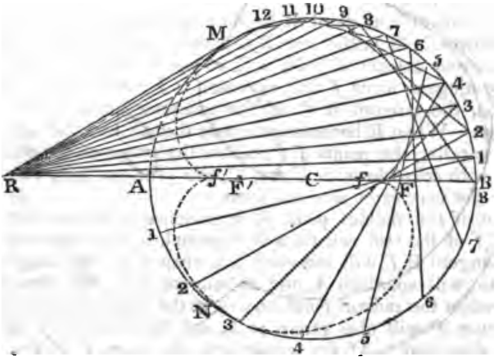
[*] See (in the College edition) the Appendix of Am. ed. chap. v.

In order to explain their mode of formation and general properties, let M B N be a concave spherical mirror, *fig.* 46., whose centre is C, and whose focus for parallel and central

Fig. 46.



rays is F. Let R M B be a diverging beam of light falling on the upper part, M B, of the mirror at the points 1, 2, 3, 4, &c. If we draw lines perpendicular to all these points from the centre C, and make the angles of reflexion equal to the angles of incidence, we shall obtain the directions and foci of all the reflected rays. The ray R 1, near the axis R B, will have its conjugate focus at *f*, between F and the centre C. The next ray, R 2, will cut the axis nearer F, and so on with all the rest, the foci advancing from *f* to B. By drawing all the reflected rays to these foci, they will be found to intersect one another as in the figure, and to form by their intersections the *caustic curve* M *f*. If the light had also been incident on the lower part of the mirror, a similar caustic shown by a dotted line would also have been formed between N and *f*. If we suppose, therefore, the point of incidence to move from M to B, the conjugate focus of any two contiguous rays, or an infinitely slender pencil diverging from R, will move along the caustic from M to *f*.

Let us now suppose the convex surface M B N of the mirror to be polished, and the radiant point R to be placed as far to the right hand of B as it is now to the left, it will be found, by drawing the incident and reflected rays, that they will diverge after reflexion; and that when continued backwards they will intersect one another, and form an imaginary caustic situated behind the convex surface, and similar to the real caustic.

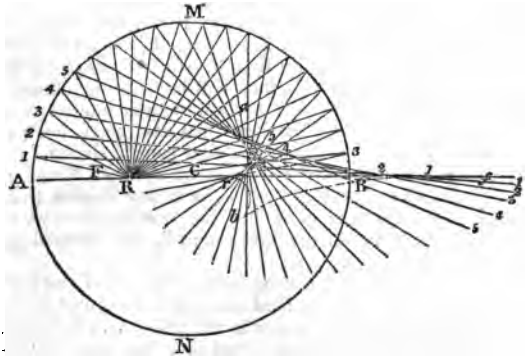
If we suppose the convex mirror M B N to be completed round the same centre, C, as at M A N, and the pencil of rays still to radiate from R, they will form the imaginary caustic M f' N smaller than M f N, and uniting with it at the points M, N.

Let the radiant point R be now supposed to recede from the mirror M B N, the line B f, which is called the *tangent* of the real caustic M f N, will obviously diminish, because the conjugate focus f will approach to F; and, for the same reason, the tangent A f' of the imaginary caustic will increase. When R becomes infinitely distant, and the incident rays parallel, the points f, f', called the *cusps* of the caustic, will both coincide with F and F′, the principal foci, and will have the very same size and form.

But if the radiant point R approaches to the mirror, the cusp f of the real caustic will approach to the centre C, and the tangent B f will increase, the cusp f' of the imaginary caustic will approach A, and its tangent A f', will diminish; and when the radiant point arrives at the circumference at A, the cusp f' will also arrive at A, and the imaginary caustic will disappear. At the same time, the cusp f of the real caustic will be a little to the right of C, and its two opposite summits will meet in the radiant point at A.

If we suppose the radiant point R now to enter within the circle A M B N, as shown in *fig.* 47., so that R C is less than

Fig. 47.

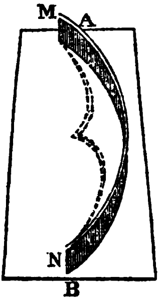


R A, a remarkable double caustic will be formed. This caustic will consist of two short ones of the common kind, *a r*, *b r*, having their common cusp at *r*, and of two long branches, *a f*, *b f*, which meet in a focus at *f*. When R C is greater than R A, the curved branches that meet at *f* behind the mirror will diverge, and have a virtual focus within the mirror. When R coincides with F, a point half-way between A and C, and the virtual principal focus of the convex mirror M A N, these curved branches become parallel lines; and when R coincides with the centre C, the caustics disappear, and all the light is condensed into a single mathematical point at C, from which it again diverges, and is again reflected to the same point.

In virtue of the principle on which these phenomena depend, a spherical mirror has, under certain circumstances, the paradoxical property of rendering rays diverging frcm a fixed point either parallel, diverging, or converging; that is, if the radiant point is a little way within the principal focus of a mirror, so that rays very near the axis are reflected into parallel lines, the rays which are incident still nearer the axis will be rendered diverging, and those incident farther from the axis will be rendered converging. This property may be distinctly exhibited by the projection of the reflected rays.

Caustic curves are frequently seen in a very distinct and beautiful manner at the bottom of cylindrical vessels of china or earthenware that happen to be exposed to the light of the sun or of a candle. In these cases the rays generally fall too obliquely on their cylindrical surface, owing to their depth; but this depth may be removed, and the caustic curves beautifully displayed, by inserting a circular piece of card or white paper about an inch or so beneath their upper edge, or by filling them to that height with milk or any white and opaque fluid.

Fig. 48.



The following method, however, of exhibiting caustic curves I have found exceedingly convenient and instructive. Take a piece of steel spring highly polished, such as a watch-spring, M N, *fig.* 48., and having bent it into a concave form as in the figure, place it vertically on its edge upon a piece of card or white paper A B. Let it then be exposed either to the rays of the sun, or those of any other luminous body, taking care that the plane of the card or the paper passes nearly through the sun; and the two caustic curves shown in the figure will be finely displayed. By varying the size of the spring, and bending it into curves of different shapes, all the variety of caustics, with their cusps and points of contrary flexure, will be finely exhibited. The steel may be bent accurately into different curves by applying a portion of its breadth to the required curves drawn upon a piece of wood, and either cut or burned sufficiently deep in the wood to allow the edge of the thin strip of metal to be inserted in it. Gold or silver foil answers very well; and when the light is strong, a thin strip of mica will also answer the purpose. The best substance of all, however, is a thin strip of polished silver.

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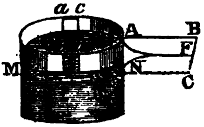
(61.) *Caustics formed by refraction.* If we expose a globe of glass filled with water, or a solid spherical lens, or even the belly of a round decanter, filled with water, to the rays of the sun, or to the light of a lamp or candle, and receive the refracted light on white paper held almost parallel to the axis of the sphere, or so that its plane passes nearly through the luminous body, we shall perceive on the paper a luminous figure bounded by two bright caustics, like *a f* and *b f*, *fig.* 47., but placed behind the sphere, and forming a sharp cusp or angle at the point *f*, which is the focus of refracted rays. The production of these curves depends upon the intersection of rays, which, being incident on the sphere at different distances from the axis, are refracted to foci at different points of the axis, and therefore cross one another. This result is so easily understood, and may be exhibited so clearly, by projecting the refracted rays, that it is unnecessary to say any more on the subject.

Some of the phenomena of caustics produced by refraction may be illustrated experimentally in the following manner:— Take a shallow cylindrical vessel of lead, M N, two or three inches in diameter, and cut its upper margin, as shown in the figure, leaving two opposite projections, *a c*, *b d*, forming each about 10° or 15° of the whole circumference. Complete the circumference by cementing on the vessel two strips of mica, so as to substitute for the lead that has been removed two transparent cylindrical surfaces. If this vessel is filled with water, or any other transparent fluid, and a piece of card or white paper, A B C D, is held almost parallel to the surface of the water, and having its plane nearly passing through the sun or the candle, the caustics A F, D F will be finely displayed. By altering the curvature of the vessel, and that of the strips of mica, many interesting variations of the experiment may be made.

Fig. 49.



PART II.

PHYSICAL OPTICS.

(62.) PHYSICAL Optics is that branch of the science which treats of the physical properties of light. These properties are exhibited in the decomposition and recomposition of white light; in its decomposition by absorption; in the inflexion or diffraction of light; in the colors of thick and thin plates; and in the double refraction and polarization of light.

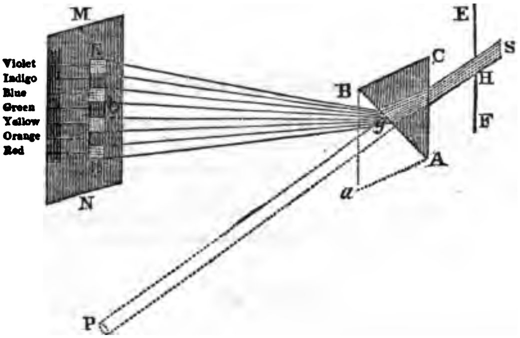
CHAP. VII.

ON THE COLORS OF LIGHT, AND ITS DECOMPOSITION.

(63.) IN the preceding chapters we have regarded light as a simple substance, all the parts of which had the same index of refraction, and therefore suffered the same changes when acted upon by transparent media. This, however, is not its constitution. White light, as emitted from the sun or from any luminous body, is composed of seven different kinds of light, viz., *red*, *orange*, *yellow*, *green*, *blue*, *indigo*, and *violet;* and this compound substance may be decomposed, or analyzed, or separated into its elementary parts, by two different processes, viz., by *refraction* and *absorption*.

The first of these processes was that which was employed by Sir Isaac Newton, who discovered the composition of white light. Having admitted a beam of the sun's light, S H, through a small hole, H, in the window-shutter, E F, of a darkened room, it will go on in a straight line and form a round white spot at P. If we now interpose a prism, B A C, whose refracting angle is B A C, so that this beam of light may fall on its first surface C A, and emerge at the same angle from its second surface B A in the direction *g* G, and if we receive the refracted beam on the opposite wall, or rather on a white screen, M N, we should expect, from the principles already laid down, that the white beam which previously fell upon P would suffer only a change in its direction, and fall somewhere upon M N, forming there a round white spot exactly similar to that at P. But this is not the case. Instead of a white spot, there will be formed upon the screen M N an oblong image K L of the sun, containing seven colors, viz. *red*, *orange*, *yellow*, *green*, *blue*, *indigo*, and *violet*, the whole beam of light diverging from its emergence out of the prism at *g*, and being bounded by the lines *g* K, *g* L. This lengthened image of the sun is called the *solar spectrum*, or the *prismatic spectrum*. If the aperture H is small, and the distance *g* G considerable, the colors of the spectrum will be very bright. The lowest portion of it at L is a brilliant *red*. This red shades off by imperceptible gradations into *orange*, the *orange* into *yellow*, the *yellow* into *green*, the *green* into *blue*, the *blue* into a pure *indigo*, and the *indigo* into a *violet*. No lines are seen across the spectrum thus produced; and it is extremely difficult for the sharpest eye to point out the boundary of the different colors. Sir Isaac Newton, however, by many trials, found the lengths of the colors to be as follows, in the kind of glass of which his prism was made. We have added the results obtained by Fraunhofer with flint glass.

Fig. 50.



	Newton.	Fraunhofer.
Red	45	56
Orange	27	27
Yellow	48	27
Green	60	46
Blue	60	48
Indigo	40	47
Violet	80	109
Total length	360	360

These colors are not equally brilliant. At the lower end, L, of the spectrum the red is comparatively faint, but grows brighter as it approaches the orange. The light increases gradually to the middle of the yellow space, where it is brightest; and from this it gradually declines to the upper or violet end, K, of the spectrum, where it is extremely faint.

(64.) From the phenomena which we have now described, Sir Isaac Newton concluded that the beam of white light, S, is compounded of light of seven different colors, and that for each of these different kinds of light, the glass, of which his prism was made, had different indices of refraction; the index of refraction for the *red* light being the least, and that of the *violet* the greatest.

If the prism is made of *crown glass*, for example, the indices of refraction for the different colored rays will be as follows:—

	Index of Refraction.		Index of Refraction.
Red	1·5258	Blue	1·5360
Orange	1·5268	Indigo	1.5417
Yellow	1·5296	Violet	1·5466
Green	1·5330		

If we now draw the prism, B A C, on a great scale, and determine the progress of the refracted rays, supposed to be incident upon the same point of the first surface C A, by using for each ray the index of refraction in the preceding table, we shall find them to diverge as in the preceding figure, and to form the different colors in the order of those in the spectrum.

In order to examine each color separately, Sir Isaac made a hole in the screen M N, opposite the centre of each colored space; and he allowed that particular color to fall upon a second prism, placed behind the hole. This light, when refracted by the second prism, was not drawn out into an oblong image as before, and was not refracted into any other colors. Hence he concluded that the light of each different color had the same index of refraction; and he called such light *homogeneous*, or *simple*, *white* light being regarded as *heterogeneous* or *compound*. This important doctrine is called the *different refrangibility of the rays of light*. The different colors as existing in the spectrum are called *primary* colors; and any mixtures or combinations of any of them are called *secondary* colors, because we can easily separate them into their primary colors by refraction through the prism.

(65.) Having thus clearly established the composition of white light, Sir Isaac also proved, experimentally, that all the seven colors, when again combined and made to fall upon the same spot, formed or *recomposed* white light. This important truth he established by various experiments; but the following method of proving it is so satisfactory, that no farther evidence seems to be wanted. Let the screen M N, *fig.* 50., which receives the spectrum, be gradually brought nearer the prism B A C, the spectrum K L will gradually diminish; but though the colors begin to mix, and encroach upon one another, yet, even when it is brought close to the face B A of the prism, we shall recognize the separation of the light into its component colors. If we now take a prism, B *a* A, shown by dotted lines, made of the same kind of glass as B A C, and having its refracting angle A B *a* exactly equal to the refracting angle B A C of the other prism; and if we place it in the opposite direction, we shall find that all the seven differently colored rays which fall upon the second prism, A B *a*, are again combined into a single beam of white light *g* P, forming a white circular spot at P, as if neither of the prisms had been interposed. The very same effect will be produced, even if the surfaces, A B, of the two prisms are joined by a transparent cement of the same refractive power as the glass, so as to remove entirely the refractions at the common surface A B. In this state the two prisms combined are nothing more than a thick piece of glass, B C A *a*, whose two sides, A C, *a* B, are exactly parallel; and the decomposition of the light by the refraction of the first surface, A C, is counteracted by the opposite and equal refraction of the second surface, *a* B; that is, the light decomposed by the first surface is recomposed by the second surface. The refraction and re-union of the rays in this experiment may be well exhibited by placing a thick plate of oil of cassia between two parallel plates of glass, and making a narrow beam of the sun's light fall upon it very obliquely. The spectrum formed by the action of the first surface will be distinctly visible, and the re-union of the colors by the second will be equally distinct. We may, therefore, consider the action of a plate of parallel glass on the sun's rays, that is, its property of transmitting them colorless, as a sufficient proof of the recomposition of light.

The same doctrine may be illustrated experimentally by mixing together *seven* different powders having the same colors as those of the spectrum, taking as much of each as seems to be proportional to the rays in each colored space. The union of these colors will be a sort of grayish-white, because it is impossible to obtain powders of the proper colors. The same result will be obtained, if we take a circle of paper and divide it into sectors of the same size as the colored spaces; and when this circle is made to revolve rapidly, the effect of all the colors when combined will be a grayish-white.

Decomposition of Light by Absorption.

(66.) If we measure the quantity of light which is reflected from the surfaces and transmitted through the substance of transparent bodies, we shall find that the sum of these quantities is always less than the quantity of light which falls upon the body. Hence we may conclude that a certain portion of light is *lost* in passing through the most transparent bodies. This loss arises from two causes. A part of the light is scattered in all directions by irregular reflexion from the imperfectly polished surface of particular media, or from the imperfect union of its parts; while another, and generally a greater portion, is *absorbed*, or stopped by the particles of the body. Colored fluids, such as black and red ink, though equally homogeneous, stop or absorb different kinds of rays, and when exposed to the sun they become heated in different degrees; while pure water seems to transmit all the rays equally,* and scarcely receives any heat from the passing light of the sun.

When we examine more minutely the action of colored glasses and colored fluids in absorbing light, many remarkable phenomena present themselves, which throw much light upon this curious subject.

If we take a piece of blue glass, like that generally used for finger glasses, and transmit through it a beam of white light, the light will be a fine deep blue. This blue is not a simple homogeneous color, like the blue or indigo of the spectrum, but is a mixture of all the colors of white light which the glass has not absorbed; and the colors which the glass has absorbed are those which the blue wants of white light, or which, when mixed with this blue, would form white light. In order to determine what these colors are, let us transmit through the blue glass the prismatic spectrum K L, *fig.* 50.; or, what is the same thing, let the observer place his eye behind the prism B A C, and look through it at the sun, or rather at a circular aperture made in the window-shutter of a dark room. He will then see through the prism the spectrum K L as far below the aperture as it was above the spot P when shown in the screen. Let the blue glass be now interposed between the eye and the prism, and a remarkable spectrum will be seen, deficient in a certain number of its differently colored rays. A particular thickness absorbs the middle of the red space, the whole of the orange, a great part of the green, a considerable part of the blue, a little of the indigo, and very little of the violet. The yellow space, which has not been much absorbed, has *increased in breadth.* It occupies part of the space formerly covered by the *orange* on one side, and part of the space formerly covered by the *green* on the other. Hence it follows, that the blue glass has absorbed the red light, which, when mixed with the yellow light, constituted *orange,* and has absorbed also the *blue* light, which, when mixed with the *yellow,* constituted the part of the *green* space next to the *yellow.* We have therefore, by absorption, decomposed *green* light into *yellow* and *blue,* and *orange* light into *yellow* and *red;* and it consequently follows, that the orange and green rays of the spectrum, though they cannot be decomposed by prismatic refraction, can be decomposed by absorption, and actually consist of two different colors possessing the same degree of refrangibility. *Difference of color is therefore not a test of difference of refrangibility,* and the conclusion deduced by Newton is no longer admissible as a general truth: "That to the same degree of refrangibility ever belongs the same color, and to the same color ever belongs the same degree of refrangibility."

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* See Note II., of Am. ed., which follows the author's Appendix.

With the view of obtaining a complete analysis of the spectrum, I have examined the spectra produced by various bodies, and the changes which they undergo by absorption when viewed through various colored media, and I find that the color of every part of the spectrum may be changed not only in intensity, but in color, by the action of particular media; and from these observations, which it would be out of place here to detail, I conclude that the solar spectrum consists of three spectra of equal lengths, viz. a *red* spectrum, a *yellow* spectrum, and a *blue* spectrum. The *primary red* spectrum has its maximum of intensity about the middle of the *red* space in the solar spectrum, the *primary yellow* spectrum has its maximum in the middle of the *yellow* space, and the *primary blue* spectrum has its maximum between the *blue* and the *indigo* space. The two minima of each of the three primary spectra coincide at the two extremities of the solar spectrum.

From this view of the constitution of the solar spectrum we may draw the following conclusions:—

1. *Red, yellow,* and *blue* light exist at every point of the solar spectrum.

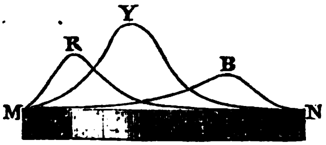
2. As a certain portion of *red, yellow,* and *blue* constitute *white* light, the color of every point of the spectrum may be

considered as consisting of the predominating color at any point mixed with white light. In the red space there is more red than is necessary to make white light with the small portions of yellow and blue which exist there; in the yellow space there is more yellow than is necessary to make white light with the red and blue; and in the part of the blue space which appears violet there is more red than yellow, and hence the excess of red forms a violet with the blue.

3. By absorbing the excess of any color at any point of the spectrum above what is necessary to form white light, we may actually cause white light to appear at that point, and this white light will possess the remarkable property of remaining white after any number of refractions, and of being decomposable only by absorption. Such a white light I have succeeded in developing in different parts of the spectrum. These views harmonize in a remarkable manner with the hypothesis of three colors, which has been adopted by many philosophers, and which others had rejected from its incompatibility with the phenomena of the spectrum.

The existence of three primary colors in the spectrum, and the mode in which they produce by their combination the seven secondary or compound colors which are developed by the prism, will be understood from *fig.* 51. where M N is the prismatic spectrum, consisting of three primary spectra of the same lengths, M N, viz. a *red*, a *yellow*, and a *blue* spectrum. The *red* spectrum has its maximum intensity at R; and this intensity may be represented by the distance of the point R from M N. The intensity declines rapidly to M and slowly to N, at both of which points it vanishes. The *yellow* spectrum has its maximum intensity at Y, the intensity declining to *zero* at M and N; and the *blue* has its maximum intensity at B, declining to nothing at M and N. The general curve which represents the total illumination at any point will be outside of these three curves, and its ordinate at any point will be equal to the sum of the three ordinates at the same point. Thus the ordinate of the general curve at the point Y will be equal to the ordinate of the yellow curve, which we

Fig. 51.



may suppose to be 10, added to that of the red curve, which may be 2, and that of the blue, which may be 1. Hence the general ordinate will be 13. Now, if we suppose that 3 parts of yellow, 2 of red, and 1 of blue make white, we shall have the color at Y equal to 3 + 2 + 1, equal to 6 parts of white mixed with 7 parts of yellow; that is, the compound tint at Y will be a bright *yellow* without any trace of red or blue. As these colors all occupy the same place in the spectrum, they cannot be separated by the prism; and if we could find a colored glass which would absorb 7 parts of the yellow, we should obtain at the point Y a *white light* which the prism could not decompose.*

CHAP. VIII.

ON THE DISPERSION OF LIGHT.

In the preceding observations, we have considered the prismatic spectrum, K L, *fig.* 50., as produced by a prism of glass having a given refracting angle, B A C. The green ray, or *g* G, which, being midway between *g* K and *g* L, is called the *mean ray* of the spectrum, has been refracted from P to G, or through an angle of deviation, P *g* G, which is called the mean refraction or deviation, produced by the prism. If we now increase the angle B A C of the prism, we shall increase the refraction. The mean ray *g* G will be refracted to a greater distance from P, and the extreme rays *g* L, *g* K, to a greater distance in the same proportion; that is, if *g* G is refracted twice as much, *g* L and *g* K will also be refracted twice as much, and consequently the length of the spectrum K L will be twice as great. For the same reason, if we diminish the angle B A C of the prism, the mean refraction and the spectrum will diminish in the same proportion; but, whatever be the angle of the prism, the length K L will always bear the same proportion to G P, the mean refraction.

Sir Isaac Newton supposed that prisms made of all substances whatever, produced spectra bearing the same proportion to the mean refraction as prisms of glass; and it is a remarkable circumstance, that a philosopher of such sagacity should have overlooked a fact so palpable, as that different bodies produced spectra whose lengths were different, when the mean refraction was the same.

The prism B A C being supposed to be made of *crown*

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* See Note III., by Am. ed., following the author's Appendix.

glass, let us take another of *flint glass* or *white crystal*, with such a refracting angle that, when placed in the position B A C, the light enters and quits it at equal angles, and refracts the mean ray to the same point G. The two prisms ought, therefore, to have the same mean refraction. But when we examine the spectrum produced by the flint glass prism, we shall find that it extends beyond K and L, and is evidently longer than the spectrum produced by the crown glass prism. Hence *flint glass* is said to have a greater *dispersive* power than crown glass, because at the same angle of mean refraction it separates the extreme rays of the spectrum, *g* L, *g* K, farther from the mean ray *g* G.

In order to explain more clearly what is the real measure of the dispersive power of a body, let us suppose that in the *crown glass* prism, B A C, the index of refraction for the extreme violet ray, *g* K, is 1·5466, and that for the extreme red ray, *g* L, 1·5258; then the difference of these indices, or ·0208, would be a measure of the dispersive power of crown glass, if it and all other bodies had the same mean refraction: but as this is far from being the case, the dispersive power must be measured by the relation between ·0208 and the mean refraction, or 1·5330, or to the excess of this above unity, viz., ·5330, to which the mean refraction is always proportional. For the purpose of making this clearer, let it be required to compare the dispersive powers of *diamond* and *crown glass*. The index of refraction of diamond for the extreme *violet* ray is 2·467, and for the extreme *red*, 2·411, and the difference of these is ·0560, nearly *three* times as great as ·0208, the same difference for crown glass; but then the difference between the sines of incidence and refraction, or the excess of the index of refraction above unity, or 1·439, is also about *three* times as great as the same difference in crown glass, viz., ·5330; and, consequently, the dispersive power of diamond is very little greater than that of crown glass. The two dispersive powers are as follows:—

		Dispersive Powers.
Crown Glass . . .	$\frac{\cdot 0308}{\cdot 5330}$	= 0·0386
Diamond	$\frac{\cdot 0560}{1\cdot 439}$	= 0·0388

This similarity of dispersive power might be proved experimentally, by taking a prism of diamond, which, when placed at B A C in *fig.* 50., produced the same mean refraction as the green ray *g* G. It would then be seen that the spectrum which it produced was of the same length as that produced by the prism of crown glass. Hence the splendid colors which distinguish diamond from every other precious stone are not owing to its high dispersive power, but to its great mean refraction.

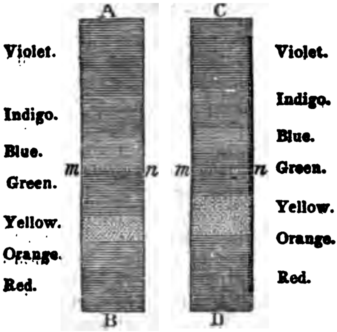
As the indices of refraction given in our table of refractive powers are nearly suited to the mean ray of the spectrum, we may, by the second column of the Table of the Dispersive Powers of Bodies, given in the Appendix, No. I., obtain the approximate indices of refraction for the extreme red and the extreme violet rays, by adding half of the number in the column to the mean index of refraction for the index of refraction of the violet, and subtracting half of the same number for the index of the red ray. The measures in the table are given for the ordinary light of day. When the sun's light is used, and when the eye is screened from the middle rays of the spectrum, the red and violet may be traced to a much greater distance from the mean ray of the spectrum.

When the index of refraction for the extreme ray is thus known, we may determine the position and length of the spectra produced by prisms of different substances, whatever be their refracting angle, whatever be the positions of the prism, and whatever be the distance of the screen on which the spectrum is received.

If we take a prism of crown glass, and another of flint glass, with such refracting angles that they produce a spectrum of precisely the same length, it will be found, that when the two prisms are placed together with their refracting angles in opposite directions, they will not restore the refracted pencil to the state of white light, as happens in the combination of two equal prisms of crown or two equal prisms of flint glass. The white light P, *fig.* 50., will be tinged on one side with *purple*, and on the other with *green* light. This is called the *secondary spectrum*, and the colors *secondary colors;* and it is manifest that they must arise from the colored spaces in the spectrum of crown glass not being equal to those in the spectrum of flint glass.

In order to render this curious property of the spectrum very obvious to the eye, let two spectra of equal length be formed by two hollow prisms, one containing *oil of cassia*, and the other *sulphuric acid.* The oil of cassia spectrum will resemble A B, *fig.* 52., and the sulphuric acid spectrum C D. In the former, the *red*, *orange*, and *yellow* spaces are less than in the latter, while the *blue*, *indigo*, and *violet* spaces are greater; the *least* refrangible rays being, as it were, contracted in the former and expanded in the latter, while the *most* refrangible rays are expanded in the one and contracted in the other. In consequence of this difference in the colored spaces, the middle or mean ray *m n* does not pass through the same color in both spectra. In the *oil of cassia* spectrum it is in the *blue* space, and in the *sulphuric acid* spectrum it is in the *green* space. As the colored spaces have not the same ratio to one another as the lengths of the spectra which they compose, this property has been called the *irrationality* of *dispersion*, or of the colored spaces in the spectrum.

Fig. 52.



In order to ascertain whether any prism contracts or expands the least refrangible rays more than another, or which of them acts most on green light, take a prism of each with such angles that they correct each other's dispersion as much as possible, or that they produce spectra of the same length. If, through the prisms placed with their refracting angles in opposite directions, we look at the bar of the window parallel to the base of the prism, we shall see its edges perfectly free from color, provided the two prisms act equally upon green light. But if they act differently on green light, the bar will have a fringe of purple on one side, and a fringe of green on the other; and the green fringe will always be on the same side of the bar as the vertex of the prism which contracts the yellow space and expands the blue and violet ones. That is, if the prisms are flint and crown glass, the uncorrected green fringe will be on the lower side of the bar when the vertex of the flint glass prism points downwards. Flint glass, therefore, has a less action upon green light than crown glass, and contracts in a greater degree the red and yellow spaces. See Appendix, No. II.

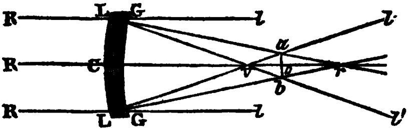
CHAP. IX.

ON THE PRINCIPLE OF ACHROMATIC TELESCOPES.

In treating of the progress of rays through lenses, it was taken for granted that the light was homogeneous, and that every ray that had the same angle of incidence had also the same angle of refraction; or, what is the same thing, that every ray which fell upon the lens had the same index of refraction. The observations in the two preceding chapters have, however, proved that this is not true, and that, in the case of light falling upon crown glass, there are rays with every possible index of refraction from 1·5258, the index of refraction for the red, to 1·5466, the index of refraction for the violet rays. As the light of the sun, by which all the bodies of nature are rendered visible, is white, this property of light, viz. the different refrangibility of its parts, affects greatly the formation of images by lenses of all kinds.

In order to explain this, let L L be a convex lens of crown glass, and R L, R L rays of white light incident upon it parallel to its axis R *r*. As each ray R L of white light consists of seven differently colored rays having different degrees of refrangibility or different indices of refraction, it is evident that all the rays which compose R L cannot possibly be refracted in the same direction, so as to fall upon one point. The extreme *red* rays, for example, in R L, R L, whose index of refraction is 1·5258, if traced through the lens by the method formerly given, will be found to have their focus in *r*, and C *r* will be the focal length of the lens for red rays. In like manner the extreme *violet* rays, which have a greater index of refraction, or 1·5466, will be refracted to a focus *v* much nearer the lens, and C *v* will be the focal length of the lens for *violet* rays. The distance *v r* is called the chromatic aberration, and the circle whose diameter is *a b* passing through the focus of the mean refrangible rays at *o*, is called the circle of least aberration.

Fig. 53.



These effects may be shown experimentally by exposing the lens L L to the parallel rays of the sun. If we receive the image of the sun on a piece of paper placed between *o* and C, the luminous circle on the paper will have a *red* border, because it is a section of the cone L *a b* L, the exterior rays of which L *a*, L *b* are red; but if the paper is placed at any greater distance than *o*, the luminous circle on the paper will have a *violet* border, because it is a section of the cone *l a b l'*, the exterior rays of which *a l*, *b l'* are violet, being a continuation of the violet rays L *v*, L *v*. As the spherical aberration of the lens is here combined with its chromatic aberration, the undisguised effect of the latter will be better seen by taking a large convex lens L L, and covering up all the central part, leaving only a small rim round its circumference at L L, through which the rays of light may pass. The refraction of the differently colored rays will be then finely displayed by viewing the image of the sun on the different sides of *a b*.

It is clear from these observations that the lens will form a violet image of the sun at *v*, a red image at *r*, and images of the other colors in the spectrum at intermediate points between *r* and *v*; so that if we place the eye behind these images, we shall see a confused image, possessing none of that sharpness and distinctness which it would have had if formed only by one kind of rays.

The same observations are true of the refraction of white light by a concave lens; only in this case the parallel rays which such a lens refracts diverge, as if they proceeded from separate foci, *v* and *r*, in front of the lens.

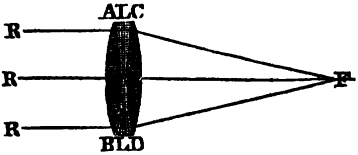
If we now place behind L L a concave lens G G of the same glass, and having its surfaces ground to the same curvature, it is obvious that since *v* is its virtual focus for violet, and *r* its virtual focus for *red* rays, if the paper is held at *a b*, the focus of the mean refrangible rays, where the *violet* and *red* rays cross at *a* and *b*, the image will be more distinct than in any other position; and when rays converge to the focus of any concave lens, they will be refracted into parallel directions; that is, the concave lens will refract these converging rays into the parallel lines G *l*, G *l*, and they will again form white light. That the *red* and *violet* rays will be thus reunited in one, viz. G *l*, may be proved by projecting them; but it is obvious also from the consideration that the two lenses L L, G G actually form a piece of parallel glass, the outer concave surface of G G being parallel to the outer convex surface of L L.

(67.) But though we have thus corrected the color produced by L L, by means of the lens G G, we have done this by a useless combination; since the two together act only like a piece of plane glass, and are incapable of forming an image. If we make the concave lens G G, however, of a longer focus than L L, the two together will act as a convex lens, and will form images behind it, as the rays G *l*, G *l* will now converge to a focus behind L L. But as the chromatic aberration of the lens G G will now be less than that of L L, the one will not correct or compensate the other; so that the difference between the two aberrations will still remain. Hence it is impossible, by means of *two lenses of the same glass*, to form an image which shall be free from color.

As Sir Isaac Newton believed that all substances whatever produced the same quantity of color, or had the same chromatic aberration when formed into lenses, he concluded that it was impossible, by the combination of a concave with a convex glass, to produce refraction without color. But we have already seen that the premises from which this conclusion was drawn are incorrect, and that bodies have different dispersive powers, or produce different degrees of color at the same mean refraction. Hence it follows that different lenses may produce the same degree of color when they have different focal lengths; so that if the lens L L is made of *crown glass*, whose index of refraction is 1·519, and dispersive power 0·036, and the lens G G of *flint glass*, whose index of refraction is 1·589, and dispersive power 0·0393, and if the focal length of the convex crown-glass lens is made 4½ inches, and that of the concave flint-glass lens 7¾ inches, they will form a lens with a focal length of 10 inches, and will refract white light to a single focus free of color. Such a lens is called an *achromatic lens*; and when used as a telescope, with another glass to magnify the colorless image which it forms of distant objects, it constitutes the *achromatic telescope*, one of the greatest inventions of the last century. Although Newton, reasoning from his imperfect knowledge of the dispersive power of bodies, pronounced such an invention to be hopeless; yet, in a short time after the death of that great philosopher, it was accomplished by a Mr. Hall, and afterwards by Mr. Dollond, who brought it to a high degree of perfection.

The image formed by an achromatic lens thus constructed would have been perfect if the equal spectra formed by the crown and flint glass were in every respect similar: but as we have seen that the colored spaces in the one are not equal to the colored spaces in the other, a secondary spectrum is left; and therefore the images of all luminous objects, when seen through such a lens, will be bordered on one side with a *purple* fringe, and on the other with a *green* fringe. If two substances could be found of different refractive and dispersive powers, and capable of producing equal spectra, in which the colored spaces were equal, a perfect achromatic lens would be produced: but, as no such substances have yet been found, philosophers have endeavored to remove the imperfection by other means; and Doctor Blair had the merit of surmounting the difficulty. He found that muriatic acid had the property of producing a primary spectrum, in which the green rays were among the most refrangible, something like C D, *fig.* 52., as in crown glass. But as muriatic acid has too low a refractive and dispersive power to fit it for being used as a concave lens along with a convex one of crown glass, he therefore conceived the idea of increasing the refractive and dispersive powers of the muriatic acid, by mixing it with metallic solutions, such as muriate of antimony; and he found he could do this to the requisite extent without altering its law of dispersion, or the proportion of the colored spaces in its spectrum. By inclosing, therefore, muriate of antimony, L L, between two convex lenses of crown glass, as A B, C D in *fig.* 54., Doctor Blair succeeded in refracting parallel rays R A, R B to a single focus F, without the least trace of secondary color. Before he discovered this property of the muriatic acid, he had contrived another, though a more complicated combination, for producing the same effect; but as he preferred the combination which we have described, and employed it in the best aplanatic object-glasses which he constructed, it is unnecessary to dwell any longer upon the subject.

Fig. 54.



In these observations, we have supposed that the lenses which are combined have no spherical aberration; but though this is not the case, the combination of concave with convex surfaces, when properly adjusted, enables us completely to correct the spherical along with the chromatic aberration of lenses.

In the course of an examination of the secondary spectra produced by different combinations, I was led to the conclusion that there may be refraction without color, by means of two prisms, and that two lenses may converge white light to one focus, even though the prisms and the lenses are made of the same kind of glass. When one prism of a different angle is thus made to correct the dispersion of another prism, a *tertiary spectrum* is produced, which depends wholly on the angles at which the light is refracted at the two surfaces of the prisms. See *Treatise on New Philosophical Instruments*, p. 400.

CHAP. X.

ON THE PHYSICAL PROPERTIES OF THE SPECTRUM.

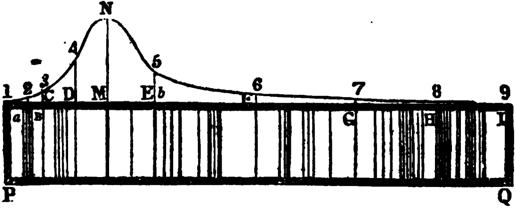
(68.) In the preceding chapter we have considered only those general properties of the solar spectrum on which the construction of achromatic lenses depends. We shall now proceed to take a general view of all its physical properties.

On the Existence of Fixed Lines in the Spectrum.

In the year 1802, Dr. Wollaston announced that in the spectrum formed by a fine prism of flint glass, free from veins, when the luminous object was a slit, the twentieth of an inch wide, and viewed at the distance of 10 or 12 feet, there were two fixed dark lines, one in the green and the other in the blue space. This discovery did not excite any attention, and was not followed out by its ingenious author.

Without a knowledge of Dr. Wollaston's observation, the late celebrated M. Fraunhofer, of Munich, by viewing through a telescope the spectrum formed from a narrow line of solar light by the finest prisms of flint glass, discovered that the surface of the spectrum was crossed throughout its whole length by dark lines of different breadths. None of these lines coincide with the boundaries of the colored spaces. They are nearly 600 in number: the largest of them subtends an angle of from 5″ to 10″. From their distinctness, and the facility with which they may be found, seven of these lines, viz. B, C, D, E, F, G, H, have been particularly distinguished by M. Fraunhofer. Of these B lies in the *red* space, near its outer end; C, which is broad and black, is beyond the middle of the *red;* D is in the *orange*, and is a strong double line, easily seen, the two lines being nearly of the same size, and separated by a bright one; E is in the *green*, and consists of several, the middle one being the strongest; F is in the *blue*, and is a very strong line; G is in the *indigo*, and H in the *violet.* Besides these lines there are others which deserve to be noticed. At A is a well defined dark line within the red space, and half-way between A and B is a group of seven or eight, forming together a dark band. Between B and C there are 9 lines; between C and D there are 30; between D and E there are 84 of different sizes. Between E and *b* there are 24, at *b* there are three very strong lines, with a fine clear space between the two widest; between *b* and F there are 52; between F and G 185; and between G and H 190, many being accumulated at G.

Fig. 55.



These lines are seen with equal distinctness in spectra produced by all solid and fluid bodies, and, whatever be the lengths of the spectra and the proportion of their colored spaces, the lines preserve the same relative position to the boundaries of the colored spaces; and therefore their proportional distances vary with the nature of the prism by which they are produced. Their number, however, their order, and their intensity are absolutely invariable, provided light coming either directly* or indirectly from the sun be employed. Similar bands are perceived in the light of the *planets* and *fixed stars*, of *colored flames*, and of the *electric spark.* The spectra from the light of *Mars* and from that of *Venus* contain the lines D, E, *b*, and F in the same positions as in sunlight. In the spectrum from the light of *Sirius*, no fixed lines could be perceived in the *orange* and *yellow* spaces; but in the *green* there was a very strong streak, and two other very strong ones in the *blue.* They had no resemblance, however, to any of the lines in planetary light. The star *Castor* gives a spectrum exactly like that of *Sirius*, the streak in the *green* being in the very same place. The streaks were also seen in the *blue*, but Fraunhofer could not ascertain their place. In the spectrum of *Pollux* there were many weak but fixed lines, which looked like those in Venus.

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* Fraunhofer found the very same lines in moonlight.

It had the line D, for example, in the very same place as in the light of the planets. In the spectrum of *Capella* the lines D and *b* are seen as in the sun's light. The spectrum of *Betalgeus* contains numerous fixed lines sharply defined, and those at D and *b* are precisely in the same places as in sun-light. It resembles the spectrum of Venus. In the spectrum of *Procyon* Fraunhofer saw the line D in the orange; but though he observed other lines, yet he could not determine their place with any degree of accuracy. In the spectrum of electric light there is a great number of bright lines. The spectrum from the light of a lamp *contains none of the dark fixed lines* seen in the spectrum from sun-light; but there is in the *orange* a bright line which is more distinct than the rest of the spectrum. It is a double line, and occurs at the same place where D is found in the solar spectrum. The spectrum from the light of a flame maintained by the blowpipe contains several distinct bright lines.*

(69.) One of the most important practical results of the discovery of these fixed lines in the solar spectrum is, that they enable us to take the most accurate measures of the refractive and dispersive powers of bodies, by measuring the distances of the lines B, C, D, &c. Fraunhofer computed the table of the indices of refraction of different substances, given in the Appendix, No. III. From the numbers in the table here referred to we may compute the ratios of the dispersive powers of any two of the substances, by the method already explained in a preceding chapter.

On the Illuminating Power of the Spectrum.

(70.) Before the time of M. Fraunhofer, the illuminating power of the different parts of the spectrum had been given only from a rude estimate. By means of a *photometer* he obtained the following results:—

The place of maximum illumination he found to be at M, *fig.* 55., so situated that D M was about one third or one fourth of D E; and therefore this place is at the boundary of the orange and yellow. Calling the illuminating power at M, where it is a maximum, 100, then the light of other points will be as follows:—

Light at the red extremity	0·0	Light at F	17·00
——— B	3·2	——— G	3·10
——— C	9·4	——— H	0·56
——— D	64·0	——— the violet extremity	0·00
Maximum light at M	100·0		
Light at E	48·0		

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* See *The Edinburgh Journal of Science*, No. XV. p. 7.

Calling the intensity of the light in the brightest space D E 100, Fraunhofer found the light to have the following intensity in the other spaces :—

Intensity of light in B C	2·1	Intensity of light in E F	32·8
———— C D	29·9	———— F G	18·5
———— D E	100·0	———— G H	3·5

From these results it follows that, in the spectrum examined by Fraunhofer, the most luminous ray is nearer the red than the violet extremity in the proportion of 1 to 3·5, and that the mean ray is almost in the middle of the *blue* space. As a great part, however, of the violet extremity of the spectrum is not seen under ordinary circumstances, these results cannot be applied to spectra produced under such circumstances.

On the Heating Power of the Spectrum.

(71.) It had always been supposed by philosophers that the heating power in the spectrum would be proportional to the quantity of light; and Landriani, Rochon, and Sennebier, found the *yellow* to be the warmest of the colored spaces. Dr. Herschel, however, proved by a series of experiments that the heating power gradually increased from the violet to the red extremity of the spectrum. He found also that the thermometer continued to rise when placed beyond the red end of the spectrum, where not a single ray of light could be perceived. Hence he drew the important conclusion, *that there were invisible rays in the light of the sun, which had the power of producing heat, and which had a less degree of refrangibility than red light.* Dr. Herschel was desirous of ascertaining the refrangibility of the extreme invisible ray which possessed the power of heating, but he found this to be impracticable; and he satisfied himself with determining that, at a point 1½ inches distant from the extreme red ray, the invisible rays exerted a considerable heating power, even though the thermometer was placed at the distance of 52 inches from the prism.

These results were confirmed by Sir Henry Englefield, who obtained the following measures:—

	Temperature.		Temperature.
Blue	56°	Red	72°
Green	58	Beyond red . .	79
Yellow	62		

When the thermometer was returned from beyond the red into the red, it fell again to 72°.

M. Berard obtained analogous measures; but he found that the maximum of heat was at the very extremity of the red rays when the bulb of the thermometer was completely covered by them, and that beyond the red space the heat was only one fifth above that of the ambient air.

Sir Humphry Davy ascribed Berard's results to his using thermometers with circular bulbs, and of too large a size; and he therefore repeated the experiments in Italy and at Geneva, with very slender thermometers, not more than one twelfth of an inch in diameter, with very long bulbs filled with air confined by a colored fluid. The result of these experiments was a confirmation of those of Dr. Herschel.*

M. Seebeck, who has more recently studied this subject, has shown that the place of maximum heat in the spectrum varies with the substance of which the prism is made. The following are his results:—

Substance of the Prism.	Colored space in which the heat is a maximum.
Water	Yellow.
Alcohol	Yellow.
Oil of turpentine	Yellow
Sulphuric acid concentrated . .	Orange.
Solution of sal-ammoniac . . .	Orange.
Solution of corrosive sublimate .	Orange.
Crown glass	Middle of the red.
Plate glass	Middle of the red.
Flint glass	Beyond the red.

The observations on alcohol and oil of turpentine were made by M. Wunsch.†

On the Chemical Influence of the Spectrum.

(72.) It was long ago noticed by the celebrated Scheele, that *muriate of silver* is rendered much blacker by the *violet* than by any of the other rays of the spectrum. In 1801, M. Ritter of Jena, while repeating the experiments of Dr. Herschel, found that the muriate of silver became very soon black *beyond the violet extremity* of the spectrum. It became a little less blackened in the violet itself, still less in the *blue*, the blackening growing less and less towards the red extremity. When muriate of silver a little blackened was used, its color was partly restored when placed in the red space, and still more in the space of the invisible rays beyond the red. Hence he concluded that there are two sets of invisible rays in the solar spectrum, one on the red side which favors oxygenation, and the other on the violet side which favors disoxygenation. M. Ritter also found that phosphorus emitted white fumes in the invisible red; while in the invisible violet, phosphorus in a state of oxygenation was instantly extinguished.

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* See *Edinburgh Encyclopædia*, vol. x. p. 69., where they were first published, as communicated to me by Sir Humphry.

† For the recent observations of Signor Melloni, see Note IV. of Am. ed. which follows author's Appendix.

In repeating the experiments with muriate of silver, M. Seebeck found that its color varied with the colored space in which it was held. In and beyond the *violet*, it was *reddish brown;* in the *blue*, it was *blue* or *bluish* grey; in the *yellow*, it was white, either unchanged or faintly tinged with *yellow;* and in and beyond the *red* it was *red*. In prisms of flint glass, the muriate was decidedly colored beyond the limits of the spectrum.

Without knowing what had been done by Ritter, Dr. Wollaston obtained the very same results respecting the action of violet light on muriate of silver. In continuing his experiments, he discovered some new chemical effects of light upon *gum guaiacum*. Having dissolved some of this gum in alcohol, and washed a card with the tincture, he exposed it in the different colored spaces of the spectrum without observing any change of color. He then took a lens 7 inches in diameter, and having covered the central part of it so as to leave only a ring of one tenth of an inch at its circumference, he could collect the rays of any color in a focus, the focal distance being about 24½ inches for yellow light. The card washed with guaiacum was then cut in small pieces, which were placed in the different rays concentrated by the lens. In the violet and blue rays it acquired a *green* color. In the *yellow* no effect was produced. In the *red* rays, pieces of the card already made *green* lost their green color, and were restored to their original hue. The guaiacum card, when placed in carbonic acid gas, could not be rendered green at any distance from the lens, but was speedily restored from green to yellow by the red rays. Dr. Wollaston also found that the back of a heated silver spoon removed the green color as effectually as the red rays.

On the Magnetizing Power of the Solar Rays.

(73.) Dr. Morichini, more than twenty years ago, announced that the violet rays of the solar spectrum had the power of magnetizing small steel needles that were entirely free from magnetism. This effect was produced by collecting the violet rays in the focus of a convex lens, and carrying the focus of these rays from the middle of one half of the needle to the extremities of that half, without touching the other half. When this operation had been performed for an hour, the needle had acquired perfect polarity. MM. Carpa and Ridolfi repeated this experiment with perfect success; and Dr. Morichini magnetized several needles in the presence of Sir H. Davy, Professor Playfair, and other English philosophers. M. Berard at Montpelier, M. Dhombre Firmas at Alais, and professor Configliachi at Pavia, having failed in producing the same effects, a doubt was thus cast over the accuracy of preceding researches.

A few years ago, Dr. Morichini's experiment was restored to credit by some ingenious experiments by Mrs. Somerville. Having covered with paper half of a sewing needle, about an inch long, and devoid of magnetism, and exposed the other half uncovered to the violet rays, the needle acquired magnetism in about two hours, the exposed end exhibiting north polarity. The indigo rays produced nearly the same effect, and the blue and green produced it in a less degree. When the needle was exposed to the yellow, orange, red, or calorific rays beyond the red, it did not receive the slightest magetism, although the exposures lasted for three days. Pieces of clock and watch springs gave similar results; and when the violet ray was concentrated with a lens, the needles, &c., were magnetized in a shorter time. The same effects were produced by exposing the needles half covered with paper to the sun's rays transmitted through glass colored *blue* with cobalt. Green glass produced the same effect. The light of the sun transmitted through blue and green riband produced the same effect as through colored glass. When the needles thus covered had hung a day in the sun's rays behind a pane of glass, their exposed ends were north poles, as formerly.

In repeating Mrs. Somerville's experiments, M. Baumgartner of Vienna discovered that a steel wire, some parts of which were polished, while the rest were without lustre, became magnetic by exposure to the white light of the sun; a north pole appearing at each polished part, and a south pole at each unpolished part. The effect was hastened by concentrating the solar rays upon the steel wire. In this way he obtained 8 poles on a wire eight inches long. He was not able to magnetize needles perfectly oxidated, or perfectly polished, or having polished lines in the direction of their lengths.

About the same time, Mr. Christie of Woolwich found that when a magnetized needle, or a needle of copper or glass, vibrated by the force of torsion in the white light of the sun, the arch of vibration was more rapidly diminished in the sun's light than in the shade. The effect was greatest on the magnetized needle. Hence he concludes that the compound solar rays possess a very sensible magnetic influence.

These results have received a very remarkable confirmation from the experiments of M. Barlocci and M. Zantedeschi. Professor Barlocci found that an armed natural loadstone, which could carry 1½ Roman pounds, had its power nearly *doubled* by twenty-four hours' exposure to the strong light of the sun. M. Zantedeschi found that an artificial horse-shoe loadstone, which carried 13½ oz., carried 3½ more by three days' exposure, and at last supported 31 oz., by continuing it in the sun's light. He found, that while the strength increased in oxidated magnets, it diminished in those which were not oxidated, the diminution becoming insensible when the loadstone was highly polished. He now concentrated the solar rays upon the loadstone by means of a lens; and he found that, both in oxidated and polished magnets, they *acquire* strength when their *north* pole is exposed to the sun's rays, and *lose* strength when the south pole is exposed. He found likewise that the augmentation in the first case exceeded the diminution in the second. M. Zantedeschi repeated the experiments of Mr. Christie on needles vibrating in the sun's light; and he found that, by exposing the north pole of a needle a foot long, the semi-amplitude of the last oscillation was 6° less than the first; while, by exposing the south pole, the last oscillation became greater than the first. M. Zantedeschi admits that he often encountered inexplicable anomalies in these experiments.*

Decisive as these results seem to be in favor of the magnetizing power both of violet and white light, yet a series of apparently very well conducted experiments have been lately published by MM. Riess and Moser,† which cast a doubt over the researches of preceding philosophers. In these experiments, they examined the number of oscillations performed in a given time *before* and *after* the needle was submitted to the influence of the violet rays. A focus of violet light concentrated by a lens 1·2 inches in diameter, and 2·3 inches in focal length, was made to traverse one half of the needle 200 times; and though this experiment was repeated with different needles, at different seasons of the year, and different hours of the day, yet the duration of a given number of oscillations was almost exactly the same after as before the experiment. Their attempts to verify the results of Baumgartner were equally fruitless; and they therefore consider themselves as entitled *to reject totally a discovery, which for seventeen years has at different times disturbed science.* "The small variations," they observe, "which are found in some of our experiments, cannot constitute a real action of the nature of that which was observed by MM. Morichini, Baumgartner, &c., in so clear and decided a manner."

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* *Edinburgh Journal of Science*, New Series, No. V., p. 76.

† Id. No. IV., p. 225.

CHAP. XI.

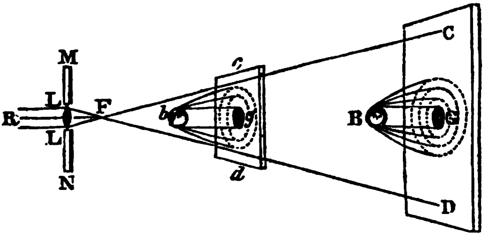
ON THE INFLEXION OR DIFFRACTION OF LIGHT.

(74.) Having thus described the changes which light experiences when refracted by the surfaces of transparent bodies, and the properties which it exhibits when thus decomposed into its elements, we shall now proceed to consider the phenomena which it presents when passing near the edges of bodies. This branch of optics is called the *inflexion* or the *diffraction* of light.

This curious property of light was first described by Grimaldi in 1665, and afterwards by Newton; but it is to the late M. Fresnel that we are indebted for a most successful and able investigation of the phenomena.

In order to observe the action of bodies upon the light which passes near them, let a lens L L, of very short focus, *fig.* 56., be fixed in the window-shutter, M N, of a dark room; and let R L L be a beam of the sun's light, transmitted through the lens. This light will be collected into a focus at F, from which it will diverge in lines F C, F D, forming a circular image of light on the opposite wall. If a small hole, about the fortieth of an inch in diameter, had been fixed in the window-shutter in place of the lens, nearly the same divergent

Fig. 56.



beam of light would have been obtained. The shadows of all bodies whatever held in this light will be found to be surrounded with three fringes of the following colors, reckoning from the shadow:—

First fringe.—Violet, indigo, pale blue, green, yellow, red.
Second fringe.—Blue, yellow, red.
Third fringe.—Pale blue, pale yellow, pale red.

In order to examine these fringes, we may either receive them on a smooth white surface as Newton did, or adopt the method of Fresnel, who looked at them with a magnifying glass, in the same manner as if they had been an image formed by a lens. This last method is decidedly the best, as it enables the observer to measure the fringes, and ascertain the changes which they undergo under different circumstances.

Let a body B be now placed at the distance B F from the focus, and let its shadow be received on the screen C D, at a fixed distance from the body B, and the following phenomena will be observed:—

1. Whatever be the nature of the body B with regard to its density or refractive power, whether it is platina or the pith of a rush, whether it is tabasheer or chromate of lead, the fringes surrounding its shadow will be the very same in magnitude and in color, and the colors will be those given above.

2. If the light R L is homogeneous light of the different colors in the spectrum, the fringes will be of the same color as the light R L; and they will be *broadest* in *red* light, smallest in *violet*, and of intermediate sizes in the intermediate colors.

3. The body B continuing fixed, let us either bring the screen C D nearer to B, or bring the lens with which we view the fringes nearer to B, so as to see them at different distances behind B. It will be found that they grow less and less as they approach the edge of B, from which they take their rise. But if we measure the distances of any one fringe from the shadow at different distances behind B, we shall find that the line joining the same point of the fringe is not a straight line, but a hyperbola whose vertex is at the edge of the body; so that the same fringe is not formed by the same light at all distances from the body, but resembles a caustic curve formed by the intersection of different rays. This curious fact we have endeavored to represent in the figure by the hyperbolic curves joining the edge of the body B and the fringes which are shown by dotted lines.

4. Hitherto we have supposed that B has been held at the same distance from F; but let it now be brought to *b*, much

nearer F, and let the screen C D be brought to *c d*, so that *b g* is equal B G. In this new position, where nothing has been changed but the distance from F, the fringes will be found greatly increased in breadth, their relative distances from each other and from the margin of the shadow remaining the same. The influence of distance from the radiant point F on the size of the fringes, or on the quantity of inflexion, is shown in the following results obtained by M. Fresnel:—

	Distance of the inflecting body B behind the radiant point F.	Distance B G or *b g* behind the body B or *b*, where the inflexion was measured.	Angular inflexion of the red rays of the first fringe.
F *b*	4 inches.	39 inches.	12′ 6″
F B	20 feet.	39 ——	3 55

When we consider that the fringes are largest in red, and smallest in violet light, it is easy to understand the cause of their colors in white light; for the colors seen in this case arise from the superposition of fringes of all the seven colors; that is, if the eye could receive all the seven differently colored fringes at once, these colors would form by their mixture the actual colors in the fringes seen by white light. Hence we see why the color of the first fringe is violet near the shadow, and red at a greater distance; and why the blending of the colors beyond the third fringe forms white light, instead of exhibiting themselves in separate tints.

Upon measuring the proportional breadths of the fringes with great care, Newton found that they were as the numbers 1, $\sqrt{\frac{1}{3}}$, $\sqrt{\frac{1}{5}}$, $\sqrt{\frac{1}{7}}$, and their intervals in the same proportion.

Besides the external fringes which surround all bodies, Grimaldi discovered within the shadows of long and narrow bodies a number of parallel streaks or fringes alternately light and dark. Their number grew smaller as the body tapered; and Dr. Young remarked that the central line was always white, so that there must always be an odd number of white stripes, and an even number of dark ones. At the angular termination of bodies these fringes widen and become convex to the central white line; and when the termination is rectangular, what are called the crested fringes of Grimaldi are produced.

The phenomena exhibited by substituting apertures of various forms in place of the body B are very interesting. When the aperture is circular, such as that formed in a piece of lead with a small pin, and when a lens is placed behind it so as to view the shadow at different distances, the aperture will be seen surrounded with distinct rings, which contract and dilate, and change their tints in the most beautiful manner. When the aperture is one thirtieth of an inch, its distance F B from the luminous point 6 feet 6 inches, and its distance from the focus of the eye-lens, or B G, 24 inches, the following series of rings was observed:—

1st order. White, pale yellow, yellow, orange, dull red.

2d order. Violet, blue, whitish, greenish yellow, yellow, bright orange.

3d order. Purple, indigo blue, greenish blue, bright green, yellow green, red.

4th order. Bluish green, bluish white, red.

5th order. Dull green, faint bluish white, faint red.

6th order. Very faint green, very faint red.

7th order. A trace of green and red.

When the aperture B is brought nearer to the eye-lens whose focus is supposed to be at G, the central white spot grows less and less till it vanishes, the rings gradually closing in upon it, and the centre assuming in succession the most brilliant tints. The following were the tints observed by Mr. Herschel; the distance between the radiant point F and the focus G of the eye-lens remaining constant, and the aperture, supposed to be at B, being gradually brought nearer to G:—

Distance of aperture B from the eye-lens.	Color of the Central spot.	Character of the rings which surround the central spot.
24 in.	White.	Rings as described above.
18	White.	First two rings confused. Red of 3d, and green of 4th order, splendid.
13·5	Yellow.	Inner rings diluted. Red and green of the outer rings good.
10	Intense orange.	All the rings much diluted.
9·25	Deep orange red.	Rings all very dilute.
9·10	Brilliant blood red.	Rings all very dilute.
8·75	Deep crimson red.	Rings all very dilute.
8·36	Deep purple.	Rings all very dilute.
8·00	Very sombre violet.	A broad yellow ring.
7·75	Intense indigo blue.	A pale yellow ring.
7·00	Pure deep blue.	A rich yellow.
6·63	Sky blue.	A ring of orange, with a sombre space.
6·00	Bluish white.	Orange red, with a pale yellow space.
5·85	Very pale blue.	A crimson red ring.
5·50	Greenish white.	Purple, with orange yellow.
5·00	Yellow.	Blue, orange.
4·75	Orange yellow.	Bright blue, orange red, pale yellow, white.
4·50	Scarlet.	Pale yellow, violet, pale yellow, white.
4·00	Red.	White, indigo, dull orange, white.
3·85	Blue.	White, yellow, blue, dull red.
3·50	Dark blue.	Orange, light blue, violet, dull orange.

When *two* small apertures are used instead of one, and the rings examined by the eye-lens as before, two systems of rings will be seen, one round each centre; but, besides the rings, there is another set of fringes which, when the apertures are equal, are parallel rectilineal fringes equidistant from the two centres, and perpendicular to the line joining these centres. Two other sets of parallel rectilineal fringes diverge in the form of a St. Andrew's cross from the middle point between the two centres, and forming equal angles between the first set of parallel fringes. If the apertures are unequal, the two systems of rings are unequal, and the first set of parallel fringes become hyperbolas, concave towards the smaller system of rings, and having the aperture in their common focus.[*]

The finest experiments on this subject are those of Fraunhofer; but a proper view of them would require more space than we can spare.[†]

CHAP. XII.

ON THE COLORS OF THIN PLATES.

(75.) When light is either reflected from the surfaces of transparent bodies, or transmitted through portions of them with parallel surfaces, it is invariably white, for all the different thicknesses of such bodies as we are in the habit of seeing. The thinnest films of blown glass, and the thinnest films of mica generally met with, will both reflect and transmit white light. If we diminish, however, the thickness of these two bodies to a certain degree, we shall find that, instead of giving white light by reflexion and transmission, the light is in both cases colored.

Mr. Boyle seems first to have observed that thin bubbles of the essential oils, spirit of wine, turpentine, and soap and water, exhibited beautiful colors; and he succeeded in blowing glass so thin as to show the same tints. Lord Brereton had observed the colors of the thin oxidated films which the action of the weather produces upon glass; and Dr. Hooke obtained films so equally thin that they exhibited over their whole surface the same brilliant color. Such pieces of mica may be produced at the edges of plates quickly detached from a mass; but they may be more readily obtained by sticking one side of a plate of mica to a piece of sealing-wax, and tearing it away with a sudden jerk. Some extremely thin films will then be left on the wax, which will exhibit the liveliest colors by reflected light. If we could produce a film of mica with only one tenth part of the thickness of that which produces a bright blue color, this film would reflect no light at all, and would appear black if viewed by reflexion against a black body. But though no such film has ever been obtained, or is likely to be obtained by any means with which we are acquainted, yet accident on one occasion produced solid fibres as thin, and actually incapable of reflecting light. This very remarkable fact occurred in a crystal of quartz of a smoky color, which was broken in two. The two surfaces of fracture were absolutely black; and the blackness appeared, at first sight, to be owing to a thin film of opaque matter which had insinuated itself into the crevice. This opinion, however, was untenable, as every part of the surface was black, and the two halves of the crystals could not have stuck together had the crevice extended across the whole section. Upon examining this specimen with care, I found that the surface was perfectly transparent by transmitted light, and that the blackness of the surfaces arose from their being entirely composed of a fine down of quartz, or of short and slender filaments, whose diameter was so exceedingly small that they were incapable of reflecting a single ray of the strongest light. The diameter of these fibres was so small, that, from principles which we shall presently explain, they could not exceed the one third of the millionth part of an inch. This curious specimen is in the cabinet of her grace the duchess of Gordon.* I have another small specimen in my own possession; and I have no doubt that fractures of quartz and other minerals will yet be found which shall exhibit a fine down of different colors depending on their size.

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[*] Herschel's *Treatise on Light*, § 735.

[‡] See *Edinburgh Encyclopædia*, art. Optics, Vol. XV., p. 556.

The colors thus produced by thinness, and hence called the *colors of thin plates*, are best observed in fluid bodies of a viscous nature. If we blow a soap-bubble, and cover it with a clear glass to protect it from currents of air, we shall observe, after it has grown thin by standing a little, a great many concentric colored rings round the top of it. The color in the centre of the rings will vary with the thickness; but as the bubble grows thinner the rings will dilate, the central spot will become white, then bluish, and then black, after which the bubble will burst, from its extreme thinness at the place of the black spot. The same change of color with the thickness may be seen by placing a thick film of an evaporable fluid upon a clean plate of glass, and watching the effects of the diminution of thickness which take place in the course of evaporation.

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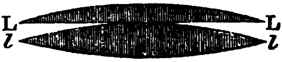
* See *Edinburgh Journal of Science*, No. I., p. 108.

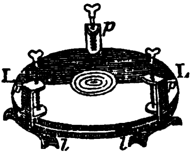
The method used by Sir Isaac Newton for producing a thin plate of air, the colors of which he intended to investigate, is shown in *fig*. 57., where L L is a plano-convex lens, the radius of whose convex surface is 14 feet, and *l l* a double convex lens, whose convex surfaces have a radius of 50 feet each. The plane side of the lens L L was placed downwards, so as to rest upon one of the surfaces of the lens *l l*. These lenses obviously touch at their middle points; and if the upper one is slowly pressed against the under one, there will be seen round the point of contact a system of circular colored rings, extending wider and wider as the pressure is increased. In order to examine these rings under different degrees of pressure, and when the lenses L L, *l l* are at different distances, three clamp-screws, *p*, *p*, *p*, should be employed, as shown in *fig*. 58., by turning which we may produce a regular and equal pressure at the point of contact.

Fig. 57.



When we look at these rings through the upper lens, so as to see those formed by the light *reflected* from the plate of air between the lenses, we may observe *seven* rings, or rather seven circular spectra or orders of colors, as described by Newton in the first two columns of the following Table; the colors being very distinct in the first three spectra, but growing more and more diluted in the others, till they almost entirely disappear in the seventh spectrum.

Fig. 58.



When we view the plate of air by looking through the under lens *l l* from below, we observe another set of rings or spectra formed in the transmitted light. Only five of these transmitted rings are distinctly seen, and their colors, as observed by Newton, are given in the third column of the following table; but they are much more faint than those seen by reflexion. By comparing the colors seen by reflexion with those seen by transmission, it will be observed that the color transmitted is always complementary to the one reflected, or which, when mixed with it, would make white light.

Table of the Colors of Thin Plates of Air, Water, and Glass.

Spectra, or Orders of Colors, reckoned from the centre.	Colors produced at the thicknesses in the last three columns.		Thicknesses in millionths of an inch.		
	Reflected.	*Transmitted.*	Air.	Water.	Glass.
First Spectrum or order of Colors.	Very black	- - - - -	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{10}{31}$
	Black	White	1	$\frac{3}{4}$	$\frac{20}{31}$
	Beginning of black	- - - - -	2	$1\frac{1}{2}$	$1\frac{2}{7}$
	Blue	Yellowish red	$2\frac{2}{5}$	$1\frac{4}{5}$	$1\frac{11}{20}$
	White	Black	$5\frac{1}{4}$	$3\frac{7}{8}$	$3\frac{2}{5}$
	Yellow	Violet	$7\frac{1}{9}$	$5\frac{1}{3}$	$4\frac{3}{5}$
	Orange	- - - - -	8	6	$5\frac{1}{6}$
	Red	Blue	9	$6\frac{3}{4}$	$5\frac{4}{5}$
Second Spectrum or order of Colors.	Violet	White	$11\frac{1}{6}$	$8\frac{3}{8}$	$7\frac{1}{5}$
	Indigo	- - - - -	$12\frac{5}{6}$	$9\frac{5}{8}$	$8\frac{2}{11}$
	Blue	Yellow	14	$10\frac{1}{2}$	9
	Green	Red	$15\frac{1}{8}$	$11\frac{1}{3}$	$9\frac{5}{7}$
	Yellow	Violet	$16\frac{2}{7}$	$12\frac{1}{5}$	$10\frac{2}{5}$
	Orange	- - - - -	$17\frac{2}{9}$	13	$11\frac{1}{6}$
	Bright red	Blue	$18\frac{1}{3}$	$13\frac{3}{4}$	$11\frac{5}{6}$
	Scarlet	- - - - -	$19\frac{2}{3}$	$14\frac{3}{4}$	$12\frac{2}{3}$
Third Spectrum or order of Colors.	Purple	Green	21	$15\frac{3}{4}$	$13\frac{11}{20}$
	Indigo	- - - - -	$22\frac{1}{10}$	$16\frac{4}{7}$	$14\frac{1}{4}$
	Blue	Yellow	$23\frac{2}{5}$	$17\frac{11}{20}$	$15\frac{1}{10}$
	Green	Red	$25\frac{1}{5}$	$18\frac{9}{10}$	$16\frac{1}{4}$
	Yellow	- - - - -	$27\frac{1}{7}$	$20\frac{1}{3}$	$17\frac{1}{2}$
	Red	Bluish green	29	$21\frac{3}{4}$	$18\frac{5}{7}$
	Bluish red	- - - - -	32	24	$20\frac{2}{3}$
Fourth Spectrum or order of Colors.	Bluish green	- - - - -	34	$25\frac{1}{2}$	22
	Green	Red	$35\frac{2}{7}$	$26\frac{1}{2}$	$22\frac{3}{4}$
	Yellowish green	- - - - -	36	27	$23\frac{2}{9}$
	Red	Bluish green	$40\frac{1}{3}$	$30\frac{1}{4}$	26
Fifth Spectrum or order of Colors.	Greenish blue	Red	46	$34\frac{1}{2}$	$29\frac{2}{3}$
	Red	- - - - -	$52\frac{1}{2}$	$39\frac{3}{8}$	34
Sixth Spectrum or order of Colors.	Greenish blue	- - - - -	$58\frac{3}{4}$	44	38
	Red	- - - - -	65	$48\frac{3}{4}$	42
Seventh Spectrum or order of Colors.	Greenish blue	- - - - -	71	$53\frac{1}{4}$	$45\frac{4}{5}$
	Ruddy white	- - - - -	77	$57\frac{3}{4}$	$49\frac{2}{3}$

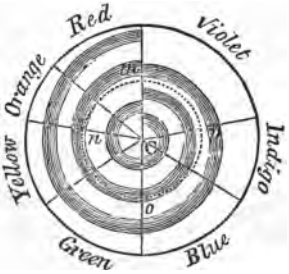
The preceding colors are those which are seen when light is reflected and transmitted nearly perpendicularly; but Sir Isaac Newton found that when the light was reflected and transmitted obliquely, the rings increased in size, the same color requiring a greater thickness to produce it. The color of any film, therefore, will descend to a color lower in, or nearer the beginning of, the scale, when it is seen obliquely.

Such are the general phenomena of the colored rings when seen by *white* light. When we place the lenses in homogeneous light, or make the different colors of the solar spectrum pass in succession over the lenses, the rings, which are always of the same color as the light, will be found to be largest in red light, and to contract gradually as they are seen in all the succeeding colors, till they reach their smallest size in the violet rays. Upon measuring their diameters, Newton found them to have the following ratio in the different colors at their boundaries:—

Extreme Red.	Orange.	Yellow.	Green.	Blue.	Indigo.	Violet.	Extreme.
1	0·924	0·885	0·825	0·763	0·711	0·681	0·630

Since white light is composed of all the preceding colors, the rings seen by it will consist of all the seven differently colored systems of rings superposed as it were, and forming, by their union, the different colors in the Table. In order to explain this, we have constructed the annexed diagram, *fig.* 59., on the supposition that each ring or spectrum has the same breadth in homogeneous light which it actually has when it is formed between surfaces nearly flat, or when the thickness of the plate varies with the distance from the point of contact.* Let us then suppose that we form such a

Fig. 59.



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* This supposition is made in order to simplify the diagram.

system of rings with the seven colors of the spectrum, and that a sector is cut out of each system, and placed, as in the figure, round the same centre C. Let the angle of the red sector be 50°, of the orange 30°, the yellow 40°, the green 60°, the blue 60°, the indigo 40°, and the violet 80°, being 360° in all, so as to complete the circle. From the centre C set off the first, second, and third rings in all the sectors, with radii corresponding to the values in the preceding small Table. Thus, since the proportional diameters of the extreme red and the extreme orange are 1 and 0·924, the middle of the red will be in the middle between these numbers, or 0·962; and consequently the proportional diameter, or the radius of the first red ring for the middle of the red space R, will be 0·962. In like manner, the radius for the orange will be 0·904, for the yellow 0·855, for the green 0·794, for the blue 0·737, for the indigo 0·696, and for the violet 0·655. Let the red rings be colored red as they appear in the experiment, the orange rings orange, and so on, each color resembling that of the spectrum as nearly as possible. If we now suppose all these colored sectors to revolve rapidly round C as a centre, the effect of them all, thus mixed, should be the production of the colored rings as seen by white light. As the diameter of each ring varies from the beginning of the red space to the end of it, and so on with all the colors, the portion of the ring in each sector should be part of a spiral, and all these separate parts should unite in forming a single spiral, the red forming the commencement, and the violet the termination of the spiral for each ring.

This diagram enables us to ascertain the composition of any of the rings seen in white light. Let it be required, for example, to determine the color of the ring at the distance C *m* from the centre, *m* being in the middle of the second red ring. Round C as a centre, and with the radius C *m*, describe a circle, *m n o p*, and it will be seen from the different colors through which it passes what is its composition. It passes nearly through the very brightest* part of the second red ring, at *m*, and through a pretty bright part of the orange. It passes nearly through the bright part of the yellow, at *n*; through the brightest part of the green; through a less bright part of the blue; through a dark part of the indigo, at *p*; and through the darkest part of the third violet ring. If we knew the exact law according to which the brightness of any fringe varied from its darkest to its brightest point, it would thus be easy to ascertain with accuracy the number of rays of each color which entered into the composition of any of the rings seen by white light.

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* In the figure, the brightest part is the most shaded.

In order to determine the thickness of the plate of air by which each color was produced, Newton found the squares of the diameters of the brightest parts of each to be in the arithmetical progression of the odd numbers, 1, 3, 5, 7, 9, &c., and the squares of the diameters of the obscurest parts in the arithmetical progression of the even numbers, 2, 4, 6, 8, 10; and as one of the glasses was plane, and the other spherical, their intervals at these rings must be in the same progression. He then measured the diameter of the fifth dark ring, and found that *the thickness of the air at the darkest part of the* FIRST *dark ring, made by perpendicular rays*, was the $\frac{1}{89,000}$ part of an inch. He then multiplied this number by the progression 1, 3, 5, 7, 9, &c., and 2, 4, 6, 8, 10, and obtained the following results:—

	Thickness of the air at the most luminous part.	Thickness of the air at the most obscure part.
FIRST Ring - - -	$\frac{1}{178,000}$ - - -	$\frac{2}{178,000}$ or $\frac{1}{89,000}$
SECOND Ring - - -	$\frac{3}{178,000}$ - - -	$\frac{4}{178,000}$
THIRD Ring - - -	$\frac{5}{178,000}$ - - -	$\frac{6}{178,000}$
FOURTH Ring - - -	$\frac{7}{178,000}$ - - -	$\frac{8}{178,000}$

When Newton admitted water between the lenses, he found the colors to become fainter, and the rings smaller; and upon measuring the thicknesses of water at which the same rings were produced, he found them to be nearly as the index of refraction for air is to the index of refraction for water, that is, nearly as 1·000 to 1·336. From these data he was enabled to compute the three last columns of the Table given in page 93, which show the thicknesses in millionth parts of an inch at which the colors are produced in plates of air, water, and glass. These columns are of extensive use, and may be regarded as presenting us with a micrometer for measuring minute thicknesses of transparent bodies by their colors, when all other methods would be inapplicable.

We have already seen that when the thickness of the film of air is about $\frac{12}{178,000}$dth of an inch, which corresponds to the seventh ring, the colors cease to become visible, owing to the union of all the separate colors forming white light; but when the rings are seen in homogeneous light, they appear in much greater numbers, a dark and a colored ring succeeding each other to a considerable distance from the point of contact. In this case, however, when the rings are formed between object glasses, the thickness of the plate of air increases so rapidly that the outer rings crowd upon one another, and cease to become visible from this cause. This effect would obviously not be produced if they were formed by a solid film whose thickness varied by slow gradations. Upon this principle, Mr. Talbot has pointed out a very beautiful method of exhibiting these rings with plates of glass and other substances even of a tangible thickness. If we blow a glass ball so thin that it bursts,* and hold any of the fragments in the light of a spirit lamp with a salted wick, or in the light of any of the monochromatic lamps which I have elsewhere described, all of which discharge a pure homogeneous yellow light, the surface of these films will be seen covered with fringes alternately yellow and black, each fringe marking out by its windings the lines of equal thickness in the glass film. Where the thickness varies slowly, the fringes will be broad and easily seen; but where the variation takes place rapidly, the fringes are crowded together, so as to require a microscope to render them visible. If we suppose any of the films of glass to be only the thousandth part of an inch thick, the rings which it exhibits will belong to the 89th order; and if a large rough plate of this glass could be got with its thickness descending to the millionth part of an inch by slow gradations, the whole of those 89 rings, and probably many more, would be distinctly visible to the eye. In order to produce such effects, the light would require to be perfectly homogeneous.

The rings seen between the two lenses are equally visible whether air or any other gas is used, and even when there is no gas at all; for the rings are visible in the exhausted receiver of an air-pump.

CHAP. XIII.

ON THE COLORS OF THICK PLATES.

(76.) THE colors of thick plates were first observed and described by Sir Isaac Newton, as produced by concave glass mirrors. Admitting a beam of solar light, R, into a dark room, through an aperture a quarter of an inch in diameter formed in the window-shutter M N, he allowed it to fall upon a glass mirror, A B, a quarter of an inch thick, quicksilvered behind, having its axis in the direction R *r*, and the radius of the curvature of both its surfaces being equal to its distance behind the aperture. When a sheet of paper was placed on the window-shutter M N, with a hole in it to allow the sun beam to pass, he observed the hole to be surrounded with *four* or *five* colored rings, with sometimes traces of a sixth

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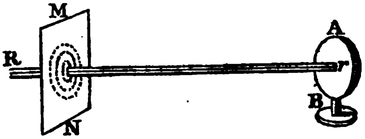
* Films of mica answer the purpose still better.

Fig. 60.



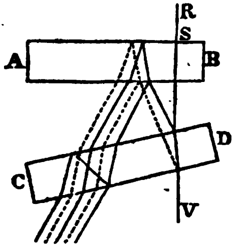
and seventh. When the paper was held at a greater or a less distance than the centre of its concavity, the rings became more dilute, and gradually vanished. The colors of the rings succeeded one another like those in the transmitted system in thin plates, as given in column 3d of the Table in page 93. When the light R was *red* the rings were red, and so on with the other colors, the rings being largest in *red* and smallest in *violet* light. Their diameters preserved the same proportion as those seen between the object glasses; the squares of the diameters of the most luminous parts (in homogeneous light) being as the numbers 0, 2, 4, 6, &c., and the squares of the diameters of the darkest parts as the intermediate numbers 1, 3, 5, 7, &c. With mirrors of greater thickness the rings grew less, and their diameters varied inversely as the square roots of the thickness of the mirror. When the quicksilver was removed, the rings became fainter; and when the back surface of the mirror was covered with a mass of oil of turpentine, they disappeared altogether. These facts clearly prove that the posterior surface of the mirror concurs with the anterior surface in the production of the rings.

When the mirror A B is inclined to the incident beam R *r*, the rings grow larger and larger as the inclination increases, and so also does the white round spot; and new rings of color emerge successively out of their common centre, and the white spot becomes a white ring accompanying them, and the incident and reflected beams always fall upon the opposite parts of this white ring, illuminating its perimeter like two mock suns in the opposite parts of an iris. The colors of these new rings were in a contrary order to those of the former.

The Duke de Chaulnes observed similar rings upon the surface of the mirror when it was covered with gauze or muslin, or with a skin of dried skimmed milk; and Sir W. Herschel noticed analogous phenomena when he scattered hair-powde in the air before a concave mirror on which a beam of light was incident, and received the reflected light on a screen.

(77.) The method which I have found to be the most simple for exhibiting these colors, is to place the eye immediately behind a small flame from a minute wick fed with oil or wax, so that we can examine them even at a perpendicular incidence. The colors of thick plates may be seen even with a common candle held before the eye at the distance of 10 or 12 feet from a common pane of crown glass in a window that has accumulated a little fine dust upon its surface, or that has on its surface a fine deposition of moisture. Under these circumstances they are very bright, though they may be seen even when the pane of glass is clean.

Fig. 61.



The colors of thick plates may, however, be best displayed, and their theory best studied, by using two plates of glass of equal thickness. The phenomena thus produced, and which presented themselves to me in 1817, are highly beautiful, and, as Mr. Herschel has shown, are admirably fitted for illustrating the laws of this class of phenomena. In order to obtain plates of exactly the same thickness, I formed out of the same piece of parallel glass two plates, A B, C D, and having placed between them two pieces of soft wax, I pressed them to the distance of about one tenth of an inch from each other; and by pressing above one piece of wax more than another, I was able to give the two plates any small inclination I chose. Let A B, C D then be a section of the two plates, thus inclined, at right angles to the common section of their surfaces, and let R S be a ray of light incident nearly in a vertical direction and proceeding from a candle, or, what is better, from a circular disc of condensed light subtending an angle of 2° or 3°. If we place the eye behind the plates, when they are parallel we shall see only an image of the circular disc; but when they are inclined, as in the figure, we shall observe in the direction V R several reflected images in a row besides the direct image. The first or the brightest of these will be seen crossed with fifteen or sixteen beautiful fringes or bands of color. The three central ones consist of blackish or whitish stripes; and the exterior ones of brilliant bands of *red* and *green* light. The direction of these bands is always parallel to the common section of the inclined plates. These colored bands increase in breadth by diminishing the inclination of the plates, and diminish by increasing their inclination. When the light of the luminous circular object falls obliquely on the first plate, so that the plane of incidence is at right angles to the section of the plates, the fringes are not distinctly visible across any of the images; but their distinctness is a maximum when the plane of incidence is parallel to that section. The reflected images of course become more bright, and the tints more vivid, as the angle of incidence becomes greater; when the angle of incidence increases from 0° to 90°, the images that have suffered the greatest number of reflexions are crossed by other fringes inclined to them at a small angle. If we conceal the bright light of the first image so as to perceive the image formed by a second reflexion within the first plate, and if we view the image through a small aperture, we shall observe colored bands across the first image far surpassing in precision of outline and richness of coloring any analogous phenomenon. When these fringes are again concealed, others are seen on the image immediately behind them, and formed by a third reflexion from the interior of the first plate.

If we bring the plate C D a little farther to the right hand, and make the ray R S fall first upon the plate C D, and be afterwards reflected back upon the first plate A B, from both the surfaces of C D, the same colored bands will be seen. The progress of the rays through the two plates is shown in the figure.

When the two plates have the form of concave and convex lenses, and are combined, as in the double and triple achromatic object glass, a series of the most splendid systems of rings are developed; and these are sometimes crossed by others of a different kind. I have not yet had leisure to publish an account of the numerous observations I have made on this curious class of phenomena.

In viewing films of blown glass in homogeneous yellow light, and even in common day-light, Mr. Talbot has observed that when two films are placed together, bright and obscure fringes, or colored fringes of an irregular form, are produced between them, though exhibited by neither of them separately.

CHAP. XIV.

ON THE COLORS OF FIBRES AND GROOVED SURFACES.

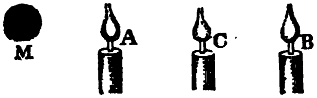
(78.) When we look at a candle or any other luminous body through a plate of glass covered with vapor or with dust in a finely divided state, it is surrounded with a corona or ring of colors, like a halo round the sun or moon. These rings increase as the size of the particles which produce them is diminished; and their brilliancy and number depend on the uniform size of these particles. Minute fibres, such as those of silk and wool, produce the same series of rings, which increase as the diameter of the fibres is less; and hence Dr. Young proposed an instrument called an *eriometer*, for measuring the diameters of minute particles and fibres, by ascertaining the diameter of any one of the series of rings which they produce. For this purpose, he selected the limit of the first red and green ring as the one to be measured. The eriometer is formed of a piece of card or a plate of brass, having an aperture about the fiftieth of an inch in diameter in the centre of a circle about half an inch in diameter, and perforated with about eight small holes. The fibres or particles to be measured are fixed in a slider, and the eriometer being placed before a strong light, and the eye assisted by a lens applied behind the small hole, the rings of colors will be seen. The slider must then be drawn out or pushed in till the limit of the red and green ring coincides with the circle of perforations, and the index will then show on the scale the size of the particles or fibres. The seed of the *lycoperdon bovista* was found by Dr. Wollaston to be the 8500dth part of an inch in diameter; and as this substance gave rings which indicated 3½ on the scale, it follows that 1 on the same scale was the 29750th part of an inch, or the 30,000dth part. The following Table contains some of Dr. Young's measurements, in thirty-thousandths of an inch:—

Milk diluted indistinct	3	Shawl wool	19
Dust of *lycoperdon bovista*	3½	Saxon wool	22
Bullock's blood	4½	Lioneza wool	25
Smut of barley	6½	Alpacca wool	26
Blood of a mare	6½	Farina of *laurestinus*	26
Human blood diluted with water	6	Ryeland Merino wool	27
		Merino South Down	28
Pus	7½	Seed of *lycopodium*	32
Silk	12	South Down ewe	39
Beaver's wool	13	Coarse wool	46
Mole's fur	16	Ditto from some worsted	60

(79.) By observing the colors produced by reflexion from the fibres which compose the crystalline lenses of the eyes of fishes and other animals, I have been able to trace these fibres to their origin, and to determine the number of poles or septa to which they are related. The same mode of observation, and the measurement of the distance of the first colored image from the white image, has enabled me to determine the diameters of the fibres, and to prove that they all taper like needles, diminishing gradually from the equator to the poles of the lens, so as to allow them to pack into a spherical superficies as they converge to their poles or points of origin. These colored images, produced by the fibres of the lens, lie in a line perpendicular to the direction of the fibres, and by taking an impression on wax from an indurated lens the colors are communicated to the wax. In several lenses I observed colored images at a great distance from the common image, but lying in a direction coincident with that of the fibres; and from this I inferred, that the fibres were crossed by joints or lines, whose distance was so small as the 11,000dth part of an inch; and I have lately found, by the use of very powerful microscopes, that each fibre has in this case teeth like those of a rack, of extreme minuteness, the colors being produced by the lines which form the sides of each tooth.

(80.) In the same class of phenomena we must rank the principal colors of mother-of-pearl. This substance, obtained from the shell of the pearl oyster, has been long employed in the arts, and the fine play of its colors is therefore well known. In order to observe its colors, take a plate of regularly formed mother-of-pearl, with its surfaces nearly parallel, and grind these surfaces upon a hone or upon a plate of glass with the powder of schistus, till the image of a candle reflected from the surfaces is of a dull reddish-white color. If we now place the eye near the plate, and look at this reflected image, C, we shall see on one side of it a prismatic image, A, glowing with all the colors of the rainbow, and forming indeed a spectrum of the candle as distinct as if it had been formed by an equilateral prism of flint glass. The blue side of this image is

Fig. 62.



next the image C, and the distance of the red part of the image is in one specimen 7° 22′; but this angle varies even in the same specimen. Upon first looking into the mother-of-pearl, the image A may be above or below C, or on any side of it; but, by turning the specimen round, it may be brought either to the right or left hand of C. The distance A C is smallest when the light of the candle falls nearly perpendicular on the surface, and increases as the inclination of the incident ray is increased. In one specimen it was 2° 7′ at nearly a perpendicular incidence, and 9° 14′ at a very great obliquity.

On the outside of the image A there is invariably seen a mass, M, of colored light, whose distance M C is nearly double A C. These three images are always nearly in a straight line, but the angular distance of M varies with the angle of incidence according to a law different from that of A. At great angles of incidence the nebulous mass is of a beautiful crimson color; at an angle of about 37° it becomes green; and nearer the perpendicular it becomes yellowish-white, and very luminous.

If we now *polish* the surface of the mother-of-pearl, the ordinary image C will become brighter and quite white, but a *second prismatic image*, B, *will start up on the other side of* C, *and at the same distance from it.*

This second image has in all other respects the same properties as the first. Its brightness increases with the polish of the surface, till it is nearly equal to that of A, the lustre of which is slightly impaired by polishing. This second image is never accompanied, like the first, with a nebulous mass M. If we remove the polish, the image B vanishes, and A resumes its brilliancy. The lustre of the nebulous mass M is improved by polishing.

If we repeat these experiments on the *opposite* side of the specimen, the very same phenomena will be observed, with this difference only, that the images A and M are on the opposite side of C.

In looking through the mother-of-pearl, when ground extremely thin, nearly the same phenomena will be observed, The colors and the distances of the images are the same; but the nebulous mass M is never seen by transmission. When the second image, B, is invisible by reflexion, it is exceedingly bright when seen by transmission, and *vice versâ.*

In making these experiments, I had occasion to fix the mother-of-pearl to a goniometer with a cement of resin and bees'-wax; and upon removing it, I was surprised to see the whole surface of the wax shining with the prismatic colors of the mother-of-pearl. I at first thought that a small film of the substance had been left upon the wax; but this was soon found to be a mistake, and it became manifest that the mother-of-pearl really impressed upon the cement its own power of producing the colored spectra. When the unpolished mother-of-pearl was impressed on the wax, the wax gave only one image, A; and when the polished surface was used, it gave both A and B: but the nebulous image M was never exhibited by the wax. The images seen in the wax are always on the opposite side of C, from what they are in the surface that is impressed upon it.

The colors of mother-of-pearl, as communicated to a soft surface, may be best seen by using black wax; but I have transferred them also to balsam of Tolu, realgar, fusible metal, and to clean surfaces of lead and tin by hard pressure, or the blow of a hammer. A solution of gum arabic or of isinglass, when allowed to indurate upon a surface of mother-of-pearl, takes a most perfect impression from it, and exhibits all the communicable colors in the finest manner, when seen either by reflexion or transmission. By placing the isinglass between two finely polished surfaces of good specimens of mother-of-pearl, we shall obtain a film of artificial mother-of-pearl, which when seen by single lights, such as that of a candle, or by an aperture in the window, will shine with the brightest hues.

If, in this experiment, we could make the grooves of the one surface of mother-of-pearl exactly parallel to the grooves in the other, as in the shell itself, the images, A and B, formed by each surface, would coincide, and only two would be observed by transmission and reflexion: but, as this cannot be done, *four images* are seen through the isinglass film, and also four by reflexion; the two new ones being formed by reflexion from the second surface of the film.

From these experiments it is obvious that the colors under our consideration are produced by a particular configuration of surface, which, like a seal, can convey a reverse impression of itself to any substance capable of receiving it. By examining this surface with microscopes, I discovered in almost every specimen a grooved structure, like the delicate texture of the skin at the top of an infant's finger, or like the section of the annual growths of wood, as seen upon a dressed plank of fir. These may sometimes be seen by the naked eye, but they are often so minute that 3000 of them are contained in an inch. The direction of the grooves is always at right angles to the line M A C B, *fig.* 62.; and hence in irregularly formed mother-of-pearl, where the grooves are often circular, and having every possible direction, the colored images A, B are irregularly scattered round the common image C. If the grooves were, accordingly, circular, the series of prismatic images, A B, would form a prismatic ring round C, provided the grooves retained the same distance. The general distance of the grooves is from the 200th to the 5000th of an inch, and the distance of the prismatic images from C increases as the grooves become closer. In a specimen with 2500 in an inch, the distance A C was 3° 41′; and in a specimen of about 5000 it was about 7° 22′.

These grooves are obviously the sections of all the concentric strata of the shell. When we use the actual surface of any stratum, none of the colors A, B are seen, and we observe only the mass of nebulous light M occupying the place of the principal image C. Hence we see the reason why the *pearl* gives none of the images A, B, why it communicates none of its colors to wax, and why it shines with that delicate white light which gives it all its value. The pearl is formed of concentric spherical strata, round a central nucleus, which Sir Everard Home conceives to be one of the ova of the fish. None of the edges of its strata are visible, and as the strata have parallel surfaces, the mass of light M is reflected exactly like the image C, and occupies its place; whereas in the mother-of-pearl it is reflected from surfaces of the strata, inclined to the general surface of the specimen which reflects the image C. The mixture of all these diffuse masses of nebulous light, of a pink and green hue, constitutes the beautiful white of the pearls. In bad pearls, where the colors are too blue or too pink, one or other of these colors has predominated. If we make an oblique section of a pearl, so as to exhibit a sufficient number of concentric strata, with their edges tolerably close, we should observe all the communicable colors of mother-of-pearl.*

These phenomena may be observed in many other shells besides that of the pearl-oyster; and in every case we may distinguish communicable from incommunicable colors, by placing a film of fluid or cement between the surface and a plate of glass. The communicable colors will all disappear from the filling up of the grooves, and the incommunicable colors will be rendered more brilliant.

(81.) Mr. Herschel has discovered in very thin plates of mother-of-pearl another pair of nebulous prismatic images, more distant from C than A and B, and also a pair of fainter nebulous images, the line joining which is always at right angles to the line joining the first pair.* These images are seen by *looking through* a thin piece of mother-of-pearl, cut parallel to the natural surface of the shell, and between the 70th and the 300dth of an inch thick. They are much larger than A and B; and Mr. Herschel found that the line joining them was always perpendicular to a veined structure which goes through its substance. The distance of the red part of the image from C was found to be 16° 29′, and the veins which produced these colors were so small that 3700 of them were contained in an inch. We have represented them in *fig*. 63. as crossing the ordinary grooves which give the communicable colors. Mr. Herschel describes them as crossing these grooves at all angles, "giving the whole surface much the appearance of a piece of twilled silk, or the larger waves of the sea intersected with minute ripplings." The second pair of nebulous images seen by transmission must arise from a veined structure exactly perpendicular to the first, though the structure has not yet been recognized by the microscope. The structure which produces the lightest pair Mr. Herschel has found to be in all cases coincident with the plane passing through the centres of the two systems of polarized rings.

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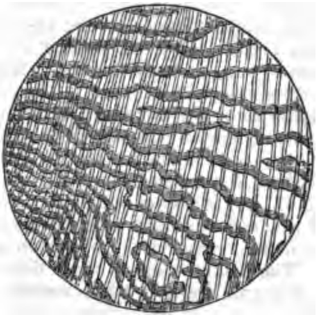
* See *Edinburgh Journal of Science*, No. XII., p. 277.

Fig. 63.



The principle of the production of color by grooved surfaces, and of the communicability of these colors by pressure to various substances, has been happily applied to the arts by John Barton, Esq. By means of a delicate engine, operating by a screw of the most accurate workmanship, he has succeeded in cutting grooves upon steel at the distance of from the 2000th to the 10,000th of an inch. These lines are cut with the point of a diamond; and such is their perfect parallelism and the uniformity of their distance, that while in mother-of-pearl we see only one prismatic image, A, on each side of the common image, C, of the candle, in the grooved steel surfaces 6, 7, or 8 prismatic images are seen, consisting of spectra, as perfect as those produced by the finest prisms. Nothing in nature or in art can surpass this brilliant display of colors; and Mr. Barton conceived the idea of forming buttons for gentlemen's dress, and articles of female ornament covered with grooves, beautifully arranged in patterns, and shining in the light of candles or lamps with all the hues of the spectrum. To these he gave the appropriate name of *Iris* ornaments. In forming the buttons, the patterns were drawn on steel dies, and these, when duly hardened, were used to stamp their impressions upon polished buttons of brass. In day-light the colors on these buttons are not easily distinguished, unless when the surface reflects the margin of a dark object seen against a light one; but in the light of the sun, and that of gas-flame or candles, these colors are scarcely if at all surpassed by the brilliant flashes of the diamond.

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* In a specimen now before us, the line joining the two faintest nebulous images is at right angles to the line joining A and B.

The grooves thus made upon steel are, of course, all transferable to wax, isinglass, tin, lead, and other substances; and by indurating thin transparent films of isinglass between two of these grooved surfaces, covered with lines lying in all directions, we obtain a plate which produces by transmission the most extraordinary display of prismatic spectra that has ever been exhibited.

(82.) In examining the phenomena produced by some of the finest specimens of Mr. Barton's skill, which he had the kindness to execute for this purpose, I have been led to the observation of several curious properties of light. In mother-of-pearl, well polished, the central image, C, of the candle or luminous object is always white, as we should expect it to be, in consequence of being reflected from the flat and polished surfaces between the grooves. In like manner, in many specimens of grooved steel the image C is also perfectly white, and the spectra on each side of it, to the amount of six or eight, are perfect prismatic images of the candle; the image A, which is nearest C, being the least dispersed, and all the rest in succession more and more dispersed, as if they were formed by prisms of greater and greater dispersive powers, or greater and greater refracting angles. These spectra contain the fixed lines and all the prismatic colors; but the *red* or least refrangible spaces are greatly *expanded*, and the *violet* or most refrangible spaces greatly *contracted*, even more than in the spectra produced by sulphuric acid.

In examining some of these prismatic images which seemed to be defective in particular rays, I was surprised to find that, in the specimens which produced them, the image C reflected from the polished original surface of the steel was itself slightly colored; that its tint varied with the angle of incidence, and had some relation to the defalcation of color in the prismatic images. In order to observe these phenomena through a great range of incidence, I substituted for the candle a long narrow rectangular aperture, formed by nearly closing the window-shutters, and I then saw at one view the state of the ordinary image and all the prismatic images. In order to understand this, let A B, *fig.* 64., be the ordinary

Fig. 64.

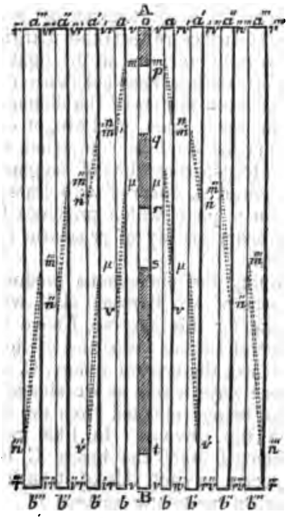


image of the aperture reflected from the flat surface of the steel which lies between the grooves, and *a b*, *a' b'*, *a'' b''*, &c., the prismatic images on each side of it, every one of these images forming a complete spectrum with all its different colors. The image A B was crossed in a direction perpendicular to its length with broad colored fringes, varying in their tints from 0° to 90° of incidence. In a specimen with 1000 grooves in an inch, the following were the colors distinctly seen at different angles of incidence:—

	Angle of incidence.
White	90° 0′
Yellow	80 30
Reddish orange	77 30
Pink	76 20
Junction of pink and blue	75 40
Brilliant blue	74 30
Whitish	71 0
Yellow	64 45
Pink	59 45
Junction of pink and blue	58 10
Blue	56° 0′
Bluish green	54 30
Yellowish green	53 15
Whitish green	51 0
Whitish yellow	49 0
Yellow	47 15
Pinkish yellow	41 0
Pink red	36 0
Whitish pink	31 0
Green	24 0
Yellow	10 0
Reddish	0 0

These colors are those of the reflected rings in thin plates. If we turn the steel plate round in azimuth, the very same colors appear at the same angle of incidence, *and they suffer no change either by varying the distance of the steel plate from the luminous aperture, or the distance of the eye of the observer from the grooves.*

In the preceding table there are four orders of colors; but in some specimens there are only three, in others two, in others one, and in some only one or two tints of the first order are developed. A specimen of 500 grooves in an inch gave only the yellow of the first order through the whole quadrant of incidence. A specimen of 1000 grooves gave only one complete order, with a portion of the next. A specimen of 3333 grooves gave only the yellow of the first order. A specimen of 5000 gave a little more than one order; and a specimen of 10,000 grooves in an inch gave also a little more than one order.

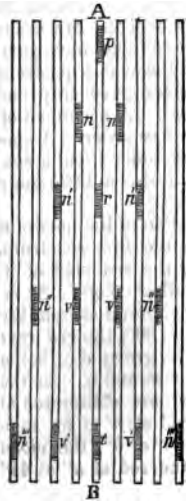
In *fig.* 64. we have represented the portion of the quadrant of incidence from about 22° to 76°. In the first spectrum, *a b a b*, *v v* is the violet side of it, and *r r* the red side of it, and between these are arranged all the other colors. At *m*, at an incidence of 74°, the violet light is obliterated from the spectrum *a b*; and at *n*, at an incidence of 66°, the *red* rays are obliterated; the intermediate colors, blue, green, &c., being obliterated at intermediate points between *m* and *n*. In the second spectrum, *a′ b′ a′ b′*, the *violet* rays are obliterated at *m′* at an incidence of 66° 20′, and the *red* at *n′* at an incidence of 56°. In the third spectrum, *a″ b″ a″ b″*, the *violet* rays are obliterated at *m″* at 57°, and the red at *n″* at 41°

35′; and in the fourth spectrum, the violet rays are obliterated at m''' at 48°, and the red at n''' at 23° 30″. A similar succession of obliterated tints takes place on all the prismatic images at a lesser incidence, as shown at $\mu\,\nu$, $\mu'\,\nu'$; the violet being obliterated at μ and μ', and the red at ν and ν', and the intermediate colors at intermediate points. In this second succession the line $\mu\,\nu$ begins and ends at the same angle of incidence as the line $m''\,n''$ in the third prismatic image $a''\,b''$, and the line $\mu'\,\nu'$ in the second prismatic image corresponds with $m'''\,n'''$ on the fourth prismatic image. In all these cases, the tints obliterated in the direction $m\,n\,\mu\,\nu$, &c., would, if restored, form a complete prismatic spectrum whose length is $m\,n\,\mu\,\nu$, &c.

Considering the ordinary image as white, a similar obliteration of tints takes place upon it. The violet is obliterated at o about 76°, leaving *pink*, or what the violet wants of white light; and the red is obliterated at p at 74°, leaving a bright blue. The violet is obliterated at q and s, and the red at r and t, as may be inferred from the preceding Table of colors.

The analysis of these curious and apparently complicated phenomena becomes very simple when they are examined by homogeneous light. The effect produced on red light is represented in *fig.* 65., where A B is the image of the narrow aperture reflected from the original surface of the steel, and the four images on each side of it correspond with the prismatic images. All these nine images, however, consist of homogeneous red light, which is obliterated, or nearly so, at the fifteen shaded rectangles, which are the minima of the new series of periodical colors which cross both the ordinary and the lateral images. The centres p, r, t, n, ν, &c., of these rectangles correspond with the points marked with the same letters in *fig.* 64.; and if we had drawn the same figure for violet light, the centres of the rectangles would have been all higher up in the figure, and would have corresponded with o, q, s, m, μ, &c. in *fig.* 64. The rectangles should have been shaded off to represent the phenomena accurately, but the only object of the figure is to show to the eye the position and relations of the minima.

Fig. 65.



If we cover the surface of the grooved steel with a fluid, so as to diminish the refractive power of the surface, we develope more orders of colors on the ordinary image, and a greater number of minima on the lateral images, higher tints being produced at a given incidence. But, what is very remarkable, in grooved surfaces when the ordinary image is perfectly white, and when the spectra are complete without any obliteration of tints, the application of fluids to the grooved surface developes colors on the ordinary image, and a corresponding obliteration of tints on the lateral images. The following Table contains a few of the results relative to the ordinary image :—

Number of grooves in an inch.	Maximum tint without a fluid.	Maximum tint with fluids.
312.	Perfectly white.	1. Water, tinge of yellow. 2. Alcohol, tinge of yellow. 3. Oil of cassia, faint reddish yellow.
3333	Gamboge yellow of the first order.	1. Water, pinkish red (first order). 2. Alcohol, reddish pink. 3. Oil of cassia, bright blue (second order).

Phenomena analogous to those above described take place upon the grooved surfaces of *gold*, *silver*, and *calcareous spar;* and upon the surfaces of *tin*, *isinglass*, *realgar*, &c., to which the grooves have been transferred from steel. For an account of the phenomena exhibited by several of these substances, I must refer the reader to the original memoir in the Philosophical Transactions for 1829.

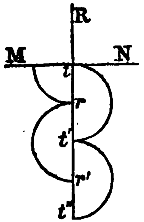
CHAP. XV.

ON FITS OF REFLEXION AND TRANSMISSION, AND ON THE INTERFERENCE OF LIGHT.

(83.) In the preceding chapters we have described a very extensive class of phenomena, all of which seem to have the same origin. From his experiments on the colors of thin and of thick plates, Newton inferred that they were produced by a singular property of the particles of light, in virtue of which they possess, at different points of their path, fits or dispositions to be reflected from or transmitted by transparent bodies. Sir Isaac does not pretend to explain the origin of these fits, or the cause which produces them; but we may form a tolerable idea of them by supposing that each particle of light, after its discharge from a luminous body, revolves round an axis perpendicular to the direction of its motion, and presenting alternately to the line of its motion an attractive and a repulsive pole, in virtue of which it will be refracted if the attractive pole is nearest any refracting surface on which it falls, and reflected if the repulsive pole is nearest that surface. The disposition to be refracted and reflected will of course increase and diminish as the distance of either pole from the surface of the body is increased or diminished. A less scientific idea may be formed of this hypothesis, by supposing a body with a sharp and a blunt end passing through space, and successively presenting its sharp and blunt ends to the line of its motion. When the sharp end encounters any soft body put in its way, it will penetrate it; but when the blunt end encounters the same body, it will be reflected or driven back.

To explain this more clearly, let R, *fig.* 66., be a ray of light falling upon a refracting surface M N, and *transmitted* by that surface. It is clear that it must have met the surface M N when it was nearer its fit of transmission than its fit of reflexion; but whether it was exactly at its fit of transmission, or a little from it, it is put, by the action of the surface, into the same state as if it had begun its fit of transmission at t. Let us suppose that, after it has moved through a space equal to $t\,r$, its fit of reflexion takes place, the fit of transmission always recommencing at $t\,t'$, &c. and that of reflexion at $r\,r'$, &c.; then it is obvious, that if the ray meets a second transparent surface at $t\,t'$, &c., it will be transmitted, and if it meets it at $r\,r'$, &c., it will be reflected. The spaces $t\,t'$, $t'\,t''$ are called the intervals of the fits of transmission, and $r\,r'$, $r'\,r''$ the intervals of the fits of reflexion. Now, as the spaces $t\,t'$, $r\,r'$, &c. are supposed equal for light of the same colors, it is manifest that, if M N be the first surface of a body, the ray will be transmitted if the thickness of the body is $t\,t'$, $t\,t''$, &c.; that is, $t\,t'$, $2\,t\,t'$, $3\,t\,t'$, $4\,t\,t'$, or any multiple whatever of the interval of a fit of easy transmission. In like manner the ray will be reflected if the thickness of the body is $t\,r$, $t\,r'$; or, since $t\,t'$ is equal to $r\,r'$, if the thickness of the body is $\frac{1}{2}\,t\,t'$, $1\frac{1}{2}\,t\,t'$, $2\frac{1}{2}\,t\,t'$, $3\frac{1}{2}\,t\,t'$. If the body M N, therefore, had parallel surfaces, and if the eye were placed above it so as to receive the rays reflected perpendicularly, it would, in every case, see the surface M N by the portion of light uniformly reflected from that surface;

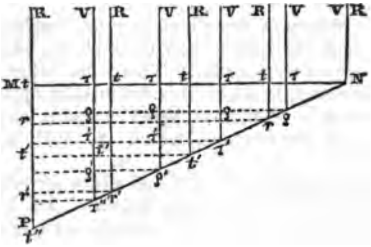
Fig. 66.



but when the thickness of the body was $t\,t'$, $2\,t\,t'$, $3\,t\,t'$, $4\,t\,t'$ or $1000\,t\,t'$, the eye would receive no rays from the second surface, because they are all transmitted; and in like manner, if the thickness was $\frac{1}{2}\,t\,t'$, $1\frac{1}{2}\,t\,t'$, $2\frac{1}{2}\,t\,t'$, or $1000\frac{1}{2}\,t\,t'$, the eye would receive *all the light* reflected from the second surface, because it is all reflected. When this reflected light meets the first surface M N, on its way to the eye, it is all transmitted, because it is then in its fit of transmission. Hence, in the first case, the eye receives no *light* from the *second* surface, and in the *second* case, it receives all the light from the *second* surface. If the body had intermediate thicknesses between $t\,t'$ and $2\,t\,t'$, &c., as $\frac{3}{4}\,t\,t'$, then a portion of the light would be reflected from the second surface, increasing as the thickness increased from $t\,t'$ to $1\frac{1}{2}\,t\,t'$, and diminishing again as the thickness increased from $1\frac{1}{2}\,t\,t'$ to $2\,t\,t'$.

But let us now suppose that the plate whose surface is M N is unequally thick, like the plate of air between the two lenses, or a film of blown glass. Let it have its thickness varying like a wedge M N P, *fig.* 67. Let $t\,t'$, $r\,r'$ be the intervals of the fits, and let the eye be placed above the wedge as before. It is quite clear that near the point N the light that falls upon the second surface N P will be all transmitted, as it is in a fit of transmission; but at the thickness $t\,r$ the light R will be reflected by the second surface, because it is then in its fit of reflexion. In like manner the light will be transmitted at t', again reflected at r', and again transmitted at t''; so that the eye above M N will see a series of dark and luminous bands, the middle of the dark ones being at N, t', t'' in the line N P, and of the luminous ones at r, r', &c. in the same line. Let us suppose that the figure is suited to red homogeneous light, $t\,t'$ being the interval of a fit for that species of rays; then in violet light, V, the interval of the fits will be less, as $\tau\,\rho$. If we therefore use violet light, the in-

Fig. 67.



terval of whose fits is $\tau\rho$, a smaller series of *violet* and *obscure bands* or fringes will be seen, whose obscurest points are at N, τ', τ'', &c., and whose brightest points are at ρ, $'\rho$, &c. In like manner, with the intermediate colors of the spectrum, bands of intermediate magnitudes will be formed, having their obscurest points between τ' and t', τ'' and t'', and their brightest points between ρ and r, ρ' and r', &c.; and when white light is used, all these differently colored bands will be seen forming fringes of the different orders of colors given in the Table in page 93. If M N P, in place of being the section of a prism, were the section of one half of a plano-concave lens, whose centre is N, and whose concave surface has an oblique direction somewhat like N P, the direction of the colored bands will always be perpendicular to the radius N M, or will be regular circles. For the same reason, the colored bands are circular in the concave lens of air between the object glasses; the same colors always appearing at the same thickness of the medium, or at the same distance from the centre.

By the same means Sir Isaac Newton explained the colors of thick plates, with this difference, that the fringes are not in that case produced by the light regularly refracted and reflected at the two surfaces of the concave mirror, but by the light irregularly scattered by the first surface of the mirror in consequence of its imperfect polish; for, as he observes, "there is no glass or speculum, how well soever polished, but, besides the light which it refracts and reflects regularly, scatters every way irregularly a faint light, by means of which the polished surface, when illuminated in a dark room by a beam of the sun's light, may be easily seen in all positions of the eye."

The same theory of fits affords a ready explanation of the phenomena of double and equally thick plates, which we have described in another chapter. There are other phenomena of colors, however, to which it is not equally applicable; and it has accordingly been, in a great measure, superseded by the doctrine of interference, which we shall now proceed to explain.

(83.) In examining the black and white stripes within the shadows of bodies as formed by inflexion, Dr. Young found that when he placed an opaque screen either a few inches before or a few inches behind one side of the inflecting body, B, *fig*. 56., so as to intercept all the light on that side by receiving the edge of the shadow on the screen, then all the fringes in the shadow constantly disappeared, although the light still passed by the other edge of the body as before. Hence he concluded that the light which passed on both sides was necessary to the production of the fringes; a conclusion which he might have deduced also from the known fact, that when the body was above a certain size, fringes never appeared in its shadow. In reasoning upon this conclusion, Dr. Young was led to the opinion, that the fringes within the shadow were produced *by the interference of the rays bent into the shadow by one side of the body* B *with the rays bent into the shadow by the other side.*

In order to explain the *law of interference* indicated in this experiment, let us suppose two pencils of light to radiate from two points very close to each other, and that this light falls upon the same spot of a piece of paper held parallel to the line joining the points, so that the spot is directly opposite the point which bisects the distance between the two radiant points. In this case they may be said to interfere with one another; because the pencils would cross one another at that spot if the paper were removed, and would diverge from one another. The spot will, therefore, be illuminated with the sum of their lights; and in this case the length of the paths of the two pencils of light is exactly the same, the spot on the paper being equally distant from both the radiant points. Now, it has been found that when there is a certain minute difference between the lengths of the paths of the two pencils of light, the spot upon the paper where the two lights interfere is still a bright spot illuminated by the sum of the two lights. If we call this difference in the lengths of their paths d, bright spots will be formed by the interference of the two pencils when the differences in the lengths of the paths are d, $2\,d$, $3\,d$, $4\,d$, &c. All this is nothing more than what is consistent with daily observation; but, what is truly remarkable and altogether unexpected, it has been clearly demonstrated that if the two pencils interfere at intermediate points, or when the difference in the lengths of the paths of the two pencils is $\frac{1}{2}\,d$, $1\frac{1}{2}\,d$, $2\frac{1}{2}\,d$, $3\frac{1}{2}\,d$, &c. instead of adding to one another's intensity, and producing an illumination equal to the sum of their lights, *they destroy each other*, and produce a dark spot. This curious property is analogous to the beating of two musical sounds nearly in unison with each other; the beats taking place when the effect of the two sounds is equal to the sum of their separate intensities, corresponding to the luminous spots or fringes where the effect of the two lights is equal to the sum of their separate intensities, and the cessation of sound between the beats when the two sounds destroy each other, corresponding to the dark spots or fringes where the two lights produce darkness.

By the aid of this doctrine the phenomena of the inflexion of light, and those of thin and thick plates, may be well explained. With regard to the interior fringes, or those in the shadow, it is clear that as the middle of the shadow is equally distant from the edges of the inflecting body B, *fig*. 56., there will be no difference in the length of the paths of the pencils coming from each side of the body, and consequently along the middle of the whole length of every narrow shadow there should be a white stripe illuminated with the sum of the two inflected pencils; but at a point at such a distance from the centre of the shadow that the difference of the two paths of the pencil from each side of the body is equal to $\frac{1}{2}$ *d*, the two pencils will destroy each other, and give a dark stripe. Hence there will be a dark stripe on each side of the central bright one. In like manner it may be shown, that at a point at such a distance from the centre of the shadow that the difference in the lengths of the paths is 2 *d*, 3 *d*, there will be bright stripes; and at intermediate points, where the difference in the lengths of the paths is $1\frac{1}{2}$ *d*, $2\frac{1}{2}$ *d*, there will be dark stripes.*

In order to explain the origin of the external fringes, both Dr. Young and M. Fresnel ascribed them to the interference of the direct rays with other rays reflected from the margin of the inflecting body; but M. Fresnel has found that the fringes exist when no such reflexion can take place; and he has, besides, shown the insufficiency of the explanation, even if such reflected rays did exist. He therefore ascribes the external fringes to the interference of the direct rays with other rays which pass at a sensible distance from the inflecting body, and which are made to deviate from their primitive direction. That such rays do exist, he proves upon the undulatory theory, which we shall afterwards explain.

The phenomena of thin plates are admirably explained by the doctrine of interference. The light reflected from the second surface of the plate interferes with the light reflected from the first, and as these two pencils of light come from different points of space, they must reach the eye with different lengths of paths. Hence they will, by their interference, form luminous fringes when the difference of the paths is *d*, 2 *d*, 3 *d*, &c., and obscure fringes when that difference is $\frac{1}{2}$ *d*, $1\frac{1}{2}$ *d*, $2\frac{1}{2}$ *d*, $3\frac{1}{2}$ *d*, &c.

In accounting for the colors of thick plates observed by Newton, the light scattered irregularly from every point of the first surface of the concave mirror falls diverging on the second surface, and being reflected from this surface in lines diverging from a point behind, they will suffer refraction in coming out of the first surface of the mirror, being made to diverge as if from a point still nearer the mirror, but behind its surface. From this last point, therefore, the screen M N, in *fig.* 60., is illuminated by the rays originally scattered on entering the first surface. But when the regularly reflected light, after reflexion from the second surface, emerges from the first, it will be scattered irregularly from each point on that surface, and radiating from these points will illuminate the paper screen M N. Every point, therefore, in the paper screen is illuminated by two kinds of scattered light, the one radiating from each point of the first surface, and the other from points behind the second surface; and hence bright and obscure bands will be formed when the differences of the lengths of their paths are such as have been already described.

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* See Note No. V., by Am. ed., following the author's Appendix.

The colors of two equally thick and inclined plates are also explicable by the law of interference. Although the light reflected by the different surfaces of the plate emerges parallel as shown in *fig.* 61., yet in consequence of the inclination of the plates it reaches the eye by paths of different lengths.

The colors of fine fibres, of minute particles, of mottled and striated surfaces, and of equidistant parallel lines, may be all referred to the interference of different portions of light reaching the eye by paths of different lengths; and though some difficulties still exist in the application of the doctrine to particular phenomena that have not been sufficiently studied, yet there can be no doubt that these difficulties will be removed by closer investigation.

As all the phenomena of interference are dependent upon the quantity d, it becomes interesting to ascertain its exact magnitude for the differently colored rays, and, if possible, to trace its origin to some primary cause. It is obvious, as Fraunhofer has remarked, that this quantity d is a real absolute magnitude, and whatever meaning we may attach to it, it is demonstrable that one half of it, in reference to the phenomena produced by it, is opposed in its properties to the other half; so that if the anterior half combines accurately with the posterior half, or interferes with it in this manner under a small angle, the effect which would have been produced by each separately is destroyed, whereas the same effect is doubled if two anterior or two posterior halves of this magnitude combine or interfere in a similar manner.

(84.) In the Newtonian theory of light, or the theory of emission, as it is called, in which light is supposed to consist of material particles emitted by luminous bodies, and moving through space with a velocity of 192,000 miles in a second, the quantity *d* is double the interval of the fits of easy reflexion and transmission; while in the undulatory theory it is equal to the breadth of an undulation or wave of light.

In the undulatory theory, an exceedingly thin and elastic medium, called ether, is supposed to fill all space, and to occupy the intervals between the particles of all material bodies. The ether must be so extremely rare as to present no appreciable resistance to the planetary bodies which move freely through it.

The particles of this ether are, like those of air, capable of being put into vibrations by the agitation of the particles of matter, so that waves or vibrations can be propagated through it in all directions. Within refracting media it is less elastic than in vacuo, and its elasticity is less in proportion to the refractive power of the body.

When any vibrations or undulations are propagated through this ether, and reach the nerves of the retina, they excite the sensation of light, in the same manner as the sensation of sound is excited in the nerves of the ear by the vibrations of the air.

Differences of color are supposed to arise from differences in the frequency of the etherial undulations; *red* being produced by a much smaller number of undulations in a given time than *blue*, and intermediate colors by intermediate numbers of undulations.

Each of these two theories of light is beset with difficulties peculiar to itself; but the theory of undulations has made great progress in modern times, and derives such powerful support from an extensive class of phenomena, that it has been received by many of our most distinguished philosophers.

In a work like this it would be in vain to attempt to give a particular account of the principles of this theory. It may be sufficient at present to state, that the doctrine of interference is in complete accordance with the theory of undulation. When similar waves are combined, so that the elevations and depressions of the one coincide with those of the other, a wave of double magnitude will be produced; whereas, when the elevations of the one coincide with the depressions of the other, both systems of waves will be totally destroyed. "The spring and neap tides," says Dr. Young, "derived from the combination of the simple soli-lunar tides, afford a magnificent example of the interference of two immense waves with each other; the spring tide being the joint result of the combination when they coincide in time and place, and the neap tide where they succeed each other at the distance of half an interval, so as to leave the effect of their difference only sensible. The tides of the port of Batsha, described and explained by Halley and Newton, exhibit a different modification of the same opposition of undulations; the ordinary periods of high and low water being altogether superseded on account of the different lengths of the two channels by which the tides arrive, affording exactly the half interval which causes the disappearance of the alternation. It may also be very easily observed, by merely throwing two equal stones into a piece of stagnant water, that the circles of waves which they occasion obliterate each other, and leave the surface of the water smooth in certain lines of a hyperbolic form, while in other neighboring parts the surface exhibits the agitation belonging to both series united."

The following Table given by Mr. Herschel contains the principal data of the undulatory theory:—

Colors of the Spectrum.	Lengths of an Undulation in parts of an Inch in air.	Number of Undulations in an Inch.	Number of Undulations in a Second.*
Extreme red .	0·0000266	37640	458,000000,000000
Red . . .	0·0000256	39180	477,000000,000000
Intermediate .	0·0000246	40720	495,000000,000000
Orange . .	0·0000240	41610	506,000000,000000
Intermediate .	0·0000235	42510	517,000000,000000
Yellow . .	0·0000227	44000	535,000000,000000
Intermediate .	0·0000219	45600	555,000000,000000
Green .	0·0000211	47460	577,000000,000000
Intermediate .	0·0000203	49320	600,000000,000000
Blue . . .	0·0000196	51110	622,000000,000000
Intermediate .	0·0000189	52910	644,000000,000000
Indigo . .	0·0000185	54070	658,000000,000000
Intermediate .	0·0000181	55240	672,000000,000000
Violet . .	0·0000174	57490	699,000000,000000
Extreme violet	0·0000167	59750	727,000000,000000

"From this Table," says Mr. Herschel, "we see that the sensibility of the eye is confined within much narrower limits than that of the ear; the ratio of the extreme vibrations being nearly 1·58 : 1, and therefore less than an octave, and about equal to a minor sixth. That man should be able to measure with certainty such minute portions of space and time, is not a little wonderful; for it may be observed, whatever theory of light we adopt, these periods and these spaces have a *real existence*, being in fact deduced by Newton from direct measurements, and involving nothing hypothetical but the names here given them."

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* Taking the velocity of light at 192,000 miles per second.

CHAP. XVI.

ON THE ABSORPTION OF LIGHT.

(85.) One of the most curious properties of bodies in their action upon light, and one which we are persuaded will yet perform a most important part in the explanation of optical phenomena, and become a ready instrument in optical researches, is their power of *absorbing* light. Even the most transparent bodies in nature, *air* and *water*, when in sufficient thickness, are capable of *absorbing* a great quantity of light. On the summit of the highest mountains, where their light has to pass through a much less extent of air, a much greater number of stars is visible to the eye than in the plains below; and through great depths of water objects become almost invisible. The absorptive power of air is finely displayed in the color of the morning and evening clouds; and that of water in the red color of the meridian sun, when seen from a diving-bell at a great depth in the sea. In both these cases, one class of rays is absorbed more readily than another in passing through the absorbing medium, while the rest make their way in the one case to the clouds, and in the other to the eye.

Nature presents us with bodies of all degrees of absorptive power, as shown in the following brief enumeration:—

Charcoal.
Coal of all kinds.
Metals in general.
Silver.
Gold.
Black hornblende.
Black pleonaste.
Obsidian.
Rock crystal.
Selenite.
Glass.
Mica.
Water and transparent fluids.
Air and gases.

Although charcoal is the most absorptive of all bodies, yet, when it exists in a minutely divided state, as in some of the gases and flames, or in a particular state of aggregation, as in the diamond, it is highly transparent. In like manner, all metals are transparent in a state of solution; and even silver and gold, when beaten into thin films, are translucent, the former transmitting a beautiful blue, and the latter a beautiful green light.*

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* See Note No. VI. of Am. ed., in the notes following the author's Appendix.

Philosophers have not yet ascertained the nature of the power by which bodies absorb light. Some have thought that the particles of light are reflected in all directions by the particles of the absorbing body, or turned aside by the forces resident in the particles; while others are of opinion that they are detained by the body, and assimilated to its substance. If the particles of light were *reflected* or merely turned out of their direction by the action of the particles, it seems to be quite demonstrable that a portion of the most opaque matter, such as charcoal, would, when exposed to a strong beam of light, become actually phosphorescent during its illumination, or would at least appear white; but as all the light which enters it is never again visible, we must believe, till we have evidence of the contrary, that the light is actually *stopped* by the particles of the body, and remains within it in the form of imponderable matter.

Some idea may be formed of the law according to which a body absorbs light, by supposing it to consist of a given number of equally thin plates, at the refracting surfaces of which there is no light lost by reflexion. If the first plate has the power of absorbing $\frac{1}{10}$th of the light which enters it, or 100 rays out of 1000; then $\frac{9}{10}$ths of the original light, or 900 rays, will fall upon the second plate; and $\frac{1}{10}$th of these, or 90, being absorbed, 810 will fall upon the third plate, and so on. Hence it is obvious that the quantity of light transmitted by any number of films is equal to the light transmitted through one film multiplied as often into itself as there are films Thus, since 900 out of 1000 rays are transmitted by one film $\frac{9}{10}\times\frac{9}{10}\times\frac{9}{10}$ equal to $\frac{729}{1000}$, or 729 rays, will be the quantity transmitted by three films; and therefore the quantity absorbed will be 271 rays. Of the various bodies which absorb light copiously, there are few that absorb all the colored rays of the spectrum in equal proportions. While certain clouds absorb the blue rays and transmit the red, there are others that absorb all the rays in equal proportions, and exhibit the sun and the moon when seen through them perfectly white. Ink diluted is a fine example of a fluid which absorbs all the colored rays in equal proportions; and it has on this account been applied by Sir William Herschel as a darkening substance for obtaining a white image of the sun. Black pleonaste and obsidian afford examples of solid substances which absorb all the colors of the spectrum proportionally.

(86.) All colored transparent bodies, however, whether solid or fluid, do not necessarily absorb the colors proportionally; for it is only in consequence of an unequal absorption that they could appear colored by transmitted light. In order to exhibit this absorptive power, take a thick piece of the blue glass that is used for finger glasses, and which is sometimes met with in cylindrical rods of about $\frac{3}{10}$ths of an inch in diameter, and shape it into the form of a wedge. Form a prismatic image of the candle, or, what is better, of a narrow rectangular aperture in the window by a prism, and examine this prismatic image through the wedge of colored glass. Through the thinnest edge the spectrum will be seen nearly as complete as before the interposition of the wedge; but as we look at it through greater and greater thicknesses, we shall see particular parts or colors of the spectrum become fainter and fainter, and gradually disappear, while others suffer but a slight diminution of their brightness. When the thickness is about the twentieth part of an inch, the spectrum will have the appearance shown in *fig*. 68., where the *middle* R *of the red space* is entirely absorbed, the *inner red* that is left is weakened in intensity; the *orange* is entirely absorbed; the *yellow* Y is left almost insulated; the *green* G on the side of the yellow is very much absorbed; and a slight absorption takes place along the green and blue space. At a greater thickness still, the *inner* red diminishes rapidly, and also the yellow, green, and blue; till, at a certain thickness, all the middle colors of the spectrum are absorbed, and nothing left but the two extreme colors, the *red* R and the *violet* V, as shown in *fig*. 69. As the red light R has much greater intensity than the violet, the glass has at this thickness the appearance of being a red glass; whereas at small thicknesses it had the appearance of being a blue glass.

Fig. 68.

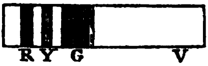
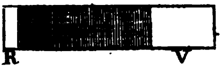


Fig. 69.



Other colored media, instead of absorbing the spectrum in the middle, attack it, some at one extremity, some at another, and others at both. Red glasses, for example, absorb the blue and violet with great force. A thin plate of native yellow orpiment absorbs the violet and refrangible blue rays very powerfully, and leaves the red, yellow, and green but little affected. Sulphate of copper attacks both ends of the spectrum at once, absorbing the red and violet rays with great avidity. In consequence of these different powers of absorption, a very remarkable phenomenon may be exhibited. If we look through the blue glass so as to see the spectrum in *fig*. 69., and then look at this spectrum again with a thin plate of sulphate of copper, which absorbs the extreme rays at R and V, the two substances thus combined will be abso lutely opaque, and not a ray of light will reach the eye. The effect is perhaps more striking if we look at a bright white object through the two media together.

(87.) In attempting to ascertain the influence of heat on the absorbing power of colored media, I was surprised to observe that it produced opposite effects upon different glasses, *diminishing* the absorbing power in some and *increasing* it in others. Having brought to a red heat a piece of purple glass, that absorbed the greater part of the green, the yellow, and the interior or most refrangible red, I held it before a strong light; and when its red heat had disappeared, I observed that the transparency of the glass was increased, and that it transmitted freely the green, the yellow, and the interior red, all of which it had formerly, in a great measure, absorbed. This effect, however, gradually disappeared, and it recovered its former absorbent power, when completely cold.

When yellowish-green glass was heated in a similar manner, it lost its transparency almost entirely. In recovering its green color, it passed through various shades of olive green; but its tint, when cold, continued less green than it was before the experiment. A part of the glass had received in cooling a polarizing structure, and this part could be easily distinguished from the other part by a difference of tint.

A plate of deep red glass, which gave a homogeneous red image of the candle, became very opaque when heated, and scarcely transmitted the light of the candle after its red heat had subsided. It recovered, however, its transparency to a certain degree; but when cold, it was more opaque than the piece from which it was broken. I have observed analogous phenomena in mineral bodies. Certain specimens of topaz have their absorbing power permanently changed by heat. In subjecting the Balas ruby to high degrees of heat, I observed that its red color changed into *green*, which gradually faded into brown as the cooling advanced, and resumed by degrees its original red color. In like manner, M. Berzelius observed the spinelle to become *brown* by heat, then to grow opaque as the heat increased, and to pass through a fine olive green before it recovered its red color. A remarkable change of absorbent power is exhibited by heating very considerably, but so as not to inflame it, a plate of yellow native orpiment, which absorbs the violet and blue rays. The heat renders it almost *blood red*, in consequence of its now absorbing the greater part of the green and yellow rays. It resumes its former color, however, by cooling. A still more striking effect may be produced with pure phosphorus, which is of a slightly yellow color, transmitting freely almost all the colored rays. When melted, and suddenly cooled, it acquired the power of absorbing all the colors of the spectrum at thicknesses at which it formerly transmitted them all. The blackness produced upon pure phosphorus was first observed by Thenard. Mr. Faraday observed, that glass tinged purple with manganese had its absorptive power altered by the mere transmission through it of the solar rays.

By the method above described of absorbing particular colors in the spectrum, I was led to propose a new method of analyzing white light. The experiments with the blue glass incontestably prove that the orange and green colors in solar light are compound colors, which, though they cannot be decomposed by the prism, may be decomposed by absorption, by which we may exhibit alone the red part of the orange and the blue part of the green, or the yellow part of the orange and the yellow part of the green; and, by submitting the other colors of the spectrum to the scrutiny of absorbent media, I was led to the conclusions respecting the spectrum which are explained in Chapter VII.

We have already seen that in the solar spectrum, as described by Fraunhofer, there are dark lines, as if rays of particular refrangibilities had been absorbed in their course from the sun to the earth. The absorption is not likely to have taken place in our atmosphere, otherwise the same lines would have been wanting in the spectra from the fixed stars, and the rays of solar light reflected from the moon and planets would probably have been modified by their atmospheres. But as this is not the case, it is probable that the rays which are wanting in the spectrum have been absorbed by the sun's atmosphere, as Mr. Herschel has supposed.

(88.) Connected with the preceding phenomena is the subject of colored flames, which, when examined by a prism, exhibit spectra deficient in particular rays, and resembling the solar spectrum examined by colored glasses. Pure hydrogen gas burns with a blue flame, in which many of the rays of light are wanting. The flame of an oil lamp contains most of the rays which are wanting in sun-light. Alcohol mixed with water, when heated and burned, affords a flame with no other rays but yellow. Almost all salts communicate to flames a peculiar color, as may be seen by introducing the powder of these salts into the exterior flame of a candle, or into the wick of a spirit lamp. The following results, obtained by different authors, have been given by Mr. Herschel:—

Salts of soda,	Homogeneous yellow.
——— potash,	Pale violet.
——— lime,	Brick red.
——— strontia,	Bright crimson.
——— lithia,	Red.
——— baryta,	Pale apple green.
——— copper,	Bluish green.

According to Mr. Herschel the muriates succeed best on account of their volatility.

CHAP. XVII.

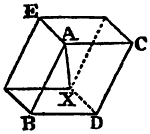
ON THE DOUBLE REFRACTION OF LIGHT.

(89.) In the preceding chapters of this work it has always been supposed, when treating of the *refraction* of light, either through surfaces, lenses, or prisms, that the transparent or refracting body had the same structure, the same temperature, and the same density in every part of it, and in every direction in which the ray could enter it. Transparent bodies of this kind are gases, fluids, solid bodies, such as different kinds of glass, formed by fusion, and slowly and equally cooled, and a numerous class of crystallized bodies, the form of whose primitive crystal is the *cube*, the *regular octohedron*, and the *rhomboidal dodecahedron*. When any of these bodies have the same temperature and density, and are not subject to any pressure, a single pencil of light incident upon any single surface of them, perfectly plane, will be refracted into a single pencil according to the law of the sines explained in Chapter III.

In almost all other bodies, including salts and crystallized minerals not having the primitive forms above mentioned; animal bodies, such as hair, horn, shells, bones, lenses of animals and elastic integuments; vegetable bodies, such as certain leaves, stalks, and seeds; and artificial bodies, such as resins, gums, jellies, glasses quickly and unequally cooled, and solid bodies having unequal density either from unequal temperature or unequal pressure;—in all such bodies a single pencil of light incident upon their surfaces will be *refracted into two different pencils*, more or less inclined to one another, according to the nature and state of the body, and according to the direction in which the pencil is incident. The separation of the two pencils is sometimes very great, and in most cases easily observed and measured; but in other cases it is not visible, and its existence is inferred only from certain effects which could not arise except from two refracted pencils. The refraction of the two pencils is called *double refraction*, and the bodies which produce it are called *doubly refracting* bodies or crystals.

As the phenomena of double refraction were first discovered in a transparent mineral substance called *Iceland spar*, *calcareous spar*, or *carbonate of lime*, and as this substance is admirably fitted for exhibiting them, we shall begin by explaining the law of double refraction as it exists in this mineral. Iceland spar is composed of 56 parts of lime, and 44 of carbonic acid. It is found in almost all countries, in crystals of various shapes, and often in huge masses; but, whether found in crystals or in masses, we can always cleave it or split it into shapes like that represented in *fig.* 70., which is called a rhomb of Iceland spar, a solid bounded by six equal and similar rhomboidal surfaces, whose sides are parallel, and whose angles B A C, A C D are 101° 55′ and 78° 5′. The inclination of any face A B C D to any of the adjacent faces that meet at A is 105° 5′, and to any of the adjacent faces that meet at X 74° 55′. The line A X, called the *axis of the rhomb* or of the crystal, is equally inclined to each of the six faces at an angle of 45° 23′. The angle between any of the three edges, BA, CA, EA, that meet at A, or of the three that meet at X, and the axis A X is 66° 44′ 46″, and the angle between any of the six edges and the faces is 113° 15′ 14″ and 66° 44′ 46″.

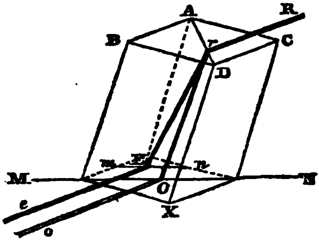
Fig. 70.



(90.) Iceland spar is very transparent, and generally colorless. Its natural faces, when it is split, are commonly even and perfectly polished; but when they are not so, we may, by a new cleavage, replace the imperfect face by a better one, or we may grind and polish any imperfect face.

Having procured a rhomb of Iceland spar like that in the figure, with smooth and well polished faces, and so large that one of the edges A B is at least an inch long, place one of its faces upon a sheet of paper, having a black line M N drawn upon it, as shown in *fig.* 71. If we then look through the upper surface of the rhomb with the eye about R, we shall probably see the line M N double; but if it is not, it will become double by turning the crystal a little round. Two lines, M N, *m n*, will then be distinctly visible; and upon turning the crystal round, preserving the same side always upon the paper, the two lines will coincide with one another, and appear to form one at two opposite points during a whole revolution of the crystal; and at two other opposite points, nearly at right angles to the former, the lines will be at their greatest distance. If we place a *black spot* at O, or a luminous aperture, such as a pin-hole in a wafer, with light passing through the hole, the spot or aperture will appear double, as at O and E; and by turning the crystal round as before, the two images will be seen separate in all positions; the one, E, revolving, as it were, round the other, O.

Fig. 71.



Let a ray or pencil of light, R r, fall upon the surface of the rhomb at r, it will be refracted by the action of the surface into two pencils, r O, r E, each of which, being again refracted at the second surface at the points O, E, will move in the directions O o, E e, parallel to one another and to the incident ray R r. The ray R r has therefore been *doubly refracted* by the rhomb.

If we now examine and measure the angle of refraction of the ray r O corresponding to different angles of incidence, we shall find that, at 0° of incidence, or a perpendicular incidence, it suffers no refraction, but moves straight through the crystal in one unbroken line; that at all other angles of incidence the sine of the angle of refraction is to that of incidence as 1 to 1·654; and that the refracted ray is always in the same plane as that of the incident ray. Hence it is obvious that the ray r O is refracted according to the *ordinary law of refraction*, which we have already explained. If we now examine in the same way the ray r E, we shall find that, at a perpendicular incidence, or one of 0°, the angle of refraction, in place of being 0°, is actually 6° 12′; that at other incidences the angle of refraction is not such as to follow the constant ratio of the sines; and, what is still more extraordinary, that the refracted ray r E is bent to one side, and lies entirely out of the plane of incidence. Hence it follows that the pencil r E is refracted according to some new and extraordinary law of refraction. The ray r O is therefore called the *ordinary ray*, and r E the *extraordinary ray.*

If we cause the ray R r to be incident in various different directions, either on the natural faces of the rhomb or on faces cut and polished artificially, we shall find that in Iceland spar there is one direction, namely, A X, along which, if the refracted pencil passes, it is not refracted into two pencils, or does not suffer double refraction. In other crystals there are two such directions, forming an angle with each other. In the former case the crystal is said to have ONE AXIS of double refraction, and in the latter case TWO AXES of double refraction. These lines are called axes of double refraction, because the phenomena are related to these lines. In some bodies there are certain planes, along which, if the refracted ray passes, it experiences no double refraction.

An axis of double refraction, however, is not, like the axis of the earth, a *fixed line* within the rhomb or crystal. It is only a *fixed direction:* for if we divide, as we can do, the rhomb A B C, *fig.* 70., into two or more rhombs, each of these separate rhombs will have their axes of double refraction; but when these rhombs are again put together, their axes will be all parallel to A X. Every line, therefore, within the rhomb parallel to A X, is an axis of double refraction; but as these lines have all one and the same direction in space, the crystal is still said to have only one axis of double refraction.

In making experiments with different crystals, it is found that in some the extraordinary ray is refracted *towards the axis* A X, while in others it is refracted *from the axis* A X. In the first case the axis is called *a positive axis of double refraction*, and in the second case *a negative axis of double refraction.*

On Crystals with one Axis of Double Refraction.

(91.) In examining the phenomena of double refraction in a great number of crystallized bodies, I found that all those crystals whose primitive or simplest form had only ONE AXIS of figure, or one pre-eminent line round which the figure was symmetrical, had also ONE AXIS of double refraction; and that their axis of figure was also the axis of double refraction. The primitive forms which possess this property are as follows:—

The rhomb with an obtuse summit.
The rhomb with an acute summit.
The regular hexahedral prism.
The octohedron with a square base.
The right prism with a square base.

(92.) The following Table contains the crystals which have one axis of double refraction, arranged under their respective primitive forms, the sign + being prefixed to those that have *positive* double refraction, and — to those that have *negative* double refraction.

Fig. 72.

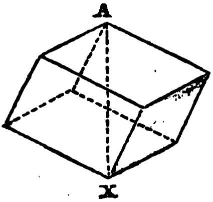


Fig. 73.

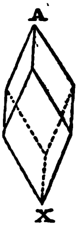
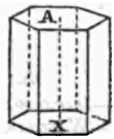


Fig. 74.



1. *Rhomb with obtuse summit, fig.* 72.

—Carbonate of lime (Iceland spar.
—Carbonate of lime and iron.
—Carbonate of lime and magnesia.
—Phosphato-arseniate of lead.
—Carbonate of zinc.
—Nitrate of soda.
—Phosphate of lead.
—Ruby silver.
—Levyne.
—Tourmaline.
—Rubellite.
—Alum stone.
—Dioptase.
—Quartz.

2. *Rhomb with acute summit, fig.* 73.

—Corundum.
—Sapphire.
—Ruby.
—Cinnabar.
—Arseniate of copper.

3. *Regular Hexahedral Prism, fig.* 74.

—Emerald.
—Beryl.
—Phosphate of lime (apatite).
—Nepheline.
—Arseniate of lead.
+Hydrate of magnesia.

4. *Octohedron with a square base, fig.* 75.

+Zircon.
+Oxide of tin.
+Tungstate of lime.
—Mellite.
—Molybdate of lead.
—Octohedrite.
—Prussiate of potash.
—Cyanide of mercury.

Fig. 75.

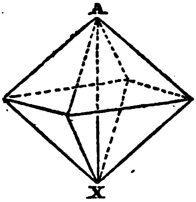
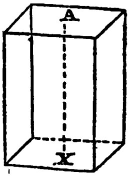


Fig. 76.



5. *Right Prism with a square base, fig*. 76.

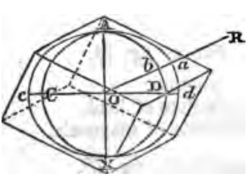
—Idocrase.
—Wernerite.
—Paranthine.
—Meionite.
—Somervillite.
—Edingtonite.
—Arseniate of potash.
—Sub-phosphate of potash.
—Phosphate of ammonia and magnesia.
—Sulphate of nickel and copper.
—Hydrate of strontia.
+Apophyllite of utoe.
+Oxahverite.
+Superacetate of copper and lime.
+Titanite.
+Ice (certain crystals).

In all the preceding crystals, and in the primitive forms to which they belong, the line A X is the axis of figure and of double refraction, or the only direction along which there is no double refraction.

On the Law of Double Refraction in Crystals with one Negative Axis.

(93.) In order to give a familiar explanation of the law of double refraction, let us suppose that a rhomb of Iceland spar is turned in a lathe to the form of a sphere, as shown in *fig*. 77., A X being the axis of both the rhomb and the sphere.

Fig. 77.



If we now make a ray pass along the axis A X, after grinding or polishing a small flat surface at A and X, perpendicular to A X, we shall find that there is no double refraction; the ordinary and extraordinary ray forming a single ray. Hence,

The index of refraction along the axis A X will be - } 1·654 for ordinary ray.
1·654 for extraordinary ray

0·000 difference.

If we do the same at any point, *a*, about 45° from the axis, we shall have

The index of refraction along the line R *a* *b* O, which is nearly perpendicular to the face of the rhomb,	1·654 for ordinary ray. 1·572 for extraordinary ray.
	0.082 difference.

If we do the same at any point of the equator C D, inclined 90° to the axis, we shall have

The index of refraction perpendicular to the axis,	1·654 for ordinary ray. 1.483 for extraordinary ray.
	0·171 difference.

Hence it follows that the index of extraordinary refraction decreases from the axis A X to the equator C D, or to a line perpendicular to the axis, where it is the least. The index of extraordinary refraction is the same at all equal angles with the axis A X; and hence, in every part of a circle described on the surface of the sphere round the pole A or X, the index of extraordinary refraction has the same value, and consequently the double refraction or separation of the rays will be the same. In crystals, therefore, with one axis of double refraction, the lines of equal double refraction are circles parallel to the equator or circle of greatest double refraction.

The celebrated Huygens, to whom we owe the discovery of the law of double refraction in crystals with one axis, has given the following method of determining the index of extraordinary refraction at any point of the sphere, when the ray of light is incident in a plane passing through the axis of the crystal A X:—

Let it be required, for example, to determine the index of refraction for the extraordinary ray R *a* *b*, *fig.* 77., A X being the axis, and C D the equator of the crystal; the ordinary index of refraction being known, and also the least extraordinary index of refraction, or that which takes place in the equator. In calcareous spar these numbers are 1·654 and 1.483. From O set off in the lines O C, O D continued, O *c*, O *d*, so that O C or O D is to O *c* or O *d* as $\frac{1}{1.654}$ is to $\frac{1}{1.483}$, or as ·604 is to ·674; and through the points A, *c*, X, *d*, draw an ellipse, whose greater axis is *c* *d*, and whose lesser axis is A X. The radius O *a* of the ellipse will be what is called the reciprocal of the index of refraction at *a*; and as we can find O *a*, either by projecting the ellipse on a large scale, or by calculation, we have only to divide 1 by O *a* to have that index. In the present case O *a* is ·636, and $\frac{1}{.636}$ is equal to 1·572, the index required.

As the index of extraordinary refraction thus found always diminishes from the pole A to the equator C D, and is always equal to the index of ordinary refraction *minus* another quantity depending on the difference between the radii of the circle and those of the ellipse, the crystals in which this takes place may be properly said to have *negative* double refraction.

In order to determine the direction of the extraordinary refracted ray, when the plane of incidence is oblique to a plane passing through the axis, the process, either by projection or calculation, is too troublesome to be given in an elementary work.

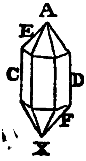
In every case the force which produces the double refraction exerts itself as if it proceeded from the axis.

Every plane passing through the axis is called a ***principal section*** of the crystal.

On the Law of Double Refraction in Crystals with one Positive Axis.

(94.) Among the crystals best fitted for exhibiting the phenomena of positive double refraction is *rock crystal* or *quartz*, a mineral which is generally found in six-sided prisms, like *fig.* 78., terminated with six-sided pyramids, E, F.

Fig. 78.



If we now grind down the summits A and X, and replace them by faces well polished, and perpendicular to the axis A X; and if we transmit a ray through these faces, so that it may pass along the axis A X, we shall find that there is no double refraction, and that the index of refraction is as follows:—

Index of refraction along the axis A X - - } 1·5484 for ordinary ray.
1·5484 for extraordinary ray

0·0000 difference.

If we now transmit the ray perpendicularly through the arallel faces E F, which are inclined 38° 20′ to the axis A X, e plane of its incidence passing through A X, we shall tain the following results:—

Index of refraction perpendicular to the faces of the pyramid - - - } 1·5484 for ordinary ray.
1·5544 for extraordinary ray.

0·0060 difference.

CHDavis.

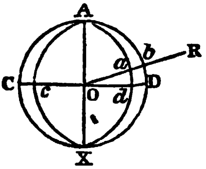
In like manner, it will be found that when the ray passes perpendicularly through the faces C D, perpendicular to the axis A X, the index of extraordinary refraction is the greatest, viz.

Index of refraction perpendicular to the faces of the prism C D - -	1·5484 for ordinary ray. 1·5582 for extraordinary ray.
	0·0098 difference.

Hence it appears that in *quartz* the index of extraordinary refraction *increases* from the pole A to the equator C D, whereas it *diminished* in calcareous spar, and the extraordinary ray appears to be *drawn* to the axis.

In this case the variation of the index of extraordinary refraction will be represented by an ellipse, A *c* X *d*, whose greater axis coincides with the axis A X of double refraction, as in *fig.* 79., and O C will be to O *c* as $\frac{1}{1\cdot5484}$ is to $\frac{1}{1\cdot5582}$, or as ·6458 is to ·6418. By determining, therefore, the radius O *a* of the ellipse for any ray R *b a*, and dividing 1 by it, we shall have the index of extraordinary refraction for that ray.

Fig. 79.



As the index of extraordinary refraction is always equal to the index of ordinary refraction, *plus* another quantity depending on the difference between the radii of the circle and the ellipse, the crystals in which this takes place may properly be said to have *positive* double refraction.

On Crystals with two Axes of Double Refraction.

(95.) The great variety of crystals, whether they are mineral bodies or chemical substances, have TWO axes of double refraction, or two directions inclined to each other along which the double refraction is nothing. This property of possessing two axes of double refraction I discovered in 1815, and I found that it belonged to all the crystals which are included in the prismatic system of Mohs, or whose primitive forms are,

A right prism, base a rectangle.
————— base a rhomb.
————— base an oblique parallelogram.
Oblique prism, base a rectangle.
————— base a rhomb.
————— base an oblique parallelogram.
Octohedron, base a rectangle
————— base a rhomb.

In all these primitive forms there is not a ***single*** pre-eminent line or axis about which the figure is symmetrical.

The following is a list of some of the most important crystals, with their primitive forms according to Haüy, and the inclination of the two lines or axes along which there is no double refraction:—

Glauberite - - -	2° or 3°		Oblique prism, base a rhomb.
Nitrate of potash -	5°	20′	Octohedron, base a rectangle,
Arragonite - - -	18	18	Octohedron, base a rectangle.
Sulphate of baryta -	37	42	Right prism, base a rectangle.
Mica - - - - - -	45	0	Right prism, base a rectangle.
Sulphate of lime -	60	0	Right prism, base an oblique parallelogram.
Topaz - - - - -	65	0	Octohedron, base a rectangle.
Carbonate of potash	80	30	Prismatic system of Mohs.
Sulphate of iron -	90	0	Oblique prism, base a rhomb.

In crystals with *one axis* of double refraction, the axis has the same position whatever be the color of the pencil of light which is used; but in crystals with *two axes*, the axes change their position according to the color of the light employed, so that the inclination of the two axes varies with differently colored rays. This discovery we owe to Mr. Herschel, who found that in *tartrate of potash and soda* (Rochelle salts) the inclination of the axis for *violet* light was about 56°, while in *red* light it was about 76°. In other crystals, such as *nitre*, the inclination of the axes for the *violet* rays is greater than for the *red* rays; but in every case the line joining the extremity of the axes for all the different rays is a straight line.

In examining the properties of *Glauberite*, I found that it had *two axes for red light* inclined about 5°, and only *one axis for violet light.*

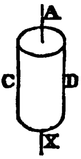
It was at first supposed that in crystals with two axes, one of the rays was refracted according to the ordinary law of the sines, and the other by an extraordinary law; but Mr. Fresnel has shown that both the rays are refracted according to laws of extraordinary refraction.

On Crystals with innumerable Axes of Double Refraction.

(96.) In the various doubly refracting bodies hitherto mentioned, the double refraction is related to one or more axes; but I have found that in *analcime* there are several planes, along which if the refracted ray passes, it will not suffer double refraction, however various be the directions in which it is incident. Hence we may consider each of these planes as containing an infinite number of axes of double refraction, or rather lines in which there is no double refraction. When the ray is incident in any other direction, so that the refracted ray is not in one of these planes, it is divided into two rays by double refraction. No other substance has yet been found possessing the same property.

On Bodies to which Double Refraction may be communicated by Heat, rapid Cooling, Pressure, and Induration.

Fig. 80.



(97.) If we take a cylinder of glass, C D, *fig*. 80., and having brought it to a red heat, roll it along a plate of metal upon its cylindrical surface till it is cold, it will acquire a *permanent* doubly refracting structure, and it will become a cylinder with *one positive axis* of double refraction, A X, coinciding with the axis of the cylinder, and along which there is no double refraction. This axis differs from that in quartz, as it is a fixed line in the cylinder, while it is only a fixed direction in the quartz; that is, any other line parallel to A X, *fig*. 80., is not an axis of double refraction, but the double refraction along that line increases as it approaches the circumference of the cylinder. The double refraction is a maximum in the direction C D, being equal in every line perpendicular to the axis, and passing through it.

If, instead of heating the glass cylinder, we had placed it in a vessel and surrounded it with boiling oil or boiling water, it would have acquired *the very same doubly refracting* structure when the heat had reached the axis A X; but this structure is only transient, as it disappears when the cylinder is uniformly heated.

If we had heated the cylinder uniformly in boiling oil, or at a fire, so as not to soften the glass, and had placed it in a cold fluid, it would have acquired a transient doubly refracting structure as before, when the cooling had reached the axis A X; but its axis of double refraction A X will now be a *negative* one, like that of calcareous spar.

Analogous structures may be produced by pressure and by the induration of soft solids, such as animal jellies, isinglass, &c.

If the cylinder in the preceding explanation is not a circular one, but has its section perpendicular to the axis everywhere an *ellipse* in place of a *circle*, it will have two axes of double refraction.

In like manner, if we use rectangular plates of glass instead of cylinders in the preceding experiments, we shall have plates with *two planes* of double refraction; a positive structure being on one side of each plane, and a negative one on the other.

If we use perfect spheres, there will be axes of double refraction along every diameter, and consequently an infinite number of them.

The crystalline lenses of almost all animals, whether they are lenses, spheres, or spheroids, have one or more axes of double refraction.

All these phenomena will be more fully explained when we treat of the colors produced by double refraction.

On Substances with Circular Double Refraction.

(98.) When we transmit a pencil of light along the axis A X, *fig.* 78., of a crystal of quartz, it suffers no double refraction; but certain phenomena, which will be afterwards described, are seen along this axis, which induced M. Fresnel to examine the light which passed along the axis. He found that it possessed a new kind of double refraction, and he distinctly observed the refraction of the two pencils. This kind of double refraction has, from its properties, been called *circular;* and it is divided into two kinds,—*positive* or right-handed, and *negative* or left-handed.

The following substances possess this remarkable property:—

Positive Substances.	*Negative Substances.*
Rock crystal, certain specimens.	
Camphor.	Rock crystal, certain specimens.
Oil of turpentine.	Concentrated syrup of sugar.
Solution of camphor in alcohol.	Essential oil of lemon.
Essential oil of laurel.	
Vapor of turpentine.	

In examining this class of phenomena, I found that the amethyst possessed in the same crystal both the positive and the negative circular double refraction. This subject will be more fully treated when we come to that of *circular polarization.**

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* For the formulæ referring to certain of the articles of this and of the subsequent chapter, see (in the College edition,) Appendix of Am. ed., Chap. VI.

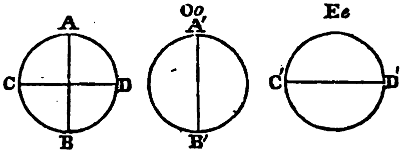
CHAP. XVIII.

ON THE POLARIZATION OF LIGHT.

If we transmit a beam of the sun's light through a circular aperture into a dark room, and if we reflect it from any crystallized or uncrystallized body, or transmit it through a thin plate of either of them, it will be reflected and transmitted in the very same manner and with the same intensity, whether the surface of the body is held above or below the beam, or on the right side or left, or on any other side of it, provided that in all these cases it falls upon the surface in the same manner; or, what amounts to the same thing, the beam of solar light has the same properties on all its sides; and this is true, whether it is white light as directly emitted from the sun, or whether it is red light, or light of any other color.

The same property belongs to light emitted from a candle, or any burning or self-luminous body, and all such light is called *common* light. A section of such a beam of light will be a circle, like A C B D, *fig*. 81., and we shall distinguish

Fig. 81.



the section of a beam of common light by a circle with two diameters, A B, C D, at right angles to each other.

If we now allow the same beam of light to fall upon a rhomb of Iceland spar, as in *fig*. 71., and examine the two circular beams O *o*, E *e*, formed by double refraction, we shall find,

1. That the beams O *o*, E *e*, have different properties on different sides; so that each of them differs, in this respect, from the beam of common light.

2. That the beam O *o* differs from E *e* in nothing, excepting that the former has the same properties at the sides A′ and B′ that the latter has at the sides C′ and D′, as shown in *fig*. 81.; or, in general, that the diameters of the beam, at the extremities of which the beam has similar properties, are at right angles to each other, as A′ B′ and C′ D′, for example.

These two beams, O *o*, E *e*, *fig*. 81., are therefore said to be *polarized*, or to be beams of *polarized* light, because they have sides or *poles* of different properties; and planes passing through the lines A B, C D, or A′ B′, C′ D′, are said to be the *planes* of *polarization* of each beam, because they have the same property, and one which no other plane passing through the beam possesses.

Now, it is a curious fact, that if we cause the two polarized beams O *o*, E *e* to be united into one, or if we produce them by a thin plate of Iceland spar, which is not capable of separating them, we obtain a beam which has exactly the same properties as the beam A B C D of common light

Hence we infer, that a beam of common light, A B C D, consists of *two* beams of polarized light, whose planes of polarization, or whose diameters of similar properties, are at right angles to one another. If O *o* is laid upon E *e*, it will produce a figure like A B C D, and we, therefore, represent common light by such a figure. If we place O *o* above E *e*, so that the planes of polarization A′ B′ and C′ D′ coincide, then we shall have a beam of polarized light twice as luminous as either O *o* or E *e*, and possessing exactly the same properties; for the lines of similar property in the one beam coincide with the lines of similar property in the other.

Hence it follows that there are three ways of converting a beam of common light, A B C D, into a beam or beams of polarized light.

1. We may separate the beam of common light, A B C D, into its two component parts, O *o* and E *e*.

2. We may turn round the planes of polarization, A B, C D, till they coincide or are parallel to each other. Or,

3. We may absorb or stop one of the beams, and leave the other, which will consequently be in a state of polarization.

The first of these methods of producing polarized light is that in which we employ a doubly refracting crystal, which we shall now consider.

On the Polarization of Light by Double Refraction.

(99.) When a beam of light suffers double refraction by a *negative* crystal, as *Iceland spar*, *fig*. 71., where the ray R *r* is incident in the plane of the principal section, or, what is the same thing, in a plane passing through the axis, the two pencils *r* O, *r* E are each polarized; the plane of polarization of the ordinary ray, *r* O, coinciding with the principal section, and the plane of polarization of the extraordinary ray, *r* E, being at right angles to the principal section. In *fig*. 82., if O be made to denote a section of the ordinary beam *r* O, *fig*. 71., E, the diameter of which is drawn at right angles to that of O, will represent a section of the extraordinary beam *r* E.

Fig. 82.

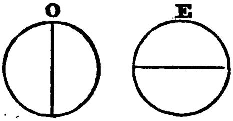
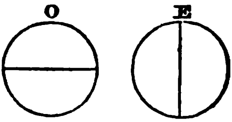


Fig. 83.



If the beam of light R *r* is incident upon a *positive* crystal, like *quartz*, O of *fig*. 83., will be the symbol of the ordinary ray, and E that of the extraordinary ray.

The phenomena which arise from this opposite polarization of the two pencils may be well seen in Iceland spar. For this purpose let A *r* X be the principal section of a rhomb of Iceland spar, *fig*. 84., through the axis A X, and perpendicular to one of the faces, and let A′ F X′ be a similar section of another rhomb, all the lines of the one being parallel to all the lines of the other. A ray of light, R *r*, incident perpendicularly at *r*, will be divided into two pencils; an ordinary one, *r* D, and an extraordinary one, *r* C. The ordinary ray falling on the second crystal at G, again suffers ordinary refraction, and emerges at K an ordinary ray, O *o*, represented by the symbol O, *fig*. 82. In like manner the extraordinary ray, *r* C, falling on the second crystal at F, again suffers extraordinary

Fig. 84.

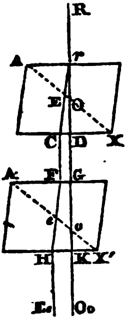
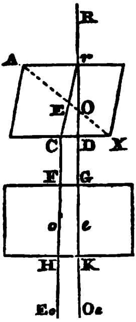


Fig. 85.



refraction, and emerges at H an extraordinary ray, E *e*, represented by E, *fig*. 82. These results are exactly the same as if the two crystals had formed a single crystal by being united at their surfaces C X, A′ G, either by natural cohesion or by a cement.

Let the upper crystal A X now remain fixed, with the same ray R *r* falling upon it, and let the second crystal A′ X′ be turned round 90°, so that its principal section is perpendicular to that of the upper one, as shown in *fig*. 85.; then the ray *r* D ordinarily refracted by the first rhomb will be extraordinarily refracted by the second, and the ray *r* C extraordinarily refracted by the first rhomb will be ordinarily refracted by the second.

The pencils or images formed from the ray R *r*, in the two positions shown in *figs*. 84. and 85., may be thus described as marked in the figures:—

O is the pencil refracted *ordinarily* by the *first* rhomb.

E is the pencil refracted *extraordinarily* by the *first* rhomb.

o is the pencil refracted *ordinarily* by the *second* rhomb.

e is the pencil refracted *extraordinarily* by the *second* rhomb.

O *o* is the pencil refracted *ordinarily* by both rhombs in *fig*. 84.

E *e* is the pencil refracted *extraordinarily* by both rhombs in *fig*. 84.

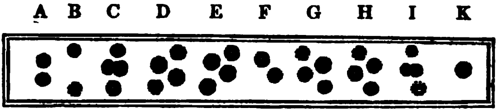
O *e* is the pencil refracted *ordinarily* by the *first*, and *extraordinarily* by the *second* rhomb in *fig*. 85.

E *o* is the pencil refracted *extraordinarily* by the *first*, and *ordinarily* by the *second* rhomb in *fig*. 85.

In both the cases shown in *figs*. 84. and 85., when the planes of the principal sections of the two rhombs are either parallel, as in *fig*. 84., or perpendicular to each other, as in *fig*. 85., the lower rhomb is not capable of dividing into two any of the pencils which fall upon it; but in every other position between the parallelism and the perpendicularity of the principal sections, each of the pencils formed by the first rhomb will be divided into two by the second.

In order to explain the appearances in all intermediate positions, let us suppose that the ray R *r* proceeds from a round aperture, like one of the circles at A, *fig*. 86., and that the eye is placed behind the two rhombs at H K, *fig*. 84., so as to see the images of this aperture. Let the two images shown at A, *fig*. 86., be the appearance of the aperture at R, seen through one of the rhombs by an eye placed behind C D, *fig*. 84., then B, *fig*. 86., will represent the images seen through the two rhombs in the position in *fig*. 84., their distance being doubled, from suffering the same quantity of double refraction twice. If we now turn the second rhomb, or that nearest the eye, from left to right, two faint images will appear, as at C, between the two bright ones, which will now be a little fainter. By continuing to turn, the four images will be all equally luminous, as at D; they will next appear as at E; and when the second rhomb has moved round 90°, as in *fig.* 85., there will be only two images of equal brightness, as at F. Continuing to turn the second rhomb, two faint images will appear, as at G; by a farther rotation, they will be all equally bright, as at H; farther on they will become unequal, as at I; and at 180° of revolution, when the planes of the principal section are again parallel, and the axes A X, A′ X′ at right angles nearly to each other, all the images will coalesce into one bright image, as at K, having double the brightness of either of those at A, B, or F, and four times the brightness of any one of the four at D and H.

Fig. 86.



If we now follow any one of the images A, B from the position in *fig.* 84., where the principal sections are inclined 0° to one another, to the position in *fig.* 85., where it disappears at F, we shall find that its brightness diminishes as the square of the cosine of the angle formed by the principal sections, while the brightness of any image, from its appearance between B and C, *fig.* 86., to its greatest brightness at F, increases as the square of the sine of the same angle.

By considering the preceding phenomena it will appear, that whenever the plane of polarization of a polarized ray, whether ordinary or extraordinary, coincides with or is parallel to the principal section, the ray will be refracted *ordinarily;* and whenever the plane of polarization is *perpendicular* to the principal section, it will be refracted *extraordinarily.* In all intermediate positions it will suffer both kinds of refraction, and will be doubly refracted; the ordinary pencil being the brightest if the plane of polarization is nearer the position of parallelism than that of perpendicularity, and the extraordinary pencil the brightest if the plane of polarization is nearer the position of perpendicularity than that of parallelism. At equal distances from both these positions, the ordinary and extraordinary images are equally bright.

(100.) It does not appear from the preceding experiments that the polarization of the two pencils is the effect of any polarizing force resident in the Iceland spar, or of any change produced upon the light. The Iceland spar has merely separated the common light into its two elements, according to a different law, in the same manner as a prism separates all the seven colors of the spectrum from the compound white beam by its power of refracting these elementary colors in different degrees. The re-union of the two oppositely polarized pencils produces common light, in the same manner as the re-union of all the seven colors produces white light.

The method of producing polarized light by double refraction is of all others the best, as we can procure by this means from a given pencil of light a stronger polarized beam than in any other way. Through a thickness of *three* inches of Iceland spar we can obtain two separate beams of polarized light *one third* of an inch in diameter; and each of these beams contains half the light of the original beam, excepting the small quantity of light lost by reflexion and absorption. By sticking a black wafer on the spar opposite either of these beams, we can procure a polarized beam with its plane of polarization either in the principal section or at right angles to it. In all experiments on this subject, the reader should recollect that every beam of polarized light, whether it is produced by the ordinary or the extraordinary refraction, or by positive or negative crystals, has always the same properties, provided the plane of its polarization has the same direction.

CHAP. XIX.*

ON THE POLARIZATION OF LIGHT BY REFLEXION.

(101.) In the year 1810, the celebrated French philosopher M. Malus, while looking through a prism of calcareous spar at the light of the setting sun reflected from the windows of the Luxembourg palace in Paris, was led to the curious discovery, that a beam of light reflected from *glass* at an angle of 56°, or from *water* at an angle of 53°, possessed the very same properties as one of the rays formed by a rhomb of calcareous spar; that is, that it was wholly polarized, having its plane of polarization coincident with or parallel to the plane of reflexion.

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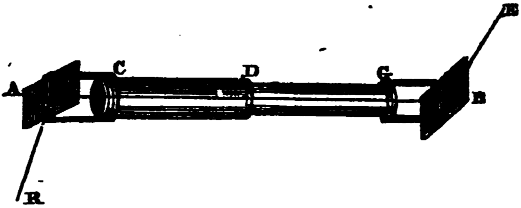
* For the formulæ relating to this chapter, see (in the College edition,) Appendix of Am. ed., Chap. VI.

This most curious and important fact, which he found to be true when the light was reflected from all other transparent or opaque bodies, excepting metals, gave birth to all those discoveries which have, in our own day, rendered this branch of knowledge one of the most interesting, as well as one of the most perfect, of the physical sciences.

In order to explain this and the other discoveries of Malus, let C D, *fig.* 87., be a tube of brass or wood, having at one end of it a plate of glass, A, not quicksilvered, and capable of turning round an axis, so that it may form different angles with the axis of the tube. Let D G be a similar tube a little smaller than the other, and carrying a similar plate of glass B. If the tube D G is pushed into C D, we may, by turning the one or the other round, place the two glass plates in any position in relation to one another.

Fig. 87.



Let a beam of light, R *r*, from a candle or a hole in the window-shutter, fall upon the glass plate A, at an angle of 56° 45′; and let the glass be so placed that the reflected ray *r s* may pass along the axis of the two tubes, and fall upon the second plate of glass B at the point *s*. If the ray *r s* falls upon the second plate B at an angle of 56° 45′ also, and if the plane of reflexion from this plate, or the plane passing through *s* E and *s r*, is at right angles to the plane of reflexion from the first plate, or the plane passing through *r* R, *r s*, the ray *r s* *will not suffer reflexion* from B, or will be so faint as to be scarcely visible. The very same thing will happen if *r s* is a ray polarized by double refraction, and having its plane of polarization in the plane passing through *r* R, *r s*. Here then we have a new property or test of polarized light,—that it will not suffer reflexion from a plate of glass B, when incident at an angle of 56° 45′, and when the plane of incidence or reflexion is at right angles to the plane of polarization of the ray. If we now turn round the tube D G with the plate B, without moving the tube C D, the last reflected ray s E will become brighter and brighter till the tube has been turned round 90°, when the plane of reflexion from B is coincident with or parallel to that from A. In this position the reflected ray s E is brightest. By continuing to turn the tube D G, the ray s E becomes fainter and fainter, till, after being turned 90° farther, the ray s E is faintest, or nearly vanishes, which happens when the plane of reflexion from B is perpendicular to that from A. After a farther rotation of 90°, the ray s E will recover its greatest brightness; and when, by a still farther rotation of 90°, the tube D G and plate B are brought back into their first position, the ray s E will again disappear. These effects may be arranged in a table, as follows:—

Inclination of the planes of the two reflexions, or the planes R r s and r s E, or azimuths of the plane r s E.	State of brightness of the image or ray s E reflected from the second plate B.
90°	Scarcely visible
At angles between 90° and 180° .	The image grows brighter and brighter
180°	Brightest
At angles between 180° and 270° .	The image grows fainter and fainter
270°	Scarcely visible
At angles between 270° and 360° .	The image grows brighter and brighter
360° or 0°	Brightest
At angles between 0° and 90° . .	The image grows fainter and fainter
90°	Scarcely visible

If we now substitute in place of the ray r s one of the polarized rays or beams formed by Iceland spar, so that its plane of polarization is in the plane R r s, it will experience the very same changes as the ray R r does when polarized by reflexion from A at an angle of 56° 45′. Hence it is manifest, that a ray reflected at 56° 45′ from glass has all the properties of polarized light as produced by double refraction.

(102.) In the preceding observations, the ray R r is supposed to be reflected only from the first surface of the glass; but Malus found that the light reflected from the second surface of the glass was polarized at the same time with that reflected from the first, although it obviously suffers reflexion at a different angle, viz. at an angle equal to the angle of refraction at the first surface.

The angle of 56° 45′, at which light is polarized by reflexion from glass, is called its maximum polarizing angle, because the greatest quantity of light is polarized at that angle. When the light was reflected at angles greater or less than 56° 45′, Malus found that a portion of it only was polarized, the remaining portion possessing all the properties of common light. The polarized portion diminished as the angle of incidence receded on either side from 56° 45′, and was nothing at 0°, or a perpendicular incidence, and also nothing at 90°, or the most oblique incidence.

In continuing his experiments on this subject, Malus found that the angle of maximum polarization varied with different bodies; and, after measuring it in various substances, he concluded *that it follows neither the order of the refractive powers nor that of the dispersive powers, but that it is a property of bodies independent of the other modes of action which they exercise upon light.* After he had determined the angles under which complete polarization takes place in different bodies, such as glass and water, he endeavored to ascertain the angle at which it took place at their separating surfaces when they were put in contact. In this inquiry, however, he did not succeed; and he remarks, "that the law according to which this last angle depends on the first two remains to be determined."

If a pencil or beam of light reflected at the maximum polarizing angle from glass and other bodies were as completely polarized as a pencil polarized by double refraction, then the two pencils would have been equally invisible when reflected from the second plate, B, at the azimuths 90° and 270°; but this is not the case: the pencil polarized by double refraction *vanishes* entirely when it passes through a second rhomb, even if it is a beam of the sun's direct light; whereas the pencil polarized by reflexion vanishes only if its light is faint, and if the plates A and B have a low dispersive power. When the sun's light is used, there is a large quantity of unpolarized light, and this unpolarized light is greatly increased when the plates A and B have a high dispersive power. This curious and most important fact was not observed by Malus.

A very pleasing and instructive variation of the general experiment shown in *fig.* 87. occurred to me in examining this subject. If, when the plates of glass A and B have the position shown in the figure where the luminous body from which the ray *s* E proceeds is invisible, we breathe gently upon the plate B, the ray *s* E will be recovered, and the luminous body from which it proceeds will be instantly visible. The cause of this is obvious: a thin film of water is deposited upon the glass by breathing, and as water polarizes light at an angle of about 53° 11′, the glass B should have been inclined at an angle of 53° 11′ to the ray *r s*, in order to be incapable of reflecting the polarized ray;* but as it is inclined 56° 45′ to the incident ray *r s*, it has the power of reflecting a portion of the ray *r s*.

If the glass B is now placed at an angle of 53° 11′ to the ray *r s*, it will then reflect a portion of the polarized ray *r s* to the eye at E; but if we breathe upon the glass B, the reflected light will disappear, because the reflecting surface is now water, and is placed at an angle of 53° 11′, the polarizing angle for water. If therefore we place two glass plates at B, the one inclined 56° 45′, and the other 53° 11′, to the beam *r s*, sufficiently large to fall upon both, the luminous object will be visible in the one but not in the other; but if we breathe upon the two plates, we shall exhibit the paradox of reviving an invisible image, and extinguishing a visible one by the same breath. This experiment will be more striking if the ray *r s* is polarized by double refraction.

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* We neglect the consideration of the separating surface of the water and glass, and suppose the glass B to be opaque.

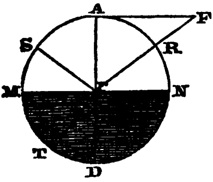
On the Law of the Polarization of Light by Reflexion.

(103.) From a very extensive series of experiments made to determine the maximum polarizing angles of various bodies, both solid and fluid, I was led, in 1814, to the following simple law of the phenomena:—

The index of refraction is the tangent of the angle of polarization.

In order to explain this law, and to show how to find the polarizing angle for any body whose index of refraction is known, let M N be the surface of any transparent body, such as *water*. From any point, *r*, draw *r* A perpendicular to M N, *fig.* 88., and round *r* as a centre describe a circle, M A N D. From A draw A F, touching the circle at A, and from any scale on which A *r* is 1 or 10 set off A F equal to 1·336 or 13·36, the index of refraction for water. From F draw F *r*, which will be the incident ray that will be polarized by reflexion from the water in the direction *r* S. The angle A *r* R will be 53° 11′, or the angle of maximum polarization for water. This angle may be obtained more readily by looking for 1·336 in the column of natural tangents in a book of logarithms, and there will be found opposite to it the corresponding angle of 53° 11′. If we calculate the angle of refraction T *r* D, corresponding to the angle of incidence A *r* R, or determine it by projection, we shall find it to be 36° 49′.

Fig. 88.



From the preceding law we may draw the following conclusions:—

1. The maximum polarizing angle, for all substances whatever, is the complement of the angle of refraction. Thus, in water, the complement of 36° 49′ is 53° 11′, the polarizing angle.

2. At the polarizing angle, the sum of the angles of incidence and refraction is a right angle, or 90°. Thus, in water, the angle of incidence is 53° 11′, and that of refraction 36° 49′, and their sum is 90°.

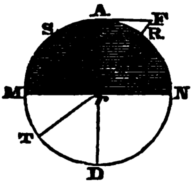
3. When a ray of light, R r, is polarized by reflexion, the reflected ray, r S, forms a right angle with the refracted ray, r T.

When light is reflected at the second surface of bodies, the law of polarization is as follows:—

The index of refraction is the cotangent of the angle of polarization.

In order to determine the angle in this case, let M N be the second surface of any body such as water. From r draw r A perpendicular to M N, *fig.* 89., and round r describe the circle M A N D. From A draw A F, touching the circle at A, and upon a scale in which r N is 1 take A F equal to ·7485, that is to $\frac{1}{1{\cdot}336}$ the reciprocal of the index of refraction,* and from F draw F r; the ray R r will be polarized when reflected in the direction r S. The maximum polarizing angle A r R will be 36° 49′, exactly equal to the angle of refraction of the first surface. Hence it follows,

Fig. 89.



1. That the polarizing angle at the second surface of bodies is equal to the complement of the polarizing angle at the first, or to the angle of refraction at the first surface. The reason is, therefore, obvious why the portions of a beam of light reflected at the first and second surfaces of a transparent parallel plate are simultaneously polarized.

2. That the angle which the reflected ray r S forms with the refracted ray r T is a right angle.

The laws of polarization now explained are applicable to the separating surfaces of two media of different refractive powers. If the uppermost fluid is water, and the undermost glass, then the index of refraction of their separating surface is equal to $\frac{1{\cdot}525}{1{\cdot}336}$, to the greater index divided by the lesser, which is 1·1415. By using this index it will be found that the polarizing angle is 48° 47′.

When the ray moves from the less refractive substance into the greater, as from water to glass, as in the preceding case, we must make use of the law and the method above explained for the *first* surface of bodies; but when the ray moves from the greater refractive body into the less, as from oil of cassia to glass, we must use the law and method for the *second* surface of bodies.

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* The tangent of an angle to radius 1, is the reciprocal of the cotangent.

If we lay a parallel stratum of water upon glass whose index of refraction is 1·508, the ray reflected from the refracting surfaces will be polarized when the angle of incidence upon the first surface of the water is 90°

(104.) The preceding observations are all applicable to white light, or to the most luminous rays of the spectrum; but, as every different color has a different index of refraction, the law enables us to determine the angle of polarization for every different color, as in the following table, where it is supposed that the most luminous ray of the spectrum is the mean one:—

		Index of Refraction.	Maximum Polarizing Angle.	Difference between the greatest and least Polarizing Angles.
WATER	Red rays	1·330	53° 4′	15′
	Mean rays	1·336	53 11	
	Violet rays	1·342	53 19	
PLATE GLASS	Red rays	1·515	56 34	21′
	Mean rays	1·525	56 45	
	Violet rays	1·535	56 55	
OIL OF CASSIA	Red rays	1·597	57 57	1° 24′
	Mean rays	1·642	58 40	
	Violet rays	1·687	59 21	

The circumstance of the different rays of the spectrum being polarized at different angles, enables us to explain the existence of *unpolarized* light at the maximum polarizing angle, or why the ray *s* E, in *fig.* 87., never wholly vanishes. If we were to use *red* light, and set the two plates at angles of 56° 34′, the polarizing angle of glass for *red* light, then the pencil *s* E would vanish entirely. But when the light is *white*, and the angle at which the plates are set is 56° 45′, or that which belongs to mean or yellow rays, then it is only the *yellow* rays that will vanish in the pencil *s* E. A small portion of *red* and a small portion of *violet* will be reflected, because the glasses are not set at their polarizing angles; and the mixture of these two colors will produce a purple color, which will be that of the unpolarized light which remains in the pencil *s* E. If we place the plates at the angle belonging to the *red* ray, then the *red* only will vanish, and the color of the unpolarized light will be bluish green. If we place the plates at the angle corresponding with the *blue* light, then the *blue* only will vanish, and the unpolarized light will be of a *reddish* cast. In *oil of cassia*, *diamond*, *chromate of lead*, *realgar*, *specular iron*, and other highly dispersive substances, the color of the unpolarized light is extremely brilliant and beautiful.

Certain doubly refracting crystals, such as *Iceland spar* *chromate of lead*, &c., have different polarizing angles on different surfaces, and in different directions on the same surface; but there is always one direction where the polarization is not affected by the doubly refracting force, or where the tangent of the polarizing angle is equal to the index of ordinary refraction.

On the partial Polarization of Light by Reflexion.

(105.) If, in the apparatus in *fig*. 87., we make the ray R *r* fall upon the plate A at an angle *greater* or *less* than 56° 45′, then the ray *s* E will not vanish entirely; but, as a considerable part of it will vanish like polarized light, Malus called it *partially polarized* light, and considered it as composed of a portion of light perfectly polarized, and of another portion in the state of common light. He found the quantity of polarized light to diminish as the angle of incidence receded from that of maximum polarization.

M. Biot and M. Arago also maintained that partially polarized light consisted partly of polarized and partly of common light; and the latter announced that, at regular angular distances above and below the maximum polarizing angle, the reflected pencil contained the same proportion of polarized light. In *St. Gobin's glass* he found that the same proportion of light was polarized at an angle of incidence of 82° 48′ as at 24° 18′; in *water* he found that the same proportion was polarized at 16° 12′ as at 86° 31′; but he remarks, "that the mathematical law which connects the value of the quantity of polarized light with the angle of incidence and the refractive power of the body has not yet been discovered."

In the investigation of this subject, I found that though there was only one angle at which light could be completely polarized by one reflexion, yet it might be polarized at any angle of incidence by a *sufficient number of reflexions*, as shown in the following Table.

BELOW THE POLARIZING ANGLE.		ABOVE THE POLARIZING ANGLE.	
No. of Reflexions.	Angle at which the Light is polarized.	No. of Reflexions.	Angle at which the Light is polarized.
1	56° 45′	1	56° 45′
2	50 26	2	62 30
3	46 30	3	65 33
4	43 51	4	67 33
5	41 43	5	69 1
6	40 0	6	70 9
7	38 33	7	71 5
8	37 20	8	71 51

In polarizing light by successive reflexions, it is not necessary that the reflexions be performed at the same angle. Some of them may be above and some below the polarizing angle, or all the reflexions may be performed at different angles.

From the preceding facts it follows as a necessary consequence, that partially polarized light, or light reflected at an angle different from the polarizing angle, has suffered a physical change, which enables it to be more easily polarized by a subsequent reflexion. The light, for example, which remains unpolarized after five reflexions at 70°, in place of being common light, has suffered such a physical change that it is capable of being completely polarized by ONE reflexion more at 70°.

This view of the subject has been rejected by M. Arago, as incompatible with experiments and speculations of his own; and, in estimating the value of the two opinions, Mr. Herschel has rejected mine as the least probable. It will be seen, however, from the following facts, that it is capable of the most rigorous demonstration.

It does not appear, from the preceding inquiries, how a beam of common light is converted into polarized light by reflexion. By a series of experiments made in 1829, I have been able to remove this difficulty. It has been long known that a polarized beam of light has its plane of polarization changed by reflexion from bodies. If its plane is inclined 45° to the plane of reflexion, its inclination will be diminished by a reflexion at 80°, still more by one at 70°, still more by one at 60°; and at the polarizing angle the plane of the polarized ray will be in the plane of reflexion, the inclination commencing again at reflexions above the polarizing angle, and increasing till at 0°, or a perpendicular incidence, the inclination is again 45°.* I now conceived a beam of common light, constituted as in *fig.* 81., to be incident on a reflecting surface, so that the plane of reflexion bisected the angle of 90° which the two planes of polarization, A B, C D, formed with each other, as shown in *fig.* 90., No. 1., where M N is the plane of reflexion, and A B, C D the planes of polarization of the beam of white light, each inclined 45° to M N. By a reflexion from glass, where the index of refraction is 1·525, at 80°, the inclination of A B to M N will be 33° 13′, as in No. 2., instead of 45°; and in like manner the inclination of C D to M N will be 33° 13′, in place of 45°; so that the inclination of A B to C D in place of 90° is 66° 26′, as in No. 2. At an incidence of 65° the inclination of A B to C D will be 25° 36′, as in No. 3.; and at the polarizing angle of 56° 45′ the planes A B, C D of the two beams will be parallel or coincident, as in No. 4. At incidences below 56° 45′ the planes will again open, and their inclination will increase till at 0° of incidence it is 90°, as in No. 1., having been 25° 36′ at an incidence of about 48° 15′, as in No. 3., and 66° 26′ at an incidence of about 30°, as in No. 2.

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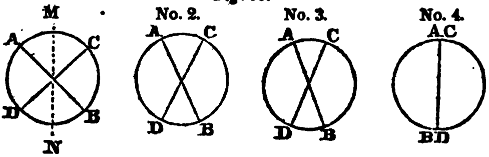
* The rule for finding the inclination is this:—Find the sum of the angles of incidence and refraction, and also their difference; divide the cosine of the former by the cosine of the latter, and the quotient will be the tangent of the inclination required.

Fig. 90.



No. 1. No. 2. No. 3. No. 4.

In the process now described, we see the manner in which *common light*, as in No. 1., is converted into *polarized light*, as in No. 4., by the action of a reflecting surface. Each of the two planes of its component polarized beams is turned round into a state of parallelism, so as to be a beam with only one plane of polarization, as in No. 4.; a mode of polarization essentially different in its nature from that of double refraction. The numbers in *fig.* 90. present us with beams of light in *different stages of polarization* from *common light* in No. 1. to *polarized light* in No. 4. In No. 2. the beam has made a certain approach to polarization, having suffered a physical change in the inclination of its planes; and in No. 3. it has made a nearer approach to it. Hence we discover the whole mystery of *partial polarization*, and we see that *partially polarized light is light whose planes of polarization are inclined at angles less than* 90° *and greater than* 0°. The influence of successive reflexions is therefore obvious. A reflexion at 80° will turn the planes, as in *fig.* 90., No. 2.; another reflexion at 80° will bring them closer; a third still closer; and so on: and though they never can by this process be brought into a state of exact parallelism, as in No. 4. (which can only be done at the polarizing angle), yet they can be brought infinitely near it, so that the beam will appear as completely polarized as if it had been reflected at the polarizing angle. The correctness of my former experiments and views is, therefore, demonstrated by the preceding analysis of common light.

It is manifest from these views that partially polarized light *does not contain a single ray of completely polarized light;* and yet if we reflect it from the second plate B, in *fig.* 87., at the polarizing angle, a certain portion of it will disappear as if it were polarized light, a result which led to the mistake of Malus and others. The light which thus disappears may be called apparently polarized light; and I have explained in another place* how we may determine its quantity at any angle of incidence, and for any refractive medium. The following Table contains some of the results for glass, whose index of refraction is 1·525. The quantity of reflected light is calculated by a rule given by M. Fresnel.

Angles of Incidence.	Inclination of the Planes of Polarization, A B, C D, *fig.* 90.	Quantity of reflected Rays out of 1000.	Quantity of polarised Rays out of 1000.
0°	90° 0′	43·23	0·
20	80 26	43·41	7·22
40	47 22	49·10	33·25
56 45′	0 0	79·5	79·5
70	37 41	162·67	129·8
80	66 26	391·7	156·6
85	78 24	616·28	123·75
90	90 0	1000·	0·

CHAP. XX.

ON THE POLARIZATION OF LIGHT BY ORDINARY REFRACTION.

(106.) ALTHOUGH it might have been presumed that the light refracted by bodies suffered some change, corresponding to that which it receives from reflexion, yet it was not until 1811 that it was discovered that the refracted portion of the beam contained a portion of polarized light.†

To explain this property of light, let R *r*, *fig.* 91., be a beam of light incident at a great angle, between 80° and 90°, on a horizontal plate of glass, No. 1.; a portion of it will be reflected at its two surfaces, *r* and *a*, and the refracted beam *a* is found to contain a small portion of polarized light.

If this beam *a* falls upon a second plate, No. 2., parallel to the first, it will suffer two reflexions; and the refracted pencil *b* will contain more polarized light than *a*. In like manner, by transmitting it through the plates Nos. 3, 4, 5, and

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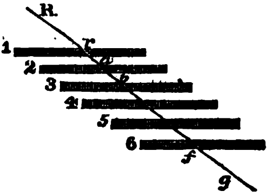
* See *Phil. Transactions*, 1830, p. 76., or Edinburgh *Journal of Science*, New Series, No. V., p. 160.

† This discovery was made by independent observation by Malus, Biot, and the author of this work.

6., the last refracted pencil, *f g*, will be found to consist entirely, so far as the eye can judge, of polarized light. But, what is very interesting, the beam *f g* is not polarized in the plane of refraction or reflexion, but in a plane at right angles to it; that is, its plane of polarization is not represented by A′ B′ *fig.* 81., as is the ordinary ray in Iceland spar, or as light polarized by reflexion, but by C′ D′ like the extraordinary ray in Iceland spar. From a great number of experiments, I found that the light of a wax candle at the distance of 10 or 12 feet was polarized at the following angles, by the following number of plates of crown glass.

Fig. 91.



No. of Plates of Crown Glass.	Observed Angles at which the Pencil is polarized.	No. of Plates of Crown Glass.	Observed Angles at which the Pencil is polarized.
8	79° 11′	27	57° 10′
12	74 0	31	53 28
16	69 4	35	50 5
21	63 21	41	45 35
24	60 8	47	41 41

It follows from the above experiments, that if we divide the number 41·84 by any number of crown glass plates, we shall have the tangent of the angle at which the beam is polarized by that number.

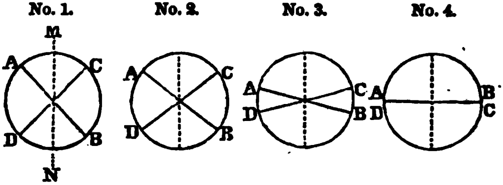
Hence it is obvious that the power of polarizing the refracted light increases with the angle of incidence, being nothing or a minimum at a perpendicular incidence, or 0°, and the greatest possible or a maximum at 90° of incidence. I found, likewise, by various experiments, that the power of polarizing the light at any given angle increased with the refractive power of the body, and consequently that a smaller number of plates of a highly refracting body was necessary than of a refracting body of low power, the angle of incidence being the same.

As Malus, Biot, and Arago considered the beams *a*, *b*, &c., before they were completely polarized, as *partially polarized*, and as consisting of a portion of polarized and a portion of unpolarized light; so, on the other hand, I concluded from the following reasoning that the *unpolarized* light had suffered a physical change, which made it approach to the state of complete polarization. For since sixteen plates are required to polarize completely a beam of light incident at an angle of 69°, it is clear that eight plates will not polarize the whole beam at the same angle, but will leave a portion *unpolarized*. Now, if this portion were absolutely unpolarized like common light, it would require to pass through other sixteen plates, at an angle of 69°, in order to be completely polarized; but the truth is, that it requires to pass through only eight plates to be completely polarized. Hence I conclude that the beam has been nearly half polarized by the first eight plates, and the polarization completed by the other eight. This conclusion, though rejected by both the French and English philosophers, is capable of rigid demonstration, as will appear from the following observations.

In order to determine the change which *refraction* produced in the plane of polarization of a polarized ray, I used prisms and plates of glass, plates of water, and a plate of a highly refractive metalline glass; and I found that a refracting surface produced the greatest change at the most oblique incidence, or that of 90°; and that the change gradually diminished to a perpendicular incidence, or 0°, where it was nothing. I found also that the greatest effect produced by a single plate of glass was about 16° 39′, at an angle of 86°; that it was 3° 54′ at an angle of 55°, 1° 12′ at an angle of 35°, and 0° at an angle of 0°.*

A beam of common light, therefore, constituted as in *fig.* 92., No. 1., with each of its planes A B, C D inclined 45° to

Fig. 92.



the plane of refraction, will have these planes opened 16° 39′

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* The rule for finding the inclination after a single refraction is as follows:—Find the difference between the angles of incidence and refraction, and take the cosine of this difference. This number will be the cotangent of the inclination required; and twice this inclination will be the inclination of A B to C D.

each, by one plate of glass at an incidence of 86°; that is, their inclination, in place of 90°, will be 123° 18′, as in No. 2. By the action of two or three plates more they will be opened wider, as in No. 3.; and by 7 or 8 plates they will be opened to near 180°, or so that A B, C D nearly coincide, as in No. 4., so as to form a single polarized beam, whose plane of polarization is perpendicular to the plane of refraction. I have shown, in another place,* that these planes can never be brought into mathematical coincidence by any number of refractions; but they approach so near to it that the pencil is, to all appearance, completely polarized with lights of ordinary strength. All the light polarized by refraction is only partially polarized, and it has the same properties as that which is partially polarized by reflexion. A certain portion of the light of a beam thus partially polarized, will disappear when reflected at the polarizing angle from the plate B, *fig.* 87.; and this quantity, which I have elsewhere shown how to calculate, is given in the following table for a *single surface* of glass, whose index of refraction is 1·525.

Angle of Incidence.	Inclination of the Planes of Polarization A B, C D, *fig.* 92.	Quantity of transmitted Rays out of 1000.	Quantity of polarized Rays out of 1000.
0°	90° 0′	956·77	0
20	90 26	956·59	7·22
40	92 0	950·90	32·2
56° 45′	94 58	920·5	79·5
70	98 56	837·33	129·8
80 40′	104 55	608·3	156·7
85	108 44	383·72	123·7
90	112 58	0	0

Although the quantity of light polarized by refraction, as given in the last column of this Table, is calculated by a formula essentially different from that by which the quantity of light polarized by reflexion was calculated; yet it is curious to see that the two quantities are precisely equal. Hence we obtain the following law:—

When a ray of common light is reflected and refracted by any surface, the quantity of light polarized by refraction is exactly equal to that polarized by reflexion.

This law is not at all applicable to plates, as it appeared to be from the experiments of M. Arago.

When the preceding method of analysis is applied to the light reflected by the second surfaces of plates, we obtain the following curious law:—

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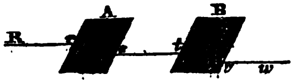
* See *Phil. Transactions*, 1830, p. 137., or *Edinburgh Journal of Science*, New Series, No. VI., p. 218.

A pencil of light reflected from the second surfaces of transparent plates, and reaching the eye after two refractions and an intermediate reflexion, contains at all angles of incidence, from 0° to the maximum polarizing angle, a portion of light polarized in the plane of reflexion. Above the polarizing angle, the part of the pencil polarized in the plane of reflexion diminishes, till the incidence becomes 78° 7′ in glass, when it disappears, and the whole pencil has the character of common light. Above this last angle the pencil contains a quantity of light polarized perpendicularly to the plane of reflexion, which increases to a maximum, and then diminishes to nothing at 90°.[*]

(107.) As a bundle of glass plates acts upon light, and polarizes it as effectually as reflexion from the surface of glass at the polarizing angle, we may substitute a bundle of glass plates in the apparatus, *fig.* 87., in place of the plates of glass A, B. Thus, if A (*fig.* 93.) is a bundle of glass plates which polarizes the transmitted ray *s t*, then, if the second bundle B is placed as in the figure, with the planes of refraction of its plates parallel to the planes of refraction of the plates of A, the ray *s t* will penetrate the second bundle; and if *s t* is incident on B at the polarizing angle, not a ray of it will be reflected by the plates of B. If B is now turned round its axis, the transmitted light *v w* will gradually diminish, and more and more light will be reflected by the plates of the bundle, till, after a rotation of 90°, the ray *v w* will disappear, and all the light will be reflected. By continuing to turn round B, the ray *v w* will re-appear, and reach its maximum brightness at 180°, its minimum at 270°, and its maximum at 0°, after having made one complete revolution.

Fig. 93.



By this apparatus we may perform the very same experiments with refracted polarized light that we did with reflected polarized light in the apparatus of *fig.* 87.

We have now described two methods of converting common light into polarized light: 1st, By separating by double refraction the two oppositely polarized beams which constitute common light; and, 2dly, By turning round, by the action of the reflecting and refracting forces, the planes of both these beams till they coincide, and thus form light polarized in one plane. Another method still remains to be noticed; namely, to disperse or absorb one of the oppositely polarized beams which constitute common light, and leave the other beam polarized in one plane. These effects may be produced by agate and tourmaline, &c.

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[*] See *Phil. Trans.* 1830, p. 145.; or *Edinburgh Journal of Science*, No. VI., p. 234, New Series.

(108.) If we transmit a beam of common light through a plate of agate, one of the oppositely polarized beams will be converted into a nebulous light in one position, and the other polarized beam in another position, so that one of the polarized beams with a single plane of polarization is left. The same effect may be produced by Iceland spar, arragonite, and artificial salts prepared in a particular manner, to produce a dispersion of one of the oppositely polarized beams.[*]

When we transmit common light through a thin plate of *tourmaline*, one of the oppositely polarized beams which constitute common light is entirely absorbed in one position, and the other in another position, one of them always remaining with a single plane of polarization.

Hence, plates of agate and tourmaline are of great use, either in affording a beam of light polarized in one plane, or in dispersing and absorbing one of the pencils of a compound beam, when we wish to analyze it, or to examine the color or properties of one of the pencils seen separately.

CHAP. XXI.

ON THE COLORS OF CRYSTALLIZED PLATES IN POLARIZED LIGHT.

(109.) THE splendid colors, and systems of colored rings, produced by transmitting polarized light through transparent bodies that possess double refraction, are undoubtedly the most brilliant phenomena that can be exhibited. The colors produced by these bodies were first discovered by independent observation, by M. Arago and the author of this volume; and they have been studied with great success by M. Biot and other authors.

In order to exhibit these phenomena, let a polarizing apparatus be prepared, similar in its nature to that in *fig.* 87.; but without the tubes, as shown in *fig.* 94., where A is a plate of glass which polarizes the ray R *r*, incident upon it at an angle of 56° 45′, and reflects it polarized in the direction *r s*, where it is received by a second plate of glass, B, whose plane of reflexion is at right angles to that of the plate A, and which reflects it to the eye at O, at an angle of 56° 45′. In order that the polarized pencil *r s* may be sufficiently brilliant, ten or twelve plates of window glass, or, what is better still, of thin and well-annealed flint glass, should be substituted in place of the single plate A. The plate or plates at A are called the *polarizing plates*, because their only use is to furnish us with a broad and bright beam of polarized light. The plate B is called the *analyzing plate*, because its use is to analyze, or separate into its parts, the light transmitted through any body that may be placed between the eye and the polarizing plate.

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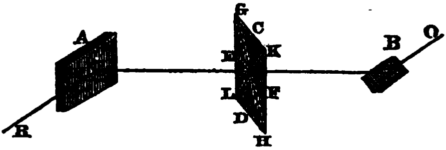
[*] See *Edinburgh Encyclopædia*, vol. xv. pp. 600, 601.; *Phil. Trans.* 1819, p. 146.

Fig. 94.



If the beam of light R *r* proceeds from the sky, which will answer well enough for common purposes, then an eye placed at O will see, in the direction O *s*, the part of the sky from which the beam R *r* proceeds. But as *r s* will be polarized light if it is reflected at 56° 45′ from A, almost none of it will be reflected to the eye at O from the plate B; that is, the eye at O will see, upon the part of the sky from which R *r* proceeds, a *black spot;* and when it does not see this black spot, it is a proof that the plates A and B are not placed at the proper inclinations to each other. When a position is found, either by moving A or B, or both, at which the black spot is darkest, the apparatus is properly adjusted.

(110.) Having procured a thin film of *sulphate of lime* or *mica*, between the 20th and the 60th of an inch thick, and which may be split by a fine knife or lancet from a mass of any of these minerals in a transparent state, expose it, as shown at C E D F, so that the polarized beam *r s* may pass through it perpendicularly. If we now apply the eye at O, and look towards the black spot in the direction O *s*, we shall see the surface of the plate of *sulphate of lime* entirely covered with the most brilliant colors. If its thickness is perfectly uniform throughout, its tint will be perfectly uniform; but if it has different thicknesses, every different thickness will display a different color—some red, some green, some blue, and some yellow, and all of the most brilliant description. If we turn the film C E D F round, keeping it perpendicular to the polarized beam, the colors will become less or more bright without changing their nature, and two lines, C D, E F at right angles will be found, so that when either of them is in the plane of reflexion *r s* O, *no colors whatever* are perceived, and the black spot will be seen as if the sulphate of lime had not been interposed, or as if a piece of common glass had been substituted for it. It will also be observed, by continuing the rotation of the sulphate of lime, that the colors again begin to appear; and reach their greatest brightness when either of the lines G H, L K, which are inclined 45° to C D, E F, are in the plane of reflexion *r s* O. The plane R *r s*, or the plane in which the light is polarized, is called the *plane of primitive polarization;* the lines C D, E F, the *neutral axes;* and G H, K L, the *depolarizing axes*, because they depolarize, or change the polarization of the polarized beam *r s*. The brilliancy or intensity of the colors increases gradually, from the position of no color, to that in which it is the most brilliant.

Let us now suppose the plate C E D F to be fixed in the position where it gives the brightest color; namely, when G H is perpendicular to the plane of primitive polarization R *r s*, or parallel to the plane *r s* O, and let the color be *red*. Let the analyzing plate B be made to revolve round the ray *r s*, beginning its motion at 0°, and preserving always the same inclination to the ray *r s*, viz. 56° 45′. The *brightest red* being now visible at 0°, when the plate B begins to move from its position shown in the figure, its brightness will gradually diminish till B has turned round 45°, when the red color will wholly disappear, and the black spot in the sky be seen. Beyond 45° a faint *green* will make its appearance, and will become brighter and brighter till it attains its greatest brightness at 90°. Beyond 90° the green becomes paler and paler till it disappears at 135°. Here the red again appears, and reaches its maximum brightness at 180°. The very same changes are repeated while the plate B passes from 180° round to its first position at 360° or 0°. From this experiment it appears, that when the film C E D F alone revolves, only *one color* is seen; and when the plate B only revolves, *two colors* are seen during each half of its revolution.

If we repeat the preceding experiment with films of different thicknesses, that give different colors, we shall find that the two colors are always *complementary* to each other, or together make *white* light.

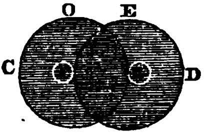
(111.) In order to understand the cause of these beautiful phenomena, let the eye be placed between the film and the plate B, and it will be seen that the light transmitted through the film is white, whatever be the position of the film. The separation of the colors is therefore produced, or the white light is analyzed, by reflexion from the plate B. Now, sulphate of lime is a doubly refracting crystal; and one of its neutral axes, C D, is the section of a plane passing through its axis, while E F is the section of a plane perpendicular to the principal section. Let us now suppose either of these planes, for example E F, to be placed, as in the figure, in the plane of polarization R *r s* of the polarized light; then this ray will not be doubled, but will pass into the ordinary ray of the crystallized film; and falling upon B, it will not suffer reflexion. In like manner, if C D is brought into the plane R *r s*, it will pass entirely into the ordinary ray, which, falling upon B, will not suffer reflexion. In these two positions of the film, therefore, it forms only a single image or beam; and as the plane of polarization of this image or beam is at right angles to the plane of reflexion from B, none of it is reflected to the eye at O. But in every other position of the doubly refracting film C E D F, it forms two images of different intensities, as may be inferred from *fig.* 86.; and when either of the depolarizing axes G H or K L is in the plane of primitive polarization, the two images are of equal brightness, and are polarized in opposite planes; one in the plane of primitive polarization, and the other at right angles to it. Now, one of these images is *red*, and the other *green*, for reasons which will be afterwards explained; and as the *green* is polarized in the plane of primitive polarization R *r s*, it does not suffer reflexion from the plate B; while the *red*, being polarized at right angles to that plane, is reflected to the eye at O, and is therefore alone seen. For a similar reason, when B is turned round 90°, the *red* will not suffer reflexion from it; while the *green* will suffer reflexion, and be transmitted to the eye at O. In this case the plate B analyzes the compound beam of white light transmitted through the film of sulphate of lime, by reflecting the half of it which is polarized in the plane of its reflexion, and refusing to reflect the other half, which is polarized in an opposite plane. If the two beams had been each white light, as they are in *thick* plates of sulphate of lime, in place of seeing two different colors during the revolution of the plate B, the reflected pencil *s* O would have undergone different variations of brightness, according as the two oppositely polarized beams of white light were more or less reflected by it; the positions of greatest brightness being those where the red and green colors were the brightest, and the darkest points being those where no color was visible.

(112.) The analysis of the white beam composed of two beams of *red* and *green* light, has obviously been effected by the power of the plate to *reflect* the one and to *transmit* or *refract* the other; but the same beam may be analyzed by various other methods. If we make it pass through a rhomb of calcareous spar sufficiently thick to separate by double refraction the *red* from the *green* beam, we shall at the same time see both the colored beams, which we could not do in the former case; the one forming the ordinary, and the other the extraordinary image. Let us now remove the plate B, and substitute for it a rhomb of calcareous spar, with its principal section in the plane of reflexion *r s* O, or perpendicular to the plane of primitive polarization R *r s*, and let the rhomb have a round aperture in the side farthest from the eye, and of such a size that the two images of the aperture, formed by double refraction, may just touch one another. Remove the film C E D F, and the eye placed behind the rhomb will see only the extraordinary image of the aperture, the ordinary one having vanished. Replace the film, with its neutral axes as in the figure, parallel and perpendicular to the plane R *r s*, and no effect will be produced; but if either of the depolarizing axes are brought into the plane R *r s*, the ordinary image of the aperture will be a brilliant *red*, and the extraordinary image a brilliant *green*; the double refraction of the rhomb having separated these two differently colored and oppositely polarized beams. By turning round the film, the colors will vary in brightness; but the same image will always have the same color. If we now keep the film fixed in the position that gives the finest colors, and move the rhomb of calcareous spar round, so that its principal section shall make a complete revolution, we shall find that, after revolving 45° from its first position, both images become white. After revolving 90°, the ordinary image that was formerly *red* is now *green*, and the extraordinary image that was formerly *green* is now *red*. The two images become again *white* at 135°, 225°, and 315°; and at 180°, the ordinary image is again *red*, and the extraordinary one *green*; and at 270°, the ordinary image is *green*, and the other *red*.

If we use a large circular aperture on the face of the rhomb, the ordinary and extraordinary images O, E will overlap each other, as in *fig.* 95.; the overlapping parts at F G being pure white light, and the parts at C and D having the colors above described. This experiment affords ocular demonstration that the two colors at C and D are complementary, and form white light.

The analysis of the compound beam transmitted by the sulphate of lime may also be effected by a plate of *agate*, or by any of the crystals, artificially prepared for the purpose of dispersing one of the component beams. The agate being placed between the eye and the film C E D F, it will disperse into nebulous light the *red* beam, and enable the *green* one to reach the eye; while in another position it will scatter the *green* beam, and allow the *red* light to reach the eye. With a proper piece of agate this experiment is both beautiful and instructive; as the nebulous light, scattered round the bright image, will be *green* when the distinct image is *red*, and *red* when the distinct image is *green*.

Fig. 95.



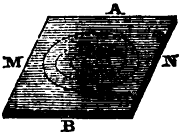
The analysis may also be effected by the absorption of *tourmaline* and other similar substances. In one position the tourmaline absorbs the *green* beam, and allows the red to pass; while in another position it absorbs the *red*, and suffers the *green* to pass. The yellow color of the tourmaline, however, is a disadvantage.

The analysis may also be performed by a bundle of glass plates, such as A or B, *fig.* 93. In one position such a bundle will *transmit* all the *red*, and reflect all the *green*; while in another position it will *transmit* all the *green*, and *reflect* all the *red*, in the opposite manner, but according to the same rules as the analyzing plate B, *fig.* 94.

(113.) In all these experiments the thickness of the sulphate of lime has been supposed such as to give a *red* and a *green* tint; but if we take a film 0·00046 of an English inch thick, and place it at C E D F in *fig.* 94., it will produce no colors at all, and the black spot in the sky will be seen, whatever be the position of the film. A film 0·00124 thick will give the *white* of the first order in Newton's scale of colors, given in p. 93; and a plate 0·01818 of an inch thick, and all plates of greater thickness, will give a white composed of all the colors. Films or plates of intermediate thicknesses between 0·00124 and 0·01818 will give all the intermediate colors in Newton's Table between the white of the first order and the white arising from the mixture of all the colors. That is, the colors reflected to the eye at O will be those in column 2d, while the colors observed by turning round the plate B will be those in column 3d; the one set of colors corresponding to the reflected tints, and the other to the transmitted tints of thin plates. In order to determine the thickness of a film of sulphate of lime which gives any particular color in the Table, we must have recourse to the numbers in the last column for glass, which has nearly the same refractive power as sulphate of lime. Suppose it is required to have the thickness which corresponds to the *red* of the first spectrum or order of colors. The number in the column for glass, opposite red, is 5⅘; then, since the *white* of the first order is produced by a film 0·00124 of an inch thick, the number corresponding to which is 3⅖ in the column for glass, we say, as 3⅖ is to 5⅘, so is 0·00124 to 0·00211, the thickness which will give the *red* of the first order. In the same manner, by having the thickness of any film of this substance, we can determine the color which it will produce.

Since the colors vary with the thickness of the plate, it is manifest, that if we could form a wedge of sulphate of lime, with its thickness varying from 0·00124 to 0·01818 of an inch, we should observe at once all the colors in Newton's Table in parallel stripes. An experiment of the same kind may be made in the following manner:—Take a plate of sulphate of lime, M N, *fig*. 96., whose thickness exceeds 0·01818 of an inch. Cement it with isinglass on a plate of glass; and placing it upon a fine lathe, turn out of it with a very sharp tool a concave or hollow surface between A and B, turning it so thin at the centre that it either begins to break or is on the eve of breaking. If the plate M N is now placed in water, the water will after some time dissolve a small portion of its substance, and polish the turned surface to a certain degree. If the plate is now held at C E D F, *fig*. 94., we shall see all the colors in Newton's Table in the form of concentric rings,

Fig. 96.



as shown in the figure. If the thickness diminishes rapidly the rings will be closely packed together, but if the turned surface is large, and the thickness diminishes slowly, the colored bands will be broad. In place of turning out the concavity, it might be done better by grinding it out, by applying a convex surface of great radius, and using the finest emery. When the plate M N is thus prepared, we may give the most perfect polish to the turned surface by cementing upon it a plate of glass with Canada balsam. The balsam will dry, and the plate may be preserved for any length of time.

By the method now described, the most beautiful patterns, such as are produced in bank-notes, &c., may be turned upon a plate of sulphate of lime 0·01818 of an inch thick, cemented to glass. All the grooves or lines that compose the pattern may be turned to different depths, so as to leave different thicknesses of the mineral, and the grooves of different depths will all appear as different colors, when the pattern is held in the apparatus in *fig.* 94. Colored drawings of figures and landscapes may in like manner be executed, by scraping away the mineral to the thickness that will give the required colors; or the effect may be produced by an etching ground, and using water and other fluid solvents of sulphate of lime to reduce the mineral to the required thicknesses. A cipher may thus be executed upon the mineral; and if we cover the surface upon which it is scratched, or cut, or dissolved, with a balsam or fluid of exactly the same refractive power as the sulphate, it will be absolutely illegible by common light, and may be distinctly read in polarized light, when placed at C E D F in *fig.* 94.

As the colors produced in the preceding experiments vary with the different thicknesses of the body which produces them, it is obvious that two films put together, as they lie in the crystal with similar lines coincident or parallel, will produce a color corresponding to the sum of their thicknesses, and not the color which arises from the mixture of the two colors which they produce separately. Thus, if we take two films of sulphate of lime, one of which gives the *orange* of the first order, whose number in the last column in Newton's Table, p. 93., is $5\frac{1}{6}$, while the other gives the *red* of the 2d order, whose number is $11\frac{5}{6}$; then by adding these numbers, we get 17, which corresponds in the Table to *greenish yellow* of the 3d order. But if the two plates are *crossed*, so that similar lines in the one are *at right angles* to similar lines in the other, then the tint or color which they produce will be that which belongs to the difference of their thicknesses. Thus, in the present case, the difference of the above numbers is $6\frac{4}{6}$, which corresponds in the Table to a *reddish violet* of the second order. If the plates which are thus crossed are equally thick, and produce the same colors, they will destroy each other's effects, and blackness will be produced; the difference of the numbers in the Table being 0. Upon this principle, we may produce colors by crossing plates of such a thickness as to give no colors separately, provided the difference of their thickness does not exceed 0·01818; for if the difference of their thickness is greater than this, the tint will be white, and beyond the limits of the Table.

If the polarized light employed in the preceding experiments is homogeneous, then the colors reflected from the plate B will always be those of the homogeneous light employed. In red light, for example, the colors or rather shades which succeed each other, with different thicknesses of the mineral, will be red at one thickness, black at another, red at another, and black at another, and so on with all the different colors.

If we place the specimen shown in *fig*. 96. in *violet* light, the rings A B will be less than in *red* light; and in intermediate colors they will be of intermediate magnitudes, exactly as in the rings of thin plates formerly described. When white light is used, all the different sets of rings are combined in the very same manner as we have already explained, in thin plates of air, and will form by their combinations the various colored rings in Newton's Table.

CHAP. XXII.

ON THE SYSTEM OF COLORED RINGS IN CRYSTALS WITH ONE AXIS.

(114.) In all the preceding experiments the film C E D F must be held at such a distance from the eye, or from the plate B, that its surface may be distinctly seen, and in the apparatus used by different philosophers this distance was considerable. In the year 1813 I adopted another method, namely, that of bringing the film or crystal to be examined as *close to the eye* as possible, a very small plate, B, not above one fourth of an inch, being interposed, as in *fig*. 94., between the crystal and the eye, to reflect the light transmitted through the crystal. By this means I discovered the systems of rings formed along the axes of crystals with one and two axes, which form the most splendid phenomena in optical science, and which by their analysis have led philosophers to the most important discoveries.

I discovered them in ruby, emerald, topaz, ice, nitre, and a great variety of other bodies, and Dr. Wollaston afterwards observed them in Iceland spar.

In order to observe the system of rings round a single axis of double refraction, grind down the summits or obtuse angles A X of a rhomb of Iceland spar, *fig.* 72., and replace them by plane and polished surfaces perpendicular to the axis of double refraction A X. But as this is not an easy operation without the aid of a lapidary, I have adopted the following method, which enables us to transmit light along the axis A X without injuring the rhomb. Let C D E F, *fig.* 97., be the principal section of the rhomb; cement upon its surfaces C D, F E, with Canada balsam, two prisms, D L K, F G H, having the angles L D K, G F H each equal to about 45°; and by letting fall a ray of light perpendicularly upon the face D L, it will pass along the axis A X, and emerge perpendicularly through the face F G. Let the rhomb thus prepared be held in the polarized beam *r s*, *fig.* 94., so that *r s* may pass along the axis A X, and let it be held as near the plate B as possible. When the eye is held very near to B, and looks along O *s* as it were through the reflected image of the rhomb C E, it will perceive along its axis A X a splendid system of colored rings resembling that shown in *fig.* 98., intersected by a rectangular black cross, A B C D, the arms of which meet at the centre of the rings. The colors in these rings are exactly the same as those in Newton's Table of colors, and consequently the

Fig. 97.

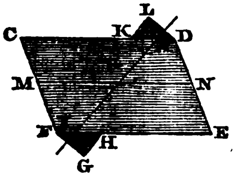
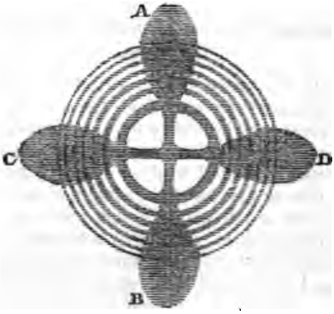
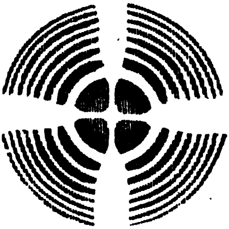


Fig. 98.



same as the system of rings seen by reflexion from the plate of air between the object glasses. If we turn the rhomb round its axis, the rings will suffer no change; but if we fix the rhomb, or hold it steadily, and turn round the plate B, then, in the azimuths 0°, 90°, 180°, and 270° of its revolution, we shall see the same system of rings; but at the intermediate azimuths of 45°, 135°, 225°, and 315°, we shall see another system, like that in *fig.* 99., in which all the colors are complementary to those in *fig.* 98., being the same as those seen in the rings formed by transmission through the plate of air. The superposition of these two systems of rings would reproduce white light.

Fig. 99.



If, in place of the glass plate B, we substitute a prism of calcareous spar, that separates its two images greatly, or a rhomb of great thickness, we shall see in the ordinary image the first system of rings, and in the extraordinary image the second system of complementary rings, when the principal section of the prism or rhomb is in the plane *r s* O as formerly described.

As the light which forms the first system of rings is polarized in an opposite plane to that which forms the second system, we may *disperse* the one system by *agate*, or *absorb* it by *tourmaline*, and thus render the other visible, the first or the second system being dispersed or absorbed according to the position of the agate or the tourmaline.

If we split the rhomb of calcareous spar, *fig.* 97., into two plates by the fissure M N, and examine the rings produced by each plate separately, we shall find that the rings produced by each plate are larger in diameter than those produced by the whole rhomb, and that the rings increase in size as the thickness of the plate diminishes. It will also be found that the circular area contained within any one ring is to the circular area of any other ring, as the number in Newton's Table corresponding to the tint of the one ring is to the number corresponding to the tint of the other.

If we use homogeneous light, we shall find that the rings are smallest in *violet* light and largest in *red* light, and of intermediate sizes in the intermediate colors, consisting always of rings of the color of the light employed, separated by black rings. In white light all the *rings* formed by the seven different colors are combined, and constitute the colored system above described, according to the principles which were fully explained in Chapter XII.

(115.) All the other crystals which have one axis of double refraction, give a similar system of rings along their axis of double refraction; but those produced by the *positive* crystals, such as *zircon*, *ice*, &c., though to the eye they differ in no respect from those of the *negative* crystals, yet possess different properties. If we take a system of rings formed by *ice* or *zircon*, and combine it with a system of rings of the very same diameter formed by Iceland spar, we shall find that the two systems destroy one another, the one being negative and the other positive; an effect which might have been expected from the opposite kinds of double refraction possessed by these two crystals.

If we combine two plates of negative crystals, such as *Iceland spar* and *beryl*, the system of rings which they produce will be such as would be formed by two plates of *Iceland spar*, one of which is the plate employed, and the other a plate which gives rings of the same size as the plate of *beryl*. But if we combine a plate of a negative crystal with a plate of a positive crystal, such as one of *Iceland spar* with one of *zircon* or *ice*, the resulting system of rings, in place of arising from the sum of their separate actions, will arise from their difference; that is, it will be equal to the system produced by a plate of *Iceland spar* whose thickness is equal to the difference of the thicknesses of the plate of Iceland spar employed, and another plate of Iceland spar that would give rings of the same size as those produced by the zircon or ice.

These experiments of combining rings are not easily made, unless we employ crystals which have external faces perpendicular to the axis of double refraction, such as the variety of Iceland spar called *spath calcaire basée*, some of the *micas* with one axis, and well crystallized plates of ice, &c. When two such plates cannot be obtained, I have adjusted the axes of the two plates so as to coincide, by placing between them, at their edges, two or three small pieces of soft wax, by pressing which in different directions, we may produce a sufficiently accurate coincidence of the systems of rings to establish the preceding conclusions.

If, when two systems of rings are thus combined, either both negative or both positive, or the one negative and the other positive, we interpose between the plates which produce them crystallized films of *sulphate of lime* or *mica*, we shall produce the most beautiful changes in the form and character of the rings. This experiment I found to be particularly splendid when the film was placed between two plates of the *spath calcaire basée* of the same thickness, and taken from the same crystal. By fixing them permanently with their faces parallel, and leaving a sufficient interval between them for the introduction of films of crystals, I had an apparatus by which the most splendid phenomena were produced. The rings were no longer symmetrical round their axis, but exhibited the most beautiful variety of forms during the rotation of the combined plates, all of which are easily deducible from the general laws of double refraction and polarization.

The table of crystals that have negative double refraction shows the bodies that have a negative system of rings; and the table of positive crystals indicates those that have a positive system of rings.

(116.) The following is the method which I have used for distinguishing whether any system of rings is positive or negative. Take a film of sulphate of lime, such as that shown at C E D F, *fig*. 94., and mark upon its surface the lines or neutral axes C D, E F as nearly as may be. Fix this film by a little wax on the surface, L D or F G, *fig*. 97., of the rhomb which produces the negative system of rings. If the film produces alone the red of the second order, it will now, when combined with the rhomb, obliterate part of the red ring of the second order, either in the two quadrants A C, B D, *fig*. 98., or in the other two, A D, C B. Let it obliterate the red in A C, B D; then if the line C D, *fig*. 94., of the film crosses these two quadrants at right angles to the rings, it will be the *principal axis* of the sulphate of lime; but if it crosses the other two quadrants, then the line E F, which crosses the quadrants A C, B D, will be the principal axis of sulphate of lime, and it should be marked as such. We shall suppose, however, that C D has been proved to be the principal axis. Then, if we wish to examine whether any other system of rings is positive or negative, we have only to cross the rings with the axis C D, by interposing the film: and if it obliterates the red ring of the second order in the quadrant which it crosses, the system will be *negative;* but if it obliterates the same ring in the other two quadrants which it does not cross, then the system will be *positive.* It is of no consequence what color the film polarizes, as it will always obliterate the tint of the same nature in the system of rings under examination.

(117.) In order to explain the formation of the systems of rings seen along the axes of crystals, we must consider the two causes on which they depend; namely, the thickness of the crystal through which the polarized light passes, and the inclination of the polarized light to the axis of double refraction or the axis of the rings. We have already shown how the tint or color varies with the thickness of the crystallized body, and how, when we know the color for one thickness, we may determine it for all other thicknesses, the inclination of the ray to the axis remaining always the same. We have now, therefore, only to consider the effect of inclination to the axis. It is obvious that along the axis of the crystal, where the two black lines A B, C D, *fig.* 98., cross each other, there is neither double refraction nor color. When the polarized ray is slightly inclined to the axis, a faint tint appears, like the blue in the first order of Newton's scale; and as the inclination gradually increases, all the colors in Newton's table are produced in succession, from the *very black* of the first order up to the *reddish white* of the seventh order. Here, then, it appears that an increase in the inclination of the polarized light to the axis corresponds to an increase of thickness; so that if the light always passed through the same thickness of the mineral, the different colors of the scale would be produced by difference of inclination alone. Now, it is found by experiment, that in the same thickness of the mineral, the numerical value of the tints, or the numbers opposite to the tints in the last column of Newton's table, vary as the square of the sine of the inclination of the polarized ray to the axis. Hence it follows, that at equal inclinations the same tint will be produced; and consequently, the similar tints will be at equal distances from the axis of the rings, or the lines of equal tint or rings will be circles whose centre is in the axis. Let us suppose that at an inclination of 30° to the axis we observe the *blue* of the second order, the numerical value of whose tint is 9 in Newton's table, and that we wish to know the tint which would be produced at an inclination of 45°. The sine of 30° is ·5, and its square ·25. The sine of 45° is ·7071, and its square .5. Then we say, as ·25 is to 9, so is ·5 to 18, which in the table is the numerical value of the *red* of the third order. If we suppose the thickness of the mineral to be increased at the inclinations 30° and 45°, then the numerical value of the tint would increase in the same proportion.

It is obvious from what has been said, that the polarizing force, or that which produces the rings, vanishes when the double refraction vanishes, and increases and diminishes with the double refraction, and according to the same law. The polarizing force, therefore, depends on the force of double refraction; and we accordingly find that crystals with high double refraction have the power of producing the same tint, either at much less thicknesses, or at much less inclinations to the axis. In order to compare the polarizing intensities of different crystals, the best way is to compare the tints which they produce at right angles to the axis where the force of double refraction and polarization is a maximum, and with a given thickness of the mineral. Thus, in the case given above, we may find the tint at right angles to the axis, by taking the square of the sine of 90°, which is 1; so that we have the following proportion: as ·25 is to 9, so is 1 to 36, the value of the maximum tint of calcareous spar at right angles to the axis, upon the supposition that a tint of the value of 9 was produced at an inclination of 30°. If we have measured the thickness of Iceland spar at which the tint 9 was produced, we are prepared to compare the polarizing intensity of Iceland spar with that of any other mineral. Thus, let us take a plate of *quartz*, and let us suppose that at an inclination of 30°, and with a thickness fifty-one times as great as that of the plate of *Iceland spar*, it produces a *yellow* of the first order, whose value is about 4. Then to find the tint at 90°, or at right angles to the axis, we say, as the square of the sine of 30°, or ·25, is to 4, so is the square of the sine of 90°, or 1, to 16, the tint at 90°, or the *green* of the third order. Now the polarizing power or intensity of the Iceland spar would have been to that of the quartz as 36 to 16, or 2¼ times as great, if the thickness of the two minerals had been the same; but as the thickness of the quartz was 51 times as great as that of the Iceland spar, the polarizing intensity of the Iceland spar will be 51 multiplied by 2¼ times, or 115 times as great as that of quartz. The intensities for various crystals have been determined by several observers, but the following have been given by Mr. Herschel:—

Polarizing Intensities of Crystals with One Axis.

	Value of highest Tint.	Thicknesses that produce the same Tint.
Iceland spar	35801	0·000028
Hydrate of strontia	1246	0·000802
Tourmaline	851	0·001175
Hyposulphate of lime	470	0·002129
Quartz	312	0·003024
Apophyllite, 1st variety	109	0·009150
Camphor	101	0·009856
Vesuvian	41	0·024170
Apophyllite, 2d variety	33	0·030374
——— 3d variety	3	0·366620

The above measures are suited to *yellow* light, and the numbers in the second column show the proportions of the thicknesses of the different substances that produce the same tint. The polarizing force of Iceland spar is so enormous at right angles to the axis, that it is almost impracticable to prepare a film of it sufficiently thin to exhibit the colors in Newton's table.

CHAP. XXIII.

ON THE SYSTEMS OF COLORED RINGS IN CRYSTALS WITH TWO AXES.

(118.) IT was long believed that all crystals had only one axis of double refraction; but, after I discovered the double system of rings in topaz and other minerals, I found that these minerals had two axes of double refraction as well as of polarization, and that the possession of two axes characterized the great body of crystals which are either formed by art, or which occur in the mineral kingdom.

The double system of rings, or rather one of the sets of the double system of rings in topaz, first presented itself to me when I was looking along the axis of topaz, which reflected a part of the light of the sky that happened to be polarized, so that they were seen without the aid either of a polarizing or an analyzing plate. In this and some other minerals, however, the axes of double refraction are so much inclined to one another, that we cannot see the two systems of rings at once. I shall therefore proceed to explain them as exhibited by *nitre*, in which I also discovered them and examined many of their properties.

Nitre, or *saltpetre*, is an artificial substance which crystallizes in six-sided prisms with angles of about 120°. It belongs to the prismatic system of Mohs, and has therefore two axes of double refraction along which a ray of light is not divided into two. These axes are each inclined about 2½° to the axis of the prism, and about 5° to each other. If, therefore, we cut off a piece of a prism of nitre with a knife driven by a smart blow from a hammer, and polish two flat surfaces perpendicular to the axis of the prism, so as to leave a thickness of the sixth or eighth of an inch, and then transmit the polarized light *r s*, *fig*. 94., along the axis of the prism, keeping the crystal as near to the plate B as possible on one side, and the eye as near it as possible on the other, we shall see the double system of rings, A B, shown in *fig*. 100., when the plane passing through the two axes of nitre is in the plane of primitive polarization, or in the plane of reflexion *r s* O, *fig*. 94., and the system shown in *fig*. 101. when the same plane is inclined 45° to either of these planes. In passing from the state of *fig*. 100. to that of *fig*. 101., the black lines assume the forms shown in *figs*. 102. and 103.

Fig. 100.

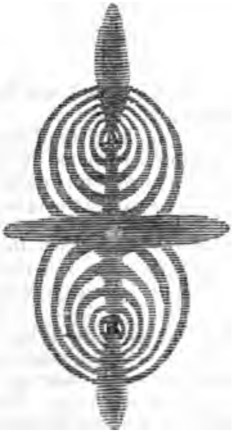
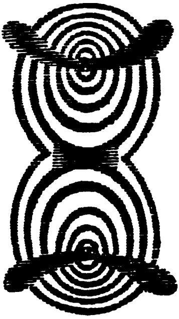


Fig. 101.



These systems of rings have, generally speaking, the same colors as those of thin plates, or as those of the systems of rings round one axis. The orders of colors commence at the centres A and B of each system; but at a certain distance, which in *fig*. 100. corresponds to the sixth ring, the rings, in place of returning and encircling each pole A and B, encircle the two poles as an ellipse does its two foci.

Fig. 102.

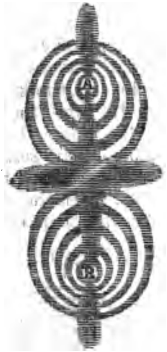
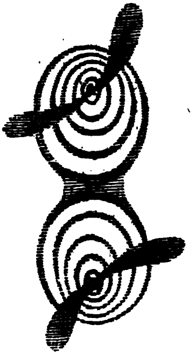


Fig. 103.



When we diminish the thickness of the plate of nitre, the rings enlarge; the fifth ring will then surround both poles. At a less thickness, the fourth ring will surround them, till at last all the rings will surround both poles, and the system will have a great resemblance to the system surrounding one axis. The place of the poles A, B never changes, but the black lines A B, C D become broad and indefinite; and the whole system is distinguished from the single system principally by the oval appearance of the rings.

If we increase the thickness of the nitre, the rings will diminish in size; the colors will lose their resemblance to those of Newton's scale; and the tints do not commence at the poles A, B, but at virtual poles in their proximity. The color of the rings within the two poles is *red*, and without them *blue*; and the great body of the rings is pink and green.

As the same color exists in every part of the same curve, the curves have been called *isochromatic lines*, or *lines of equal tint*. The lines or axes along which there is no double refraction or polarization, and whose poles are A, B, *fig*. 100., have been called *optical axes*, or *axes of no polarization*, or *axes of compensation*, or *resultant axes*; because they have been found not to be real axes, but lines along which the opposite actions of other two real axes have been compensated, or destroy one another.

(119.) In various crystallized bodies, such as *nitre* and *arragonite*, where the inclination of the *resultant axes*, A, B, *fig*. 100., is small, the two systems of rings may be easily seen at the same time; but when the inclination of the resultant axes is great, as in *topaz*, *sulphate of iron*, &c., we can only see one of the systems of rings, which may be done most advantageously by grinding and polishing two parallel faces perpendicular to the axis of the rings. In *mica* and *topaz*, and various other crystals, the plane of most eminent cleavage is equally inclined to the two resultant axes; so that in such bodies the systems of rings may be readily found and easily exhibited.

Let M N, for example, *fig*. 104., be a plate of topaz, cut or split so as to have its face perpendicular to the axis of the prism in which this body crystallizes. If we place this plate, *fig*. 104., in the apparatus *fig*. 94. so that the polarized ray *r s*, *fig*. 94., passes along the line A B *e* E, *fig*. 104., and if the eye receives this ray when reflected from the analyzing plate B, it will see in the direction of that ray a system of oval rings, like that in *fig*. 105. In like manner, if the polarized light is transmitted along the line C B *d* D, the eye will see another system perfectly similar to the first. The lines A B *e* E and C B *d* D are, therefore, the resultant axes of topaz. The angle A B C will be found equal to about 121° 16′; but if we compute the inclination of the refracted rays B *d*, B *e*, we shall find it, or the angle *d* B *e*, to be only 65°; which is, therefore, the inclination of the *optical or resultant axes* of topaz.

Fig. 104.

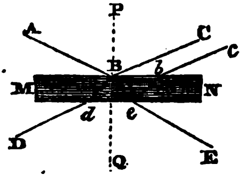
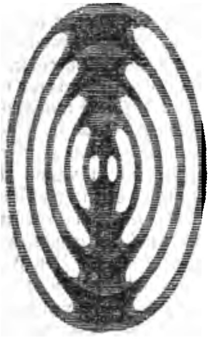


Fig. 105.



If we suppose the plate of nitre fixed in any of the positions which give any of the rings shown in *figs.* 100, 101, 102, or 103., then, if we turn round the plate B, we shall observe in the azimuths of 90° and 270° a system of rings complementary to each, in which the black cross in *fig.* 100. and the black hyperbolic curves in *figs.* 101. 103. are white, all the other dark parts light, and the *red green*, the *green red*, &c. as in the single system of rings with one axis.

In the preceding observations we have supposed the polarization of the incident light, and the analysis of the transmitted light, to be necessary to the production of the rings; but in certain cases they may be shown by common light with the analyzing plate, or by polarized light without the analyzing plate B, and in some cases without either the light being polarized or analyzed. If in topaz, for example, *fig.* 104., we allow common light to fall in the direction A B, so as to be refracted along B *e*, one of the resultant axes, and subsequently reflected at *e* from the second surface, and reaching the eye at *c*, we shall see, after reflexion from the analyzing plate, the system of rings in *fig.* 105.; or if A B is polarized light, the rings will be seen by the eye at *c* without an analyzing plate. There are several other curious phenomena seen under these circumstances, which I have described in the *Phil. Transactions* for 1814, p. 203. 211.

I have found some crystals of nitre which exhibit their rings without the use either of polarized light or an analyzing plate; and Mr. Herschel has found the same property in some crystals of carbonate of potash.

(120.) When the preceding phenomena are seen by polarized *homogeneous* light, in place of white light, the rings are bright curves, separated by dark intervals; the curves having always the color of the light employed. In many crystals the difference in the size of the rings seen in different colors is not very great, and the poles A, B of the two systems do not greatly change their place; but Mr. Herschel found that there were crystals, such as *tartrate of potash* and *soda*, in which the variation in the size of the rings was enormous, being *greatest* in *red*, and *least* in *violet* light, and in which the distance A B, *figs.* 100. 101., or the inclination of the resultant axes, varied from 56° in *violet* light to 76° in *red*, the inclination having intermediate values for intermediate colors, and the centres of all the different systems lying in the line A B. When all these systems of rings are combined, as they are in using white light, the system of rings which they form is exceedingly irregular, the two oval centres, or the halves of the first order of colors, being drawn out with long spectra or tails of red, green, and violet light, and the ends of all the other rings being red without the resultant axes, and blue within.

Mr. Herschel found other crystals in which the rings are *smallest* in *red*, and *largest* in *blue* light, and in which the inclination of the axes or A B is *least* in *red*, and *greatest* in *violet* light.

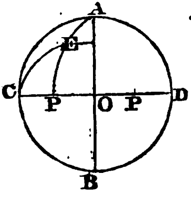
In all crystals of this kind, the deviation of the tints, or the colors of the rings seen in white light, from Newton's Table is very considerable, and may be calculated from the preceding principles. This deviation I found to be very great, even in crystals with one axis of double refraction and one system of rings, such as *apophyllite* where the rings have scarcely any other tints than a succession of *greenish yellow*, and *reddish purple* ones. By viewing these rings in homogeneous light, Mr. Herschel has found that the system is a negative one for the rays at the one end of the spectrum, a positive one for the rays at the other end of the spectrum, and that there are no rings at all in yellow light.

A similar and equally curious anomaly I have found in *glauberite*, which is a crystal which has two axes of double refraction, or two systems of rings for red light, and one negative system for violet light.

(121). All the singularities of these phenomena disappear, and may be rigorously calculated by supposing the *resultant axes* of crystals where there are two, or the single axis where there is one, with a system of rings deviating from Newton's scale, as merely apparent axes, or axes of compensation, produced by the opposite action of *two* or more rectangular axes, the principal one of which is the line bisecting the angle formed by the two resultant axes. Upon this principle, I have shown that all the phenomena presented by such crystals may be computed with as much accuracy as we can compute the motions of the heavenly bodies.

The method of doing this may be understood from the following observations. Let A C B D, *fig.* 106., be a crystal with two axes turned into a sphere. Let P, P be the poles of the axes, O the point bisecting them, and A B a line passing through O, and perpendicular to C D, a line passing through P, P. Let us suppose an axis to pass through O, perpendicular to the plane A C B D, then we may account for all the phenomena of such crystals, by supposing the axis at O to be the principal one,

Fig. 106.



and the other axis to be along either of the diameters A B or C D. If we take C D, then the axes O and C D must be both of the same name, either both *positive* or both *negative ;* but if we take A B, the axes must be one *positive* and the other *negative ;* or, what is perhaps the simplest supposition for illustration, we shall suppose the two rectangular axes which produce all the phenomena to be A B, C D, either both positive or both negative, leaving out the one at O. Supposing A O B, C P P D to be projections of great circles of the sphere, then P, P are the points where the axis A B destroys the effect of the axis C D; that is, where the tints produced by each axis must be equal and opposite. Now, if we suppose the arch C P to be 60°, then, since A P is 90°, it follows that the axis C D produces at 60° the same tint that A B does at 90°, and consequently the polarizing intensity of C D will be to that of A B as the square of the sine of 90° is to the square of the sine of 60°, or as 1 to 0·75, or as 100 to 75. The polarizing force of each axis being thus determined, it is easy to find the tint which will be produced by each axis separately at any given inclination to the axis, by the method formerly explained. Let E be any point on the surface of the sphere, and let the tints produced at that point be 9, or the *blue* of the second order, by C D, and 16, or the *green* of the third order, by A B. Let the inclination of the planes passing through A E, C E, or the spherical angle C E A be determined, then the tint at the point E will correspond to the diagonal of a parallelogram whose sides are 9 and 16, and whose angle is double the angle C E A. This law, which is general, and applies also to double refraction, has been confirmed by Biot and Fresnel, the last of whom has proved that it coincides rigorously with the law deduced from the theory of waves.

If the axes A B, C D are equal, it follows that they will produce the same tint at equal inclinations; that is, they will compensate each other only at one point, viz. O, and will produce round O a system of colored rings, the very same as if O were a single axis of double refraction of an opposite name to A B, C D. If the axis A B has exactly the same proportional action that C D has upon each of the differently colored rays, a compensation will take place for each color exactly at O, the centre of the resultant systems of rings, and the colors will be exactly those of Newton's scale. But if each axis exercises a different proportional action upon the colored rays, a compensation will take place at O for some of the rays (for violet, for example), while the compensation for red will take place on each side of O; consequently, in such a case the crystal will have one axis for *violet* light, and *two axes* for *red* light, like glauberite.

The phenomena of *apophyllite* may, in a similar manner, be explained by two equal negative axes, A B, C D, and a positive axis at O.

According to this method of combining the action of different rectangular axes, it follows that three equal and rectangular axes, either all positive or all negative, will destroy one another at every point of the sphere, and thus produce the very same effect as if the crystal had no double refraction and polarization at all. Upon this principle I have explained the absence of double refraction in all the crystals which form the tessular system of Mohs, each of the primitive forms of which has actually three similarly situated and rectangular axes. If one of these axes is not precisely equal to the other, and the crystallization not perfectly uniform, traces of double refraction will appear, which is found to be the case in *muriate of soda*, *diamond*, and other bodies of this class.

(122.) The following table contains the polarizing intensities of some crystals with *two* axes, as given by Mr. Herschel:—

Polarizing Intensities of Crystals with Two Axes.

	Value of highest Tint.	Thicknesses that produce the same Tint.
Nitre - - - - - - - - - - -	7400	0·000135
Anhydrite, inclination of axes 43° 48′	1900	0·000526
Mica, inclination of axes 45° - - -	1307	0·000765
Sulphate of baryta - - - - - - -	521	0·001920
Heulandite (white), inclination of axes 54° 17′ - - - - - - -	249	0·004021

CHAP. XXIV.

INTERFERENCE OF POLARIZED LIGHT.—ON THE CAUSE OF THE COLORS OF CRYSTALLIZED BODIES.

(123.) Having thus described the principal phenomena of the colors produced by regularly crystallized bodies that possess *one* or *two* axes of double refraction, we shall proceed to explain the cause of these remarkable phenomena.

Dr. Young had the great merit of applying the doctrine of interference to explain the colors produced by double refraction. When a pencil of light falls upon a thin plate of a doubly refracting crystal, it is separated into two, which move through the plate with different velocities, corresponding to the different indices of refraction for the ordinary and extraordinary ray. In calcareous spar, the ordinary ray moves with greater velocity than the extraordinary one; and therefore they ought to interfere with one another, and in homogeneous light produce rings consisting of bright and dark circles round the axis of double refraction. According to this doctrine, however, the rings ought to be produced in common as well as in polarized light; but as this was not the case, Dr. Young's ingenious hypothesis was long neglected. The subject was at last taken up by Messrs. Fresnel and Arago, who displayed great address in their investigation of the subject, and succeeded in showing how the production of the rings depended on the polarization of the incident pencil and its subsequent analysis by a reflecting plate or a doubly refracting prism.

The following are the laws of the interference of polarized light as discovered by MM. Fresnel and Arago:—

1. *When two rays polarized in the same plane interfere with each other, they will produce by their interference fringes of the very same kind as if they were common light.*

This law may be proved by repeating the experiments on the inflexion of light, mentioned in Chap. XI., in polarized in place of common light; and it will be found that the very same fringes are produced in the one case as in the other.

2. *When two rays of light are polarized at right angles to each other, they produce no colored fringes in the same circumstances under which two rays of common light would produce them. When the rays are polarized at angles intermediate between* 0° *and* 90°, *they produce fringes of intermediate brightness, the fringes being totally obliterated at* 90°, *and recovering their greatest brightness at* 0°, as in Law 1.

In order to prove this law, MM. Fresnel and Arago adopted several methods, the simplest of which is the following, employed by the latter. Having made *two* fine slits in a thin plate of copper, he placed the copper behind the focus F of a lens, as in *fig.* 56., and received the shadow of the copper upon the screen C D, where the fringes produced by the interference of the rays passing through the two slits were visible. In order, however, to observe the fringes more accurately, he viewed them with an eye-glass, as formerly described. He next prepared a bundle of transparent plates, like either of those shown at A and B, *fig.* 93., made of fifteen thin films of *mica* or *plane glass*, and he divided this bundle into two, by a sharp cutting instrument. At the line of division these bundles had as nearly as possible the same thickness, and they were capable of polarizing completely light incident upon them at an angle of 30°. These bundles were then placed before the slits so as to receive and transmit the rays from the focus F at an incidence of 30°, and through portions of the mica in each bundle that were very near to each other previous to their separation. The bundles were also fixed to revolving frames, so that, by turning either bundle round, their planes of polarization could be made either parallel or at right angles to each other, or could be inclined at any intermediate angle. When the bundles were placed so as to polarize the rays in parallel planes, the fringes were formed by the slits exactly as when the bundles were removed; but when the rays were polarized at 90°, or at right angles to each other, the fringes wholly disappeared. In all intermediate positions the fringes appeared with intermediate degrees of brightness.

3. *Two rays originally polarized at right angles to each other may be subsequently brought into the same plane of polarization, without acquiring the power of forming fringes by their interference.*

If, in the preceding experiment, a doubly refracting crystal be placed between the eye and the copper slits, having its principal section inclined 45° to either of the planes of polarization of the interfering rays, each pencil will be separated into two equal ones polarized in two rectangular planes, one of which planes is the principal section itself. Two systems of fringes ought, therefore, to be produced; one system from the interference of the *ordinary ray from the right hand slit* with that of the *ordinary ray from the left hand slit*, and another system from the interference of the *extraordinary ray from the right hand slit* with the *extraordinary ray from the left hand slit*; but no such fringes are produced.

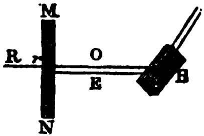
4. *Two rays polarized at right angles to each other, and afterwards brought into similar planes of polarization, produce fringes by their interference like rays of common light, provided they belong to a pencil, the whole of which was originally polarized in the same plane.*

5. *In the phenomena of interference produced by rays that have suffered double refraction, a difference of half an undulation must be allowed, as one of the pencils is retarded by that quantity from some unknown cause.*

The second of these laws affords a direct explanation of the fact which perplexed Dr. Young, that no fringes are observed when light is transmitted through a thin plate possessing double refraction. The two pencils thus produced do not form fringes by their interference, ***because they are polarized in opposite planes.***

The production of the fringes by the action of doubly refracting crystals on polarized light may be thus explained, Let M N, *fig*. 107., be a section of the plate of sulphate of lime, C E D F, *fig*. 94., and B the analyzing plate. Let R *r* be a polarized ray incident upon the plate M N, and let O and E be the ordinary and extraordinary rays produced by the double refraction of the plate M N. When the plate M N is in such a position that either of its neutral axes C D, E F, *fig*. 94., are in the plane of primitive polarization of the ray R *r*, *fig*. 107., then one of the pencils will not suffer reflexion by the plate B, and consequently only one of the rays will be reflected. Hence it is obvious that no colors can be produced by interference, because there is only one ray. But in every other position of the plate M N, the two rays, O *s*, E *s*, will be reflected by the plate B; and being polarized by the plate in the same plane, they will, by Law 1., interfere, and produce a color or a fringe corresponding to the retardation of one of the rays within the plate, arising from the difference of their velocities. If we call *d* the interval of retardation within the plate M N, we must ***add*** to it half an undulation to get the real interval, as one of the rays passes from the ordinary to the extraordinary state. If we now suppose the plate B to make a revolution of 90°, M N remaining fixed, then the ray E will be reduced to the ordinary state; and consequently we must subtract half an undulation from *d*, the interval of retardation within the plate, to have the real difference of the intervals of retardation. Hence the two intervals of retardation will differ by a whole undulation; and consequently the color produced when the plate B has been turned round 90°, will be complementary to that which is produced when the plate B has the position shown in *fig*. 107.

Fig. 107.



If we suppose the rays E and O to be received upon and analyzed by a prism of Iceland spar, we shall have two ordinary rays interfering to form the colors in one image, and two extraordinary rays interfering to produce the complementary colors in the other image.

CHAP. XXV.

ON THE POLARIZING STRUCTURE OF ANALCIME.

(124.) In a preceding chapter I have mentioned the very remarkable double refraction which is possessed by analcime. This mineral, which is also called *cubizite*, has been regarded by mineralogists as having the cube for its primitive form; but if this were correct, it should have exhibited no double refraction. Analcime has certainly no cleavage planes, and it must be regarded at present as forming in this respect as great an anomaly in crystallography as it does in optics by its extraordinary optical phenomena.

The most common form of the analcime is the solid called the *icositetrahedron*, which is bounded by twenty-four equal and similar trapezia; and we may regard it as derived from the cube, by cutting off each of its angles by three planes equally inclined to the three faces which contain the solid angle. If we now conceive the cube to be dissected by planes passing through all the twelve diagonals of its six faces, each of these planes will be found to be a plane of no double refraction, or polarization; that is, a ray of polarized light transmitted in any direction whatever, provided it is in one of these planes, will exhibit none of the polarized tints when the crystal is placed in the apparatus, *fig*. 94. These planes of no double refraction are shown by dark lines in *figs*. 108. and 109. If the polarized ray is incident in any direction which is out of these planes, it will be divided into two pencils, and exhibit the finest tints, all of which are related to the planes of no double refraction. The double refraction is sufficiently great to admit a distinct separation of the images when the incident ray passes through any pair of the four planes which are adjacent to the three axes of the solid, or of the cube from which it is derived. The least refracted image is the extraordinary one; and consequently the double refraction is *negative* in relation to the axes to which the doubly refracted ray is perpendicular.

Fig. 108.

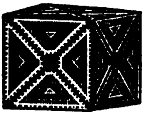
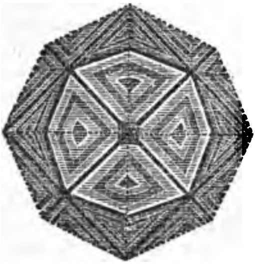


Fig. 109.



In all other doubly refracting crystals, each particle has the same force of double refraction; but in the analcime, the double refraction of each particle varies with the square of its distance from the planes already described.

The beautiful distribution of the tints shown in *figs.* 108. and 109. cannot, of course, be exhibited to the eye at once, but are deduced by transmitting polarized light in every direction through the mineral.

In several of the crystals, the tints rise to the third and fourth order; but when the crystals are very small, the tints do not exceed the white of the first order. The tints are exactly those of Newton's scale, which indicates that they are not the result of opposite and dissimilar actions. In *figs.* 108. and 109. the tints are represented by the faint shaded lines having their origin from the planes where the double refraction disappears.

The preceding property of analcime is a simple and easily applied mineralogical character, which would identify the most shapeless fragment of the mineral.

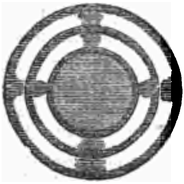
The abbé Hauy first observed in this mineral its property of yielding no electricity by friction, and derived the name of analcime from its want of this property. When we consider that the crystal is intersected by numerous planes, in which the ether does not exist at all, or has its properties neutralized by opposite actions, we may ascribe to this cause the difficulty with which friction decomposes the natural quantity of electricity residing in the mineral.

CHAP. XXVI.

ON CIRCULAR POLARIZATION.

(125). In all crystals with one axis there is neither double refraction nor polarization along the axis; and this is indicated in the system of rings, by the disappearance of all light in the centre of the rings at the intersection of the black cross. When we examine, however, the system of rings produced by a plate of rock crystal whose faces are perpendicular to the axis, we find that the black cross is obliterated within the inner ring, which is occupied with a uniform tint of red, green, or blue, according to the thickness of the plate. This effect will be seen in *fig.* 110. M. Arago first observed these colors in 1811. He found that when they were analyzed by a prism of Iceland spar, the two images had complementary colors, and that the colors changed, descending in Newton's scale as the prism revolved; so that if the color of the extraordinary image was *red*, it became in succession *orange*, *yellow*, *green*, and *violet*. From this result he concluded, that the differently colored rays had been polarized in different planes, by passing along the axis of the rock crystal. In this state of the subject, it was taken up by M. Biot, who investigated it with much sagacity and success.

Fig. 110.



Let C E D F be the plate of quartz, *fig.* 94., along whose axis a polarized ray, *r s*, is transmitted. When the eye is placed at O, above the analyzing plate fixed as in the figure, it will see, for example, a circular red space in the centre of the rings. If we turn the quartz round its axis, no change whatever takes place; but if we turn the plate B from right to left, through an angle of 100° for example, we shall observe the *red* change to *orange*, *yellow*, *green*, and *violet*, the latter having a dark purple tinge. If we now cut from the same prism of rock crystal another plate of twice the thickness, and place it in the apparatus, the plate B remaining where it was left, we shall find that its tint is different from that of the former plate; but by turning the plate B 100° farther, we shall again bring the tint to its least brightness, viz., a sombre violet. By a plate thrice as thick, the least brightness will be obtained by turning the plate B 100° farther, and so on, till, when the thickness is very great, the plate B may have made several complete revolutions. Now, it might happen that a thickness had been taken, so that the rotation of B which produced the sombre violet was 360°, or terminated in the point 0°, from which it set out, which would have perplexed the observer, if he had not made the succession of experiments which we have mentioned.

This phenomenon will be better understood, by supposing that we take a plate of *quartz* $\frac{1}{25}$th of an inch thick, and use the different homogeneous rays of the spectrum in succession. Beginning with *red*, we shall find that the *red* light in the centre of the rings has its *maximum brightness* when the plate B is at 0° of azimuth, as in *fig.* 94. If we turn B from *right* to *left*, the *red* tint will gradually decrease, and after a rotation of 17½° the red tint will *wholly vanish*, having reached its minimum. With a plate $\frac{2}{25}$ths thick, the red will vanish at 35°, every additional thickness of the 25th of an inch requiring an additional rotation of 17½°. If the light is *violet*, the same thickness, viz., $\frac{1}{25}$th of an inch, will require a rotation of 41° to make it vanish, every additional 25th of an inch of thickness requiring a rotation of 41° more.

(126.) The rotations for different colors corresponding to 1 millimetre, or $\frac{1}{25}$th of an inch of quartz, are as follows:—

Homogeneous Ray.	Arcs of Rotation.	Homogeneous Ray.	Arcs of Rotation.
Extreme red	17° 30′	Limit of green and blue	30° 03′
Mean red	19 00	Mean blue	32 19
Limit of red and orange	20 29	Limit of blue and indigo	34 34
Mean orange	21 24	Mean indigo	36 07
Limit of orange and yellow	22 19	Limit of indigo and violet	37 41
Mean yellow	24 00	Mean violet	40 53
Limit of yellow and green	25 40	Extreme violet	44 05
Mean green	27 51		

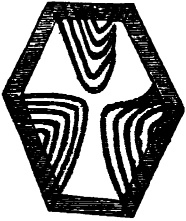
Upon trying various specimens of quartz, M. Biot found that there were several in which the very same phenomena were produced by turning the plate B from *left* to *right*. Hence, in reference to this property, quartz may be divided into *right-handed* and *left-handed* quartz.

From these interesting facts it follows, that, in passing along the axis of quartz, polarized light comports itself, at its egress from the crystal, as if its planes of polarization revolved in the direction of a spiral within the crystal, in some specimens from *right to left*, and in others from *left to right*. "To conceive this distinction," says Mr. Herschel, "let the reader take a common cork-screw, and holding it *with the head towards him*, let him turn it in the usual manner as if to penetrate a cork. The head will then turn the same way as the plane of polarization of a ray, in its progress *from* the spectator through a *right-handed* crystal, may be conceived to do. If the thread of the cork-screw were reversed, or were what is termed a *left-handed* thread, then the motion of the head as the instrument advances would represent that of the plane of polarization in a *left-handed* specimen of rock crystal."

From the opposite characters of these two varieties of quartz, it follows, that if we combine a plate of *right-handed* with a plate of *left-handed* quartz, the result of the combination will be that of a plate of the thickest of the two, whose thickness is equal to the difference of the two thicknesses. Thus, if a plate $\frac{1}{25}$th of an inch thick of *right-handed quartz* is combined with a plate $\frac{4}{25}$ths thick of *left-handed quartz*, the same colors will be produced as if we used a plate $\frac{3}{25}$ths of an inch thick of *left-handed quartz*. When the thicknesses are equal, the plates of course destroy each other's effects, and the system of rings with the black cross will be distinctly seen.

(127.) In examining the phenomena of circular polarization, in the *amethyst*, I found that it possessed the power in the same specimen of turning the planes of polarization both from *right to left* and from *left to right*, and that it actually consisted of *alternate strata of right and left-handed quartz, whose planes were parallel* to the axis of double refraction of the prism. When we cut a plate perpendicular to the axis of the prism, we therefore cut across these strata, as shown in *fig.* 111., which exhibits sections of the strata which occur opposite the three alternate faces of the six-sided prism. The *shaded* lines are those which turn the planes of polarization from *right to left*, while the intermediate unshaded ones and the three unshaded sectors turn them from *left to right*. These strata are not united together like the parts of certain composite crystals, whose dissimilar faces are brought into mechanical contact; for the right and left-handed strata destroy each other at the middle line between each stratum, and each stratum has its maximum polarizing force in its middle line, the force diminishing gradually to the lines of junction.

Fig. 111.



In some specimens of *amethyst* the thickness of these strata is so minute, that the action of the right-handed stratum extends nearly to the central line of the left-handed stratum, and *vice versâ*, so as nearly to destroy each other; and hence in such specimens we see the system of colored rings with the black cross almost entirely uninfluenced by the tints of *circular* polarization. A vein of amethyst, therefore, $\frac{1}{25}$th of an inch thick in the direction of the axis, may be so thin in a direction perpendicular to the axis that the arc of rotation for the red ray may be 0°; and we shall have the curious phenomenon of a plate which polarizes circularly only the most refrangible rays of the spectrum. By a greater degree of thinness in the strata, the plate would be incapable of polarizing circularly the yellow ray; and by a greater thinness still, there would be no action on the violet light. These feeble actions, however, might be rendered visible at great thicknesses of the mineral.

We may therefore conclude that the axes of rotation in amethyst vary from 0° to each of the numbers in the preceding table, according to the thickness of the strata.

The coloring matter of the amethyst I have found to be curiously distributed in reference to these views; but I must refer to the original memoir for farther information.*

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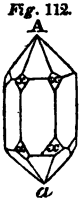
* *Edinburgh Transactions*, vol. ix. p. 139.

M. Biot maintained that this remarkable property of quartz resided in its ultimate particles, and accompanied them in all their combinations. I have found, however, that it is not possessed by *opal*, *tabasheer*, and other silicious bodies, and that it disappears in *melted quartz.* Mr. Herschel also found that it does not exist in a solution of silica in potash.

Fig. 112.



Hitherto no connexion could be traced between the right and left-handed structure in quartz, and the crystalline form of the specimens which possessed these properties. Mr. Herschel, however, discovered that the plagiedral quartz which contains unsymmetrical faces, *x x x*, *fig.* 112., turns the planes of polarization in the same direction in which these faces lean round the summits A *x x*, *a x x*.

Circular Polarization in Fluids.

(128.) The remarkable property of polarizing light circularly occurs in a feeble degree in certain fluids, in which it was discovered by M. Biot and Dr. Seebeck. Mr. Herschel has found it in camphor in a solid state, and I have discovered it in certain specimens of unannealed glass. If we take a tube six or seven inches long, and fill it with oil of turpentine, and place it in the apparatus, *fig.* 94., so that polarized light transmitted through the oil may be reflected to the eye from the plate B, we shall observe the complementary colors and a distinct rotation of the plane of polarization from *right to left.* Other fluids have the property of turning the planes of polarization from *left to right*, as shown in the following table, which contains the results of M. Biot's experiments.

Crystals which turn the Planes from Right to Left.

	Arc of Rotation for every 25th of an inch in Thickness.	Relative Thicknesses that produce the same Effect.
Rock crystal - - - - - - - - - -	18° 25′	1
Oil of turpentine - - - - - - - - - -	0 16	68½
Solution of 1753 parts of artificial camphor in 17359 of alcohol - - - - - - -	0 01	
Essential oil of laurel.		
——————— turpentine.		

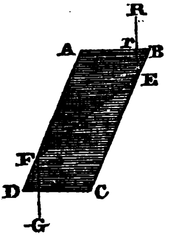
Crystals which turn the Planes from Left to Right.

	Arc of Rotation for every 25th of an inch in Thickness.	Relative Thicknesses that produce the same Effect.
Rock crystal - - - - - - - - - - -	18° 25′	1
Essential oil of lemons - - - - - - -	0 26	38
Concentrated syrup (from sugar) - - - -	0 33	4½

In examining these phenomena, M. Fresnel discovered that in quartz they were produced by the interference of two pencils formed by double refraction along the axis of the quartz. He succeeded in separating these two pencils, which differ both from common and polarized light. They differ from polarized light, because when either of them is doubled by a doubly refracting crystal, the pencil or image never vanishes during the revolution of the crystal. They differ from common light, because when they suffer two total reflexions from glass, at an angle of about 54°, the one will emerge polarized in a plane inclined 45° to the right, and the other in a plane 45° to the left, of the plane of total reflexion. M. Fresnel has also discovered the following properties of a circularly polarized ray:—When it is transmitted through a thin doubly refracting plate parallel to its axis, it is divided into two pencils with complementary colors; and these colors will be an exact quarter of a tint, or an order of colors, either higher or lower in Newton's scale, than the color which the same crystallized plate would have given by polarized light. M. Fresnel also proved that a circularly polarized ray, when transmitted along the axis of rock crystal, will not exhibit the complementary colors when analyzed.

(129.) In the prosecution of this curious subject, M. Fresnel discovered the following method of producing a ray possessing all the above properties, and therefore exactly similar to one of the pencils produced by circular double refraction. Let A B C D, *fig*. 113., be a parallelopiped of crown glass, whose index of refraction is 1·510, and whose angles A B C, A D C are each 54½°. If a common polarized ray, R *r*, is incident perpendicularly upon A B, and emerges perpendicularly from C D, after having suffered two total reflexions at E and F, at angles of 54½°; and if these reflexions are performed in a plane inclined 45° to the plane of polarization of the ray, the emergent ray F G will have all the properties of a circularly polarized ray, resembling in every respect one of those produced by double refraction along the axis of rock crystal. But as this circularly polarized ray may be restored to a single plane of polarization, inclined 45° to the plane of reflexion, by two total reflexions at 54½°, it follows, and I have verified the result by observation, that if the parallelopiped A B C D is sufficiently long, the pencil will emerge circularly polarized,

Fig. 113.



at 2, 6, 10, 14, 18 reflexions, and polarized in a single plane after 4, 8, 12, 16, 20 reflexions.

M. Fresnel proved that the ray R *r* would emerge at G, circularly polarized by three total reflexions at 69° 12′, and four total reflexions at 74° 42′. Hence, according to the preceding reasoning, the ray will be circularly polarized by 9, 15, 21, 27, &c. reflexions at 69° 12′, and restored to common polarized light at 6, 12, 18, and 24 reflexions at the same angle; and it will be circularly polarized by 12, 20, 28, 36, &c. reflexions at 74° 42′, and be restored to common polarized light by 8, 16, 24, 32, &c. reflexions.

I have found that circular polarization can be produced by 2½, 7½, 12½, &c. reflexions, or any other number which is a multiple of 2½; for though we cannot see the ray in the middle of a reflexion, yet we can show it when it is restored to a single plane of polarization, at 5, 10, 15 reflexions.* When we use homogeneous light, we find that the angle at which circular polarization is produced is different for the differently colored rays; and hence these different rays cannot be restored to a single plane of polarization at the same angle of reflexion. Complementary colors will therefore be produced, such as I described long ago, and which, I believe, have not been observed by any other person.† These colors are essentially different from those of common polarized light, and will be understood when we come to explain those of elliptical polarization.

CHAP. XXVII.

ON ELLIPTICAL POLARIZATION, AND ON THE ACTION OF METALS UPON LIGHT.

On Elliptical Polarization.

(130.) THE action of metals upon light has always presented a troublesome anomaly to the philosopher. Malus at first announced that they produced no effect whatever; but he afterwards found that the difference between transparent and metallic bodies consisted in this,—that the former reflect all the light which they polarize in one plane, and refract all the light which they polarize in an opposite plane; while *metallic bodies reflect what they polarize* in both planes. Before I was acquainted with any of the experiments of Malus, I had found* that light was modified by the action of metallic bodies; and that, in all the metals which I tried, a great portion of light was polarized in the plane of incidence. In February, 1815, I discovered the curious property possessed by *silver* and *gold* and other metals, of dividing polarized rays into their complementary colors by successive reflexions: but I was misled by some of the results into the belief, that a reflexion from a metallic surface had the same effect as a certain thickness of a crystallized body; and that the polarized tints varied with the angle of incidence, and rose to higher orders, by increasing the number of reflexions. M. Biot, in repeating my experiments, and in an elaborate investigation of the phenomena,† was misled by the same causes, and has given a lengthened detail of experiments, formulæ, and speculations, in which all the real phenomena are obscured and confounded. Although I had my full share in this rash generalization, yet I never viewed it as a correct expression of the phenomena, and I have repeatedly returned to the subject with the most anxious desire of surmounting its difficulties. In this attempt I have succeeded; and I have been enabled to refer all the phenomena of the action of metals to a new species of polarization, which I have called *elliptical polarization*, and which unites the two classes of phenomena which constitute *circular* and *rectilineal* polarization.

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* See *Phil. Transactions*, 1830, p. 301.

† See *Phil. Transactions*, 1830, p. 309. 325.

(131.) In the action of metals upon common light, it is easy to recognize the fact announced by Malus, that the light which they reflect is polarized in different planes. I have found that the pencil polarized in the plane of reflexion is always more intense than that polarized in the perpendicular plane. The difference between these pencils is least in silver, and greatest in galena, and consequently the latter polarizes more light in the plane of reflexion than silver. The following table shows the effect which takes place with other metals:—

Order in which the Metals polarize most Light in the Plane of Reflexion.

Galena.
Lead.
Grey cobalt.
Arsenical cobalt.
Iron pyrites.
Antimony.
Steel.
Zinc.
Speculum metal.
Platinum.
Bismuth.
Mercury.
Copper.
Tin plate.
Brass.
Grain tin.
Jewellers' gold.
Fine gold.
Common silver.
Pure silver.
Total reflexion from glass.

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* *Treatise on New Philos. Instruments*, p. 347. and Preface.

† *Traité de Physique*, tom. iv. p. 579. 600.

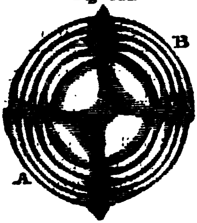
By increasing the number of reflexions, the whole of the incident light may be polarized in the plane of reflexion. Eight reflexions from plates of steel, between 60° and 80°, polarize the whole light of a wax candle ten feet distant. An increased number of reflexions [above 36] is necessary to do this with pure silver; and in total reflexions from glass, where the circular polarization begins, and where the two pencils are equal, the effect cannot be produced by any number of reflexions.

In order to examine the action of metals upon polarized light, we must provide a pair of plates of each metal, flatly ground and highly polished, and each at least 1½ inch long and half an inch broad. These parallel plates should be fixed upon a goniometer, or other divided instrument, so that one of the plates can be made to approach to or recede from the other, and so that their surfaces can receive the polarized ray at different angles of incidence. In place of giving the plates a motion of rotation round the polarized ray, I have found it better to give the plane of polarization of the ray a motion round the plates, so that the planes of reflexion and of polarization may be set at any required angle. The ray reflected from the plates one or more times is then analyzed, either by a plate of glass or a rhomb of Iceland spar.

When the plane of reflexion from the plates is either *parallel* or *perpendicular* to the plane of primitive polarization, the reflected light will receive no peculiar modification, excepting what arises from their property of polarizing a portion of light in the plane of reflexion. But in every other position of the plane of reflexion, and at every angle of incidence, and after any number of reflexions, the pencil will have received particular modifications, which we shall proceed to explain. One of these, however, is so beautiful and striking, as to arrest our immediate attention. When the plates are *silver* or *gold*, the most brilliant complementary colors are seen in the ordinary and extraordinary images, changing with the angle of incidence and the number of reflexions. These colors are most brilliant when the plane of reflexion is inclined 45° to the plane of incidence, and they vanish when the inclination is 0° or 90°. All the other metals in the table, p. 191, give analogous colors; but they are most brilliant in silver, and diminish in brilliancy from silver to galena.

In order to investigate the cause of these phenomena, let us suppose *steel* plates to be used, and the plane of the polarized ray to be inclined 45° to the plane of reflexion. At an incidence of 75° the light has suffered some physical change, which is a maximum at that angle. It is not polarized light, because it does not vanish during the revolution of the analyzing plate. It is neither partially polarized light nor common light; because, when we reflect it a second time at 75°, it is restored to light polarized in one plane. If we transmit the light reflected from steel at 75° along the axis of Iceland spar, the system of rings shown in *fig.* 98. is changed into the system shown in *fig.* 114., as if a thin film of a crystallized body which polarizes the blue of the first order had crossed the system. If we substitute for the calcareous spar films of sulphate of lime which give different tints, we shall find that these tints are increased in value by a quantity nearly equal to a quarter of a tint, according as the metallic action coincides with or opposes that of the crystal. It was on the authority of this experiment that I was led to believe that metals acted like crystallized plates. And when I found that the colors were better developed and more pure after successive reflexions, I rashly concluded, as M. Biot also did after me, that each successive reflexion corresponded to an additional thickness of the film. In order to prove the error of this opinion, let us transmit the light reflected 2, 4, 6, 8 times from steel at 75° along the axis of Iceland spar, and we shall find that the system of rings is perfect, and that the whole of the light is polarized in one plane; a result absolutely incompatible with the supposition of the tints rising with the number of reflexions. At 1, 3, 5, 7, 9, 11 reflexions, the ligh. when transmitted along the axis of Iceland spar will produce an effect equal to nearly a quarter of a tint, beyond which it never rises.

Fig. 114.



I now conceived that light reflected 1, 3, 5, 7, 9 times from steel at 75° resembled circularly polarized light. In circularly polarized light produced by *two* total reflexions from glass, the ray originally polarized + 45° to the plane of reflexion is, by the two reflexions at the same angle, restored to light polarized — 45° to the plane of reflexion; whereas in *steel*, a ray polarized + 45°, and reflected *once* from steel at 75°, is restored by another reflexion at 75° to light polarized — 17°.

With different metals the same effect is produced, but the inclination of the plane of polarization of the restored ray is different, as the following table shows:—

Total Reflexions.	Inclination of restored Ray.	Total Reflexions.	Inclination of restored Ray.
From glass - - -	45° 0′	Bismuth - - - -	21° 0′
Pure silver - - -	39 48	Speculum metal -	21 0
Common silver - -	36 0	Zinc - - - - - -	19 10
Fine gold - - - -	35 0	Steel - - - - - -	17 0
Jewellers' gold - -	33 0	Iron pyrites - - -	14 0
Grain tin - - - -	33 0	Antimony - - -	16 15
Brass - - - - -	32 0	Arsenical cobalt -	13 0
Tin Plate - - - -	31 0	Cobalt - - - - - -	12 30
Copper - - - - -	29 0	Lead - - - - - -	11 0
Mercury - - - -	26 0	Galena - - - - -	2 0
Platinum - - - -	22 0	Specular iron, - -	0 0

In total reflexions, or in circular polarization, the circularly polarized ray is restored to a single plane by the same number of reflexions and *at the same angle* at which it received circular polarization, whatever be the inclination of the plane of the second pair of reflexions to the plane of the first pair; but in metallic polarization, the angle at which the second reflexion restores the ray to a single plane of polarization varies with the inclination of the plane of the second reflexion to the plane of the first reflexion. In the case of total reflexions, this angle varies as the radii of a circle; that is, it is always the same. In the case of metallic polarization, it varies as the radii of an ellipse. Thus, when the plane of the polarized ray is inclined 45° to the plane of primitive polarization, the ray reflected once at 75° will be restored to polarized light at an incidence of 75°; but when the two planes are parallel to one another, the restoration takes place at 80°; and when they are perpendicular, at 70°; and at intermediate angles, at intermediate inclinations. For these reasons, I have called this kind of polarization *elliptic polarization.*

We have already seen that light polarized + 45° is elliptically polarized by 1, 3, 5, 7 reflexions from steel at 75°, and restored to a single plane of polarization by 2, 4, 6, 8 reflexions at the same angle; and we have stated that the ray restored by two reflexions has its plane of polarization brought into the state of — 17°. The following are the inclinations of this plane to the plane of reflexion, by different numbers of reflexions from steel and silver:—

No. of Reflexions.	Inclination of the Plane of the polarized Ray.		No. of Reflexions.	Inclination of the Plane of the polarized Ray.	
	Steel.	*Silver.*		*Steel.*	*Silver.*
2	—17° 0′	—38° 15′	10	— 0° 9′	—16° 56′
4	+ 5 22	+ 31 52	12	+ 0 3	+ 13 30
6	— 1 38	—26 6	18	— 0 0	— 6 42
8	+ 0 30	+ 21 7	36	+ 0 0	+ 0 47

These results explain in the clearest manner why *common light* is polarized by *steel* after eight reflexions, and by silver not till after thirty-six reflexions. Common light consists of two pencils, one polarized + 45°, and the other — 45°; and steel brings these planes of polarization into the plane of reflexion after eight reflexions, while silver requires more than thirty-six reflexions to do this.

(132.) The angles at which elliptical polarization is produced by one reflexion may be considered as the maximum polarizing angles of the metal, and their tangents may be considered as the indices of refraction of the different metals, as shown in the following table:—

Name of Metal.	Angle of Maximum Polarization.	Index of Refraction.
Grain tin - - - -	78° 30′	4·915
Mercury - - - -	78 27	4·893
Galena - - - -	78 10	4·773
Iron pyrites - - -	77 30	4·511
Grey cobalt - - -	76 56	4·309
Speculum metal -	76 0	4·011
Antimony melted -	75 25	3·844
Steel - - - - -	75 0	3·732
Bismuth - - - -	74 50	3·689
Pure silver - - -	73 0	3·271
Zinc - - - - -	72 30	3·172
Tin plate hammered	70 50	2·879
Jewellers' gold - -	70 45	2·864

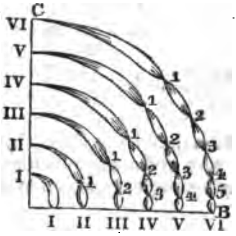
Elliptical polarization may be produced by a sufficient number of reflexions at any given angle, either above or below the maximum polarizing angle, as shown in the following table for *Steel*: -

Number of Reflexions at which Elliptical Polarization is produced.	Number of Reflexions at which the Pencil is restored to a single Plane.	Observed Angle of Incidence.
3 9 15 &c.	6 12 18 &c.	86° 0′
2½ 7½ 12½ &c.	5 10 15 &c.	84 0
2 6 10 &c.	4 8 12 &c.	82 20
1½ 4½ 7½ &c.	3 6 9 &c.	79 0
1 3 5 &c.	2 4 6 &c.	75 0
1½ 4½ 7½ &c.	3 6 9 &c.	67 40
2 6 10 &c.	4 8 12 &c.	60 20
2½ 7½ 12½ &c.	5 10 15 &c.	56 25
3 9 15 &c.	6 12 18 &c.	52 20

When the number of reflexions is an integer, it is easily understood how an elliptically polarized ray begins to retrace its course, and to recover its state of polarization in a single plane, by the same number of reflexions by which it lost it; but it is interesting to observe, when the number of reflexions is 1½, 2½, or any other mixed number, that the ray must have acquired its elliptical polarization in the middle of the second and third reflexion; that is, when it had reached its greatest depth within the metallic surface it then begins to resume its state of polarization in a single plane, and recovers it at the end of 3, 5, and 7, reflexions. A very remarkable effect takes place when one reflexion is made on one side of the maximum polarizing angle, and one on the other side. A ray that has received partial elliptical polarization by one reflexion at 85° does not acquire more elliptic polarization by a reflexion at 54°, but it retraces its course and recovers its state of single polarization.

By a method which it would be out of place to explain here, I have determined the number of points of restoration which can occur at different angles of incidence from 0° to 90°, for any number of reflexions; and I have represented them in *fig.* 115., where the arches I, I., II, II., &c. represent the quadrant of incidence, for *one*, *two*, &c. reflexions; C being the point of 0°, and B that of 90° of incidence. In the quadrant, I, I. there is no point of restoration. In II, II. there is only *one* point or node of restoration, viz. at 73° in *silver*. In III, III. there are two points of restoration, because a ray elliptically polarized by one and a half reflexion will be restored by three reflexions at 63° 43′ beneath the maximum polarizing angle, and at 79° 40′ above that angle. It may also be shown that for IV. reflexions there are 3 points of restoration, for V. reflexions 4 points; and for VI. reflexions 5 points, as shown in the figure. The loops or double curves are drawn to represent the intensity of the elliptic polarization which has its minimum at 1, 2, 3, &c., and its maximum in the middle of the unshaded parts. If we now use homogeneous light, we shall find that the loops have different sizes in the different colored rays, and that their minima and maxima are different.

Fig. 115.



Hence, in the VIth quadrant, C B for example, there will be 6 loops of all the different colors, viz. C 1; 1, 2; 2, 3; 3, 4, &c.; overlapping one another, and producing by their mixture those beautiful complementary colors which have already been mentioned. For a more full account of this curious branch of the subject of polarization, I must refer the reader to the *Philosophical Transactions*, 1830; or to the *Edinburgh Journal of Science*, Nos. VII. and VIII. new series, April, 1831.

CHAP. XXVIII.

ON THE POLARIZING STRUCTURE PRODUCED BY HEAT, COLD, COMPRESSION, DILATATION, AND INDURATION.

The various phenomena of double refraction, and the systems of polarized rings with *one* and two axes of double refraction, and with planes of no double refraction, may be produced either *transiently* or *permanently*, in glass and other substances, by *heat* and *cold*, *rapid cooling*, *compression and dilatation*, and *induration*.

1. *Transient Influence of Heat and Cold.*

(1.) *Cylinders of glass with one positive axis of double refraction.*

(133.) If we take a cylinder of glass, from half an inch to an inch in diameter, or upwards, and about half an inch or more in thickness, and transmit heat from its circumference to its centre, it will exhibit when exposed to polarized light, in the apparatus, *fig.* 94., a system of rings with a black cross, exactly similar to those in *fig* 98.; and the complementary system shown in *fig.* 99. will appear by turning round the plate B 90°. In this case we must hold the cylinder at the distance of 8 or 10 inches from the eye, when the rings will appear as it were in the inside of the glass. If we cover up any portion of the surface of the glass cylinder, we shall hide a corresponding portion of the rings, so that the cylinder has its single axis of double refraction *fixed* in the axis of its figure, and not in every possible direction parallel to that axis as in crystals.

By crossing the rings with a plate of sulphate of lime, as formerly explained, we shall find that it depresses the tints in the two quadrants which the axis of the plate crosses; and consequently that the system of rings is *negative*, like that of calcareous spar.

As soon as the heat reaches the axis of the cylinder, the rings begin to lose their brightness, and when the heat is uniformly diffused through the glass, they disappear entirely.

(2.) *Cylinders of glass with a negative axis of double refraction.*

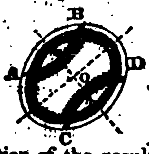
(134.) If a similar cylinder of glass is heated uniformly in boiling oil, or otherwise brought to a considerably high temperature, and is made to cool rapidly by surrounding its circumference with a good conductor, it will exhibit a similar system of rings, which will all vanish when the glass is uniformly cold. By crossing these rings with sulphate of lime, they will be found to be *positive*, like those of ice and zircon; or the same thing may be proved by combining this system of rings with the preceding system, when they will be found to destroy one another.

In both these systems of rings, the numerical value of the tint or color at any one point varies as the square of the distance of that point from the axis. By placing thin films of sulphate of lime between two of these systems of rings, very beautiful systems may be produced.

(3.) *Oval plates of glass with two axes of double refraction.*

(135.) If we take an oval plate A B D C, *fig*. 116., and perform with it the two preceding experiments, we shall find that it has in both cases *two axes* of double refraction, the principal axis passing through O, being *negative* when it is heated at its circumference, and *positive* when cooled at its circumference. The curves A B, C D, correspond to the black ones in *fig*. 101., and the distance *m n* to the inclination of the resultant axes. The effect shown in *fig*. 116. is that which is produced by inclining *m n* 45° to the plane of primitive polarization; but when *m n* is in the plane of primitive polarization, or perpendicular to it, the curves A B, C D, will form a black cross, as in *fig*. 100.

Fig. 116.



In all the preceding experiments, the *heat* and *cold* might have been introduced and conveyed through the glass from each extremity of the axis of the cylinder or plate. In this case the phenomena would have been exactly the same, but the axes that were formerly *negative* will now be *positive*, and *vice versâ*.

(4.) *Cubes of glass with double refraction.*

(136.) When the shape of the glass is that of a cube, the rings have the form shown in *fig.* 117. and when it is a parallelopiped with its length about three times its breadth, the rings have the form shown in *fig.* 118. the curves of equal tint near the angles being *circles*, as shown in both the figures.

Fig. 117.

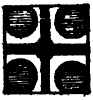
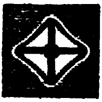


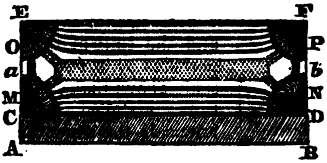
Fig. 118.



(5.) *Rectangular plates of glass with planes of no double refraction.*

(137.) If a well annealed rectangular plate of glass, E F D C, is placed with its lower edge C D on a piece of iron A B D C *fig.* 119., nearly red hot, and the two together are placed in the apparatus, *fig.* 94., so that C D may be inclined 45° to the plane of primitive polarization, and that polarized light may reach the eye at O from every part of the glass, we shall observe the following phenomena. The instant that the heat enters the surface C D, fringes of brilliant colors will be seen parallel to C D, and almost at the same time before the heat has reached the upper surface E F, or even the central line *a b*, similar fringes will appear at E F. Colors at first *faint blue*, and then *white*, *yellow*, *orange*, &c., all spring up at *a b*; and these central colors will be divided from those at the edges by two dark lines, M N, O P, in which there is neither double refraction nor polarization. These lines correspond with the black curves in *fig.* 101. and *fig.* 116., and the structure between M N and O P is *negative*, like that of calcareous spar; while the structures without M N and O P are *positive*, like those of zircon. The tints thus developed are those of Newton's scale, and are compounded of the different

Fig. 119.



sets of tints that would be given in each of the homogeneous rays of the spectrum.

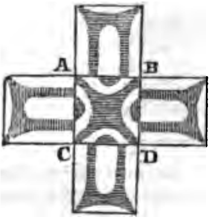
In these plates there is obviously an infinite number of axes in the planes passing through M N, O P, and all the tints, as well as the double refraction, can be calculated by the very same laws as in regular crystals, *mutatis mutandis.*

If the plate E F D C is *heated* equally all round, the fringes are produced with more regularity and quickness; and if the plate, first heated in oil or otherwise, is cooled equally all round, it will develope the same fringes, but the central ones at *a b* will in this last case be *positive*, and the outer ones at E F and C D *negative.*

Similar effects to those above described may be produced in similar plates of rock salt, obsidian, fluor spar, copal, and other solids that have not the doubly refracting structure.

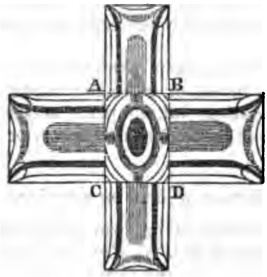
A series of splendid phenomena are produced by crossing similar or dissimilar plates of glass when their fringes are developed. When *similar* plates of glass, or those in which the fringes are produced by heat, as in *fig.* 119., are crossed, the curves or lines of equal tint at the square of intersection, A B C D, *fig.* 120., will be hyperbolas. The tint at the centre will be the difference of the central tints of each of the two plates, and the tints of the succeeding hyperbolas will rise gradually in the scale above that central tint. If the tints produced by each plate are precisely the same, and the plates of the same shape, the central tints will destroy each other, the hyperbolas will be equilateral ones, and the tints will gradually rise from the zero of Newton's scale.

Fig. 120.



When dissimilar plates are crossed, as in *fig.* 121., viz. one in which the fringes are produced by heat with one in which they are produced by cold, the lines of equal tint in the square of intersection A B C D (*fig.* 121.), will be *ellipses.* The tints in the centre will be equal to the sum of the separate tints, and the tints formed by the combination of the external fringes will be equal to their difference. If the plates and their tints are perfectly equal, the lines of equal tint will be *circles.* The beauty of these combinations can be understood only from colored drawings. When the plates are combined lengthwise, they add to or subtract from each other's effect, according as similar or dissimilar fringes are opposed to one another.

Fig. 121.



(6.) *Spheres of glass, &c. with an infinite number of axes of double refraction.*

(138.) If we place a sphere of glass in a glass trough of hot oil, and observe the system of rings, while the heat is passing to the centre of the sphere, we shall find it to be a regular system, exactly like that in *fig.* 98.; and it will suffer no change by turning the sphere in any direction. Hence the sphere has an infinite number of *positive* axes of double refraction, or one along each of its diameters.

If a very hot sphere of glass is placed in a glass trough of cold oil, a similar system will be produced, but the axes will all be *negative.*

(7.) *Spheroids of glass with one axis of double refraction along the axis of revolution and two axes along the equatorial diameters.*

(139.) If we place an oblate spheroid in a glass trough of hot oil, we shall find that it has *one* axis of *positive* double refraction along its shorter axis, or that of revolution; but if we transmit the polarized light along any of its equatorial diameters, we shall find that it has two axes of double refraction, the black curves appearing as in *fig.* 116. when the axis of revolution is inclined 45° to the plane of primitive polarization, and changing into a cross when the axis is parallel or perpendicular to the plane of primitive polarization.

The very same phenomena will be exhibited with a prolate spheroid, only the black cross opens in a different plane when the two axes are developed.

Opposite systems of rings will be developed in both these cases, if hot spheroids are plunged in cold oil.

The reason of using oil is to enable the polarized light to pass through the spheres or spheroids without refraction. The oil should have a refractive power as near as possible to that of the glass.

A number of very curious phenomena arise from heating and cooling glass tubes, or cylinders, along their axes; the most singular variations taking place according as the heat and cold are applied to the circumference, or to the axis, or to both.

(8.) *Influence of heat on regular crystals.*

(140.) The influence of uniform heat and cold on regular crystals is very remarkable. M. Fresnel found that heat dilates sulphate of lime less in the direction of its principal axis than in a direction perpendicular to it; and professor Mitscherlich has found that Iceland spar is dilated by heat in the direction of its axis of double refraction, while in all directions at right angles to this axis it contracts; so that there must be some intermediate direction in which there is neither contraction nor dilatation. Heat brings the rhomb of Iceland spar nearer to the cube, and diminishes its double refraction.

In applying heat to *sulphate of lime*, professor Mitscherlich found that the two resultant axes (P, P, *fig*. 106.) gradually approach as the heat increases, till they unite at O, and form a single axis. By a still farther increase of heat they open out on each side towards A and B. A very curious fact of an analogous kind I have found in *glauberite*, which has *one* axis of double refraction for *violet*, and *two* axes for red light. With a heat below that of boiling water, the two resultant axes (P, P, *fig*. 106.) unite at O, and, by a slight increase of heat, the resultant axes again open out, one in the direction O A, and the other in the direction O B. By applying cold, the single axis for violet light at O opened out into two at P and P. At a certain temperature the violet axis also opened out into two, in the plane A B.

2. *On the permanent Influence of sudden Cooling.*

(141.) In March, 1814, I found that glass melted and suddenly cooled, such as prince Rupert's drops, possessed a permanent doubly refracting structure;[*] and in December, 1814, Dr. Seebeck published an account of analogous experiments with cubes of glass. Cylinders, plates, cubes, spheres, and spheroids of glass, with a permanent doubly refracting structure, may be formed by bringing the glass to a red heat, and cooling it rapidly at its circumference, or at its edges. As these solid bodies often lose their shape in the process, the symmetry of their structure is affected, and the system of rings or fringes injured; so that the phenomena are not produced so perfectly as during the transient influence of heat and cold. It is often necessary, too, to grind and polish the surfaces afresh: an operation during which the solids are often broken, in consequence of the state of constraint in which the particles are held.

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[*] Letter to Sir Joseph Banks, April 8. 1814. *Phil. Trans.* 1814.

An endless variety of the most beautiful optical figures may be produced by cooling the glass upon metallic patterns (metals being the best conductors) applied symmetrically to each surface of the glass, or symmetrically round its circumference. The heat may be thus drawn off from the glass in lines of any form or direction, so as to give any variety whatever to its structure, and, consequently, to the optical figure which it produces when exposed to polarized light.

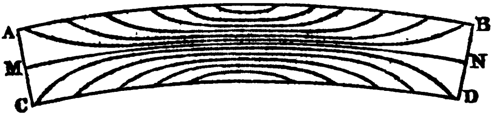
(142.) In all doubly refracting crystals the form of the rings is independent of the external shape of the crystal; but in glass solids that have received the doubly refracting structure, either transiently or permanently, from heat, the rings depend entirely on the external shape of the solid. If, in *fig.* 119., we divide the rectangular plate E F D C into two equal parts through the line *a b*, each half of the plate will have the same structure as the whole, viz. a negative and two positive structures, separated by two dark neutral lines. In like manner, if we cut a piece of a tube of glass, by a notch, through its circumference to its centre, or if we alter the shape of cylindrical plates and spheres, &c., by grinding them into different external figures, we produce a complete change upon the optical figures which they had previously exhibited.

3. *On the Influence of Compression and Dilatation.*

(143.) If we could compress and dilate the various solids above mentioned with the same uniformity with which we can heat and cool them, we should produce the same doubly refracting structures which have been described, compression and dilatation always producing opposite structures.

The influence of compression and dilatation may be well exhibited by taking a strip of glass, A B D C, *fig.* 122., and bending it by the force of the hands. When it is held in the apparatus, *fig.* 94., with its edge A B inclined 45° to the plane of primitive polarization, the whole thickness of the glass will be covered with colored fringes, consisting of a negative set separated from a positive set by the dark neutral line M N. The fringes on the *convex* side A B are *negative*, and those on the *concave* side *positive*. As the bending force increases, the tints increase in number; and as it diminishes, they diminish in number, disappearing entirely when the plate of glass recovers its shape. The tints, which are those of Newton's scale, vary with their distances from M N; and when two such plates as that shown in *fig.* 122. cross each other, they produce in the square of intersection *rectilineal* fringes parallel to the diagonal of the square which joins the angles where the two concave and the two convex sides of the plates meet.

Fig. 122.



When a plate of bent glass is made to cross a plate crystallized by heat, and suddenly cooled, the fringes in the square of intersection are parabolas, whose vertex will be towards the *convex* side of the bent plate, if the principal axis of the other plate is *positive*, but towards the *concave* side, if that axis is *negative*.

The effects of compression and dilatation may be most distinctly seen by pressing or dilating plates or cylinders of calves'-feet jelly or soft isinglass.

By the application of compressing and dilating forces, I have been able to alter the doubly refracting structure of regularly crystallized bodies in every direction, increasing or diminishing their tints according to the direction in which the forces were applied.*

The most remarkable influence of pressure, however, is that which it produces on a mixture of resin and white wax. In all the cases hitherto mentioned of the artificial production of double refraction, the phenomena are related to the shape of the mass in which the change is induced: but I have been able to communicate to the compound above mentioned a double refraction, similar to that which exists in the particles of crystals. The compressed mass has a single axis of double refraction in every parallel direction, and the colored rings are produced by the inclination of the refracted ray to the axis according to the same law as in regular crystals. If we remove the compressed film, any portion of it will be found to have one axis of double refraction like portions of a film of any crystal with one axis. The important deductions which this experiment authorizes will be noticed at the conclusion of this part of the work.

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* See *Edinburgh Transactions*, vol. viii. p. 281.

4. *On the Influence of Induration.*

(144.) In 1814 I had occasion to make some experiments on the influence of induration in communicating double refraction to soft solids. When isinglass is dried in a glass trough of a circular form, it exhibits a system of tints with the black cross exactly like *negative* crystals with one axis. When a thin cylindrical plate of isinglass is indurated at its circumference, it produces a system of rings with one *positive* axis. If the trough in the first of these experiments and the plate in the second are oval, two axes of double refraction will be exhibited.

When jelly placed in rectangular troughs of glass is gradually indurated, we have a positive and a negative structure developed, and these are separated by a black neutral line. If the bottom of the trough is taken out, so as to allow the induration to go on at two parallel surfaces, the same fringes are produced as in a rectangular plate of glass heated in oil, and subsequently cooled.

Spheres and spheroids of jelly may be made by proper induration to produce the same effects as spheres and spheroids of glass when heated or cooled. The lenses of almost all animals possess the doubly refracting structure. In some there is only one structure, which is generally positive. In others there are two structures, a positive and a negative one; and in many there are three structures, a negative between two positive, and a positive between two negative structures. In some instances we have two structures of the same name together. By the process of induration we may remove entirely the natural structure of the lens, especially when it is spherical or spheroidal, and superinduce the structure arising from induration. I have now before me a *spheroidal* lens of the *boneto* fish, with one beautiful system of rings along the axis of the spheroid, and two systems along the equatorial diameters. I have also several indurated lenses of the cod, that display in the finest manner their doubly refracting structure.

CHAP. XXIX.

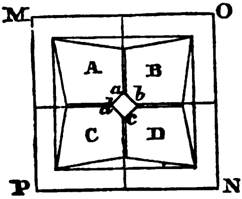
PHENOMENA OF COMPOSITE OR TESSELATED CRYSTALS.

(145.) In all regularly formed doubly refracting crystals, the separation of the two images, the size of the rings, and the value of the tints, are exactly the same in all parallel directions. If two crystals, however, have grown together with their axes inclined to one another, and if we cut a plate out of these united crystals so that the eye cannot distinguish it from a plate cut out of a single crystal, the exposure of such a crystal to polarized light will instantly detect its composite nature, and will exhibit to the eye the very line of junction. This will be obvious upon considering that the polarized ray has different inclinations to the axis of each crystal, and will therefore produce different tints at these different inclinations. Hence the examination of a body in polarized light furnishes us with a new method of *discovering structures which cannot be detected by the microscope*, or any other method of observation.

A very fine example of this is exhibited in the *bipyramidal sulphate of potash*, which Count Bournon and other crystallographers regarded as one simple crystal, whose primitive form was the bipyramidal dodecahedron, like the crystal shown in *fig.* 112. But by cutting a plate perpendicular to the axis of the pyramid, and exposing it to polarized light, I found it to be composed of several crystals, all united so as to form the regular figure above represented. The crystal has two axes of double refraction, and the plane passing through the two axes of one, is inclined 60° to the plane passing through the two axes of each of the other two. So that when we incline the plate, each of the three combined crystals displays different colors. I have found many remarkable structures of this kind in the mineral kingdom, and among artificial salts; but two of these are so interesting as to merit particular notice.

(146.) The apophyllite from Faroe generally crystallizes in right-angled square prisms, and splits with great facility into plates by planes perpendicular to the axis of the prism. If we remove with a sharp knife the *uppermost* slice, or the *undermost*, it will be found to have one axis of double refraction, and to give the single system of rings shown in *fig*. 98. If we remove other slices in the same manner, we shall find that when exposed to polarized light they exhibit the curious tesselated structure shown in *fig*. 123. The outer case, M O N P, consists of a number of parallel veins or plates. In the centre is a small lozenge, ***a b c d***, with one axis of double refraction, and round it are four crystals, A, B, C, D, with two axes of double refraction, the plane passing through the axes of A and D being perpendicular to the plane passing through the axes of B and C; and the former plane being in the direction M N, and the latter in the direction O P.

Fig. 123.



When the polarized light is transmitted through the faces of certain prisms, the beautiful tesselated figure shown in *fig*. 124 is exhibited, all the differently shaded parts shining with the most splendid colors. As the prism has everywhere the same thickness, it is obvious that the doubly refracting force varies in different parts of the crystal; but this variation takes place in such a symmetrical manner in relation to the sides and ends of the prism, as to set at defiance all the recognized laws of crystallography.

Fig. 124.

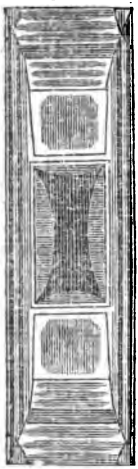
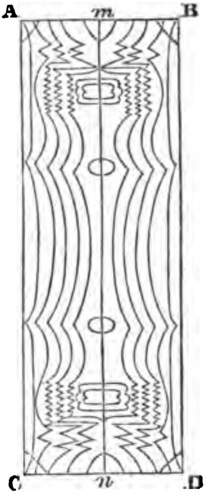


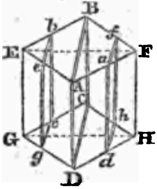
Fig. 125.



With the view of observing the form of the lines of equal color, I immersed the crystal in oil, and transmitted the polarized light in a direction parallel to a diagonal of the prism; the effect then exhibited is shown in *fig*. 125., where A B C D is the crystal; A C, and B D, its edges, where the thickness is nothing, and ***m n*** the edge through which the diagonal of the prism passes. Now, it is obvious, that if this had been a regular crystal, the lines of equal tint or of equal double refraction would have been all ***straight lines parallel to*** A C or B D; but in the apophyllite they present the most singular irregularities, all of which are, however, symmetrically related to certain fixed points within the crystal. In the middle of the crystal, half way between m and n, there are only *five* fringes or orders of colors; at points equi-distant from this there are *six* fringes, the sixth returning into itself in the form of an *oval*. At other two equidistant points near m and n, the 3d, 4th, and 5th fringes are singularly serrated, and the 6th and 7th fringes return into themselves in the form of a *square;* beyond this, near m and n, there are only four fringes, in consequence of the fifth returning into itself.

(147.) A composite structure of a very different kind, but extremely interesting from the effects which it produces, is exhibited in many crystals of Iceland spar, which are intersected by parallel films or veins of various thicknesses, as shown in *fig.* 126. These thin veins or strata are perpendicular to the short diagonals E F, G H of the faces of the rhomb, and parallel to the edges E G, F H. When we look perpendicularly through the faces A E B F, D G C H, the light will not pass through any of the planes *e b c g*, A B C D, *a f h d*, and consequently we shall only see two images of any object just as if the planes were not there. But if we look through any of the other two pair of parallel faces, we shall observe the two common images at their usual distance; and at a much greater distance, two secondary images, one on each side of the common images. In some cases there are *four*, and in other cases *six*, secondary images, arranged in two lines; one line being on each side of the common images, and perpendicular to the line joining their centres. When the interrupting planes are numerous, and especially when they are also found perpendicular to the short diagonals of the other two faces of the rhomb that meet at B, the obtuse summit, the secondary images are extremely numerous, and sometimes arranged in pyramidal heaps of singular beauty, vanishing, and reappearing, and changing their color and the intensity of their light, by every inclination of the plate. If the light of the luminous object is polarized, the phenomena admit of still greater variations. When the strata or veins are thick, the images are not colored, but have merely at their edges the colors of refracted light.

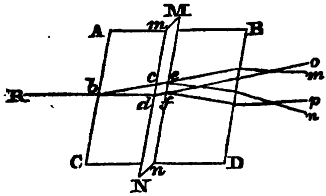
Fig. 126.



Malus considered these phenomena as produced by fissures or cracks within the crystal, and he regarded the colors as those of thin plates of air or space; but I have found that they are produced by veins or twin crystals firmly united together so as to resist separation more powerfully than the natural cleavage planes, and I have found this both crystallographically, by measuring the angles of the veins, and optically, by observing the system of rings seen through the veins alone.

This composite structure will be understood from *fig.* 127., where A B D C is the principal section of a rhomb of Iceland

Fig. 127.



spar whose axis is A D. The form and position of one of the intersecting veins or rhomboidal plates, is shown at M *m* N *n*, but greatly thicker than it actually is; the angles A *m* M, and D *n* N, being 141° 44′. A ray of common light R *b*, incident on the face A C at *b*, will be refracted in the lines *b c*, *b d*. These rays entering the vein M *m* N *n*, at *c* and *d*, will be again refracted doubly; but as the vein is so thin as to produce the complementary colors of polarized light by the interference of the two pencils which compose each of the pencils *c e*, *d f*, these colors will depend on the thickness of the vein M N, and on the inclination of the ray to the axis of the plate M N. These double pencils will emerge from the vein at *e*, *f*, and will be refracted again as in the figure into the pencils *e m*, *e n*, *f o*, *f p*; the colors of *e n*, *f o*, being complementary to those of *e m*, *f p*. That the multiplication and color of the images are owing to the causes now explained may be proved ocularly, as I have done, by dividing rhombs of calcareous spar, and inserting between them, or in grooves cut in a single plate of calcareous spar, a thin film of sulphate of lime or mica. In this way all the phenomena of the natural compound crystal may be reproduced in the artificial one, and we may give great variety to the phenomena by inserting thin films in different azimuths round the polarized pencils *b c*, *b d*, and at different inclinations to the axis of double refraction.

The compound crystal shown in *fig.* 127. is in reality a natural polarizing apparatus. The part of the rhomb A *m* N C, polarizes the incident light R *b*. The vein M N is the thin crystallized vein whose colors are to be examined; and the part B M *n* D, is the analyzing rhomb.

Various other minerals and artificial crystals are intersected with analogous veins, and produce analogous phenomena. There are several composite crystals which exhibit remarkable peculiarities of structure, and display curious optical phenomena by polarized light. The Brazilian topaz is one of those which is worthy of notice, and whose properties I have explained by colored drawings, in the second volume of the *Cambridge Transactions.*

For a full account of the properties of composite crystals, and of the multiplication of images by the crystals of calcareous spar that are intersected by veins, we refer the reader to the *Edinburgh Transactions*, vol. ix. p. 317., and the *Phil. Trans.*, 1815, p. 270.; or to the *Edinburgh Encyclopedia*, art. OPTICS.

CHAP. XXX

ON THE DICHROISM, OR DOUBLE COLOR, OF BODIES; AND THE ABSORPTION OF POLARIZED LIGHT.

(148.) IF a crystallized body has a different color in different directions when common light is transmitted through its substance, it is said to possess *dichroism*, which signifies two colors. Dr. Wollaston observed this property long ago in the *muriate of palladium* and *potash*, which appeared of a *deep red* color along the axis, and of a *vivid green* in a transverse direction; and M. Cordier observed the same change of color in a mineral called *iolite*, to which Haüy gave the name of *dichroite*. Mr. Herschel has observed a similar fact in a variety of *sub-oxysulphate of iron*, which is of a deep *blood red* color along the axis, and of a *light green* color perpendicular to the axis. In examining this class of phenomena, I have found that they depend on the absorption of light, being regulated by the inclination of the incident ray to the axis of double refraction, and on a difference of color in the two pencils formed by double refraction.

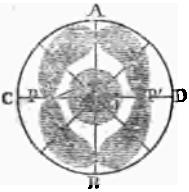
In a rhomb of *yellow* Iceland spar, the extraordinary image was of an *orange yellow* color, while the ordinary image was *yellowish white* along the axis. The color and intensity of the two pencils were the same, and the difference of color and intensity increased with the inclination to the axis. When the two images overlapped each other, their combined color was the same at all angles with the axis, and this color was that of the mineral. If we expose the rhomb to polarized light, its color will be *orange yellow* in the position where the ordinary image vanishes, and yellowish white in the position where the extraordinary image vanishes. The crystals in the following Table possess the same properties, the ordinary and extraordinary images having the colors opposite to their names:—

Colors of the two Images in Crystals with ONE AXIS.

Names of Crystals.	Principal Section in Plane of Polarization.	Principal Section, perpendicular to Plane of Polarization.
Zircon.	Brownish white.	Deeper brown.
Sapphire.	Yellowish green.	Blue.
Ruby.	Pale yellow.	Bright pink.
Emerald.	Yellowish green.	Bluish green.
Emerald.	Bluish green.	Yellowish green.
Beryl, blue.	Bluish white.	Blue.
Beryl, green.	Whitish.	Bluish green.
Beryl, yellowish green.	Pale yellow.	Pale green.
Rock crystal, nearly transparent.	Whitish.	Faint brown.
Rock crystal, yellow.	Yellowish white.	Yellow.
Amethyst.	Blue.	Pink.
Amethyst.	Greyish white.	Ruby red,
Amethyst.	Reddish yellow.	Bluish green.
Tourmaline.	Greenish white.	Bluish green.
Rubellite.	Reddish white.	Faint red.
Idocrase.	Yellow.	Green.
Mellite.	Yellow.	Bluish white.
Apatite lilac.	Bluish.	Reddish.
Apatite olive.	Bluish green.	Yellowish green.
Phosphate of lead	Bright green.	Orange yellow.
Iceland spar.	Orange yellow.	Yellowish white.
Octohedrite.	Whitish brown.	Yellowish brown.

(149.) When the crystals have two axes of double refraction, the absorption of the incident rays produces a variety of phenomena, at and near the two resultant axes. These phenomena are finely displayed in *iolite.* This mineral, which crystallizes in six and twelve-sided prisms, is of a *deep blue* color when seen along the axis, and of a *brownish yellow* when seen in a direction perpendicular to the axis of the prism. When we look along the resultant axes which are inclined 62° 50′ to one another, we see a system of rings which are pretty distinct when the plate is thin; but when it is thick, and when the plane passing through the axes is in the plane of primitive polarization, branches of blue and white light are seen to diverge in the form of a cross from the centre of the system of rings. This curious effect is shown in *fig.* 128., where P, P′, are the centres of the two systems of rings, O the principal negative axis of the crystal, and C D the plane passing through the axes. The blue branches, which are shaded in the figure, are tipped with purple at their summits P, P′, and are separated by whitish light in some specimens,

Fig. 123.



and by bluish light in others. From P and P′ to O, the white or yellowish light becomes more and more blue, and at O it is quite blue; while from P and P′ to C and D it becomes more and more yellow, and at C and D it is quite yellow, the yellow being almost equally bright in the plane A C B D, perpendicular to the principal axis O. When the plane C D is perpendicular to the plane of primitive polarization, the poles P, P′ are marked with patches of white or yellowish light, but everywhere else the light is a deep blue.

When examined by common light, we find that the *ordinary* image is *brownish yellow* at C and D, and the extraordinary one *faint blue;* the former acquiring some blue rays, and the latter some yellow ones from C to D, and from A to B where there is still a great difference in the color of the images. The yellow image becomes fainter from A to P and P′, and from B to P and P′, where it changes into *blue*, the feeble blue image being gradually reinforced by other blue rays till the intensity of the two blue images is nearly equal. The faint blue image increases in intensity from C to P, and from D to P′, and the yellow one acquires an accession of blue light, and becomes bluish white from P and P′ to O; the ordinary image is whitish, and the other a deep blue; but the whiteness gradually diminishes towards O, where the two images are almost equally blue. The following table will show that this property exists in many other crystals:—

Colors of the two Images in Crystals with TWO AXES.

Names of Crystals.	Plane of Axis in Plane of Polarization.	Plane of Axis perpendicular to Plane of Polarization.
Topaz blue.	White.	Blue.
—— green.	White.	Green.
—— greenish blue	Reddish grey.	Blue.
—— pink.	Pink.	White.
—— pink yellow.	Pink.	Yellow.
—— yellow.	Yellowish white.	Orange.
Sulphate of baryta.		
——— yellowish purple.	Lemon yellow.	Purple.
——— yellow.	Lemon yellow.	Yellowish white.
——— orange yellow	Gamboge yellow.	Yellowish white.
Cyanite.	White.	Blue.
Dichroite.	Blue.	Yellowish white.
Cymophane.	Yellowish white.	Yellowish.
Epidote olive green.	Brown.	Sap green.
——— whitish green	Pink white.	Yellowish white.
Mica.	Reddish brown.	Reddish white.

The following table shows the color of the images in crystals with two axes which have not been examined.

Names of Crystals.	Axis of Prism in Plane of Polarization.	Axis of Prism perpendicular to Plane of Polarization.
Mica.	Blood red.	Pale greenish yellow.
Acetate of copper.	Blue.	Greenish yellow.
Muriate of copper.*	Greenish white.	Blue.
Olivine.	Bluish green.	Greenish yellow.
Sphene.	Yellow.	Bluish.
Nitrate of copper.	Bluish white.	Blue.
Chromate of lead.	Orange.	Blood red.
Staurotide.	Brownish red.	Yellowish white.
Augite.	Blood red.	Bright green.
Anhydrite.	Bright pink.	Pale yellow.
Axinite.	Reddish white.	Yellowish white.
Diallage.	Brownish white.	White.
Sulphur.	Yellow.	Deeper yellow.
Sulphate of strontia.	Blue.	Bluish white.
———— cobalt.	Pink.	Brick red.
Olivine.	Brown.	Brownish white.

In the last nine crystals in the preceding table, the tints are not given in relation to any fixed line.

The following list contains the colors of the two pencils, in crystals, whose number of axes is not yet known.

Phosphate of iron.	Fine blue.†		Bluish white.	
Actynolite.	Green.		Greenish white.	
Precious opal.	Yellow.		Lighter yellow.	
Serpentine.	Dark green.		Lighter green.	
Asbestos.	Greenish.		Yellowish.	
Blue carbonate of copper.	Violet blue.		Greenish blue.	
Octohedrite (one axis.)	Whitish brown.		Yellowish brown.	
Chloride of gold and sodium.	Lemon yellow.	Axis in Plane of Polarization.	Deep orange.	Axis perpendicular to Plane of Polarization.
———— and ammonium.	Lemon yellow.		Deep orange.	
———— and potassium.	Lemon yellow.		Deep orange.	

(150.) By the application of heat to certain crystals, I have been able to produce a permanent difference in the color of the two pencils formed by double refraction. This experiment may be made most easily on Brazilian topaz. In one of these topazes, in which one of the pencils was *yellow* and the other *pink*, I found that a red heat acted more powerfully upon the extraordinary than upon the ordinary pencil, discharging the yellow color entirely from the one, and producing only a slight change upon the pink tint of the other. When the topaz was hot, it was perfectly colorless, and, during the process of cooling, it gradually acquired a pink tint, which could not be modified or renewed by the most intense heat. In various topazes, the color of whose two pencils was exactly the same, heat discharges more of the color from one pencil than the other, and thus gives them the power of absorbing light in reference to the axes of double refraction.

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* The colors are given in relation to the short diagonal of its rhomboidal base.

† When the axis of the prism is in the plane of polarization.

General Observations on Double Refraction.

(151.) The various facts which have been explained in the preceding chapters, enable us to form very plausible opinions respecting the origin and nature of the doubly refracting structure. The particles of bodies reduced to a state of fluidity by heat, and prevented by the same cause from combining into a solid body, exhibit no double refraction; and, in like manner, the particles of crystallized bodies, including metals when existing in a state of solution, exhibit no double refraction. As soon, however, as *cooling* in the one case, and *evaporation* in the other, permits the particles to combine in virtue of their mutual affinities, these particles have, subsequent to the action of the forces by which they combine, acquired the doubly refracting structure. This effect may be accounted for in two ways; either by supposing that the particles have originally a doubly refracting structure, or that they have no trace of such a structure. On the first of these suppositions, we must ascribe the disappearance of the double refraction in the fluid mass, and, in the solution, to the opposite action of the particles, which must have had an axis in every possible direction; but as no double refraction is visible, it is more philosophical to suppose that none exists in the particles. On the second supposition, then, that the particles have no doubly refracting structure, it is easily understood how it may be produced by the compression of any two particles brought together by attraction; for each particle will have an axis of double refraction in the direction of the line joining their centres, as if they had been compressed by an external force. By following out this idea, which I have done elsewhere,[*] I have shown how the various phenomena may be explained by the different attractive forces of three rectangular axes, which may produce a single negative axis, a single positive axis, or two axes, either both positive or both negative, or the one positive and the other negative. The influence of heat, in changing the intensity of the two axes of sulphate of lime, and in removing one of the axes, or in creating a new one, admits of an easy explanation on these principles.

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[*] *Phil. Transactions*, 1829, or *Edinburgh Journal of Science*, new series vol. vi. p. 328—337.

PART III.

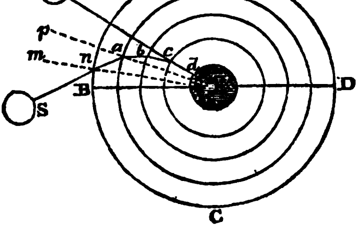
ON THE APPLICATION OF OPTICAL PRINCIPLES TO THE EXPLANATION OF NATURAL PHENOMENA.

CHAP. XXXI.

ON UNUSUAL REFRACTION.

(152.) THE atmosphere in which we live is a transparent mass of air possessing the property of refracting light. We learn from the barometer that its density gradually diminishes as we rise in the atmosphere, and, as we know from direct experiment that the refractive power of air increases with its density, it follows, that the refractive power of the atmosphere is greatest at the earth's surface, and gradually diminishes till the air becomes so rare as scarcely to be able to produce any effect upon light. When a ray of light falls obliquely upon a medium thus varying in density, in place of being bent at once out of its direction, it will be gradually more and more bent during its passage through it, so as to move in a curve line, in the same manner as if the medium had consisted of an infinite number of strata of different refractive powers. In order to explain this, let E, *fig*. 129., be

Fig 129.



the earth, surrounded with an atmosphere A B C D, consisting of four concentric strata of different densities and different refractive powers. The index of refraction for air at the earth's surface being 1·000,294, let us suppose that the index of the other three strata is 1·000,200, 1·000,120, 1·000,050. Let B E D be the horizon, and let a ray S *n*, proceeding from the sun under the horizon, fall on the outer stratum at *n*, whose index of refraction is 1·000,050. Drawing the perpendicular E *n m*, find by the rule formerly given the angle of refraction, E *n a*, corresponding to the angle of incidence S *n m*. When the ray *n a* falls on the second stratum at *a*, whose index of refraction is 1·000,120, we may in like manner, by drawing a perpendicular E *a p*, find the refracted ray *a b*. In the same way, the refracted rays *b c* and *c d* may be found. The same ray S *n* will therefore have been refracted in a polygonal line *n a b c d*, and as it reaches the eye in the direction *c d*, the sun will be seen in the direction *d c* S′, elevated above the horizon, by the refraction of the atmosphere, when it is still below it. In like manner it might be shown that the sun appears above the horizon by refraction, when he is actually below it at sunset.

Although the rays of light move in straight lines *in vacuo* and in all media of uniform density, yet, on the surface of the globe, the rays proceeding from a distant object, must necessarily move in a curve line, because they must pass through portions of air of different densities and refractive powers. Hence it follows that, excepting in a vertical line, no object, whether it is a star or planet beyond our atmosphere, or is actually within it, is seen in its real place.

Excepting in astronomical and trigonometrical observations, where the greatest accuracy is necessary, this refraction of the atmosphere does not occasion any inconvenience. But since the density of the air and its refractive power vary greatly when heated or cooled, great local heats or local colds will produce great changes of refractive power, and give rise to optical phenomena of a very interesting kind. Such phenomena have received the name of *unusual refraction*, and they are sometimes of such an extraordinary nature as to resemble more the effects of magic than the results of natural causes.

(153.) The elevation of coasts, mountains, and ships, when seen over the surface of the sea, has long been observed and known by the name of *looming*. Mr. Huddart described several cases of this kind, but particularly the very interesting one of an *inverted image* of a ship seen beneath the real ship. Dr. Vince observed at Ramsgate a ship, whose topmasts only were seen above the horizon; but he at the same time observed, in the field of the telescope through which he was looking, two images of the complete ship in the air, both directly above the ship, the uppermost of the two being erect, and the other inverted. He then directed his telescope to another ship whose hull was just in the horizon, and he observed a complete inverted image of it; the mainmast of which just touched the mainmast of the ship itself. The first of these two phenomena is shown in *fig*. 130. in which A is the real ship, and B, C the images seen by unusual refraction. Upon looking at another ship, Dr. Vince saw inverted images of some of its parts which suddenly appeared and vanished, "first appearing," says he, "below, and running up very rapidly, showing more or less of the masts at different times as they broke out, resembling in the swiftness of their breaking out the shooting of a beam of the aurora borealis." As the ship continued to descend, more of the image gradually appeared, till the image of the whole ship was at last completed, with the mainmasts in contact. When the ship descended still lower, the image receded from the ship, but no second image was seen. Dr. Vince observed another case, shown in *fig*. 131., in which the sea was distinctly seen between the ships B, C. As the ship A came above the horizon, the image C gradually disappeared, and during this time the image B descended; but the ship did not seem so near the horizon as to bring the mainmasts together. The two images were visible when the whole ship was beneath the horizon.

Fig. 130.

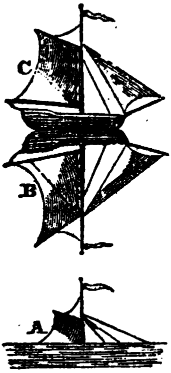
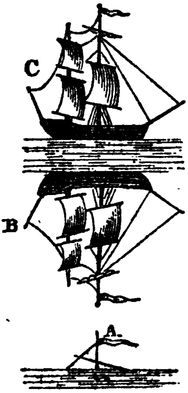


Fig. 131.



Captain Scoresby, when navigating the Greenland seas, observed several very interesting cases of unusual refraction. On the 28th of June, 1820, he saw from the mast-head eighteen sail of ships at the distance of about twelve miles. One of them was drawn out, or lengthened, in a vertical direction; another was contracted in the same direction; one had an inverted image immediately above it; and other two had two distinct inverted images above them, accompanied with two images of the strata of ice. In 1822, Captain Scoresby recognized his father's ship, the Fame, by its inverted image in the air, *although the ship itself was below the horizon.* He afterwards found that the ship was seventeen miles beyond the horizon, and its distance thirty miles. In all these cases, the image was directly above the object; but on the 17th of September, 1818, MM. Jurine and Soret observed a case of unusual refraction, where the image was on one side of the object. A bark about 4000 toises distant was seen approaching Geneva by the left bank of the lake, and at the same moment there was seen above the water an image of the sails, which, in place of following the direction of the bark, receded from it, and seemed to approach Geneva by the right bank of the lake; the image sailing from east to west, while the bark was sailing from north to south. The image was of the same size as the object when it first receded from the bark, but it grew less and less as it receded, and was only one-half that of the bark when the phenomenon ceased.

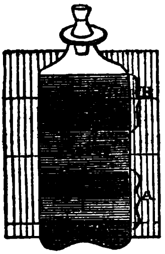
While the French army was marching through the sandy deserts of Lower Egypt, they saw various phenomena of unusual refraction, to which they gave the name of *mirage.* When the surface of the sand was heated by the sun, the land seemed to be terminated at a certain distance by a general inundation. The villages situated upon eminences appeared to be so many islands in the middle of a great lake, and under each village there was an inverted image of it. As the army approached the boundary of the apparent inundation, the imaginary lake withdrew, and the same illusion appeared round the next village. M. Monge, who has described these appearances in the *Mémoires sur l'Egypte,* ascribes them to reflexion from a reflecting surface, which he supposes to take place between two strata of air of different densities.

One of the most remarkable cases of mirage was observed by Dr. Vince. A spectator at Ramsgate sees the tops of the four turrets of Dover Castle over a hill between Ramsgate and Dover. Dr. Vince, however, on the 6th of August, 1806, at seven P. M., saw *the whole of Dover Castle,* as if it had been brought over and placed on the Ramsgate side of the hill. The image of it was so strong that the hill itself was not seen through the image.

The celebrated *fata morgana,* which is seen in the straits of Messina, and which for many centuries astonished the vulgar and perplexed philosophers, is obviously a phenomenon of this kind. A spectator on an eminence in the city of Reggio, with his back to the sun and his face to the sea, and when the rising sun shines from that point whence its incident ray forms an angle of about 45° on the sea of Reggio, sees upon the water numberless series of pilasters, arches, castles well delineated, regular columns, lofty towers, superb palaces with balconies and windows, villages and trees, plains with herds and flocks, armies of men on foot and on horseback, all passing rapidly in succession on the surface of the sea. These same objects are, in particular states of the atmosphere, seen in the air, though less vividly; and when the air is hazy, they are seen on the surface of the sea, vividly colored, or fringed with all the prismatic colors.

(154.) That the phenomena above described are generally produced by refraction through strata of air of different densities may be proved by various experiments. In order to illustrate this, Dr. Wollaston poured into a square phial (*fig.* 132.) a small quantity of clear *syrup*, and above this he poured an equal quantity of *water*, which gradually combined with the syrup, as seen at A. The word *Syrup* upon a card held behind the bottle appeared erect when seen through the pure syrup, but inverted, as represented in the figure, when seen through the mixture of water and syrup. Dr. Wollaston then put nearly the same quantity of *rectified spirit of wine* above the *water*, as in the same figure at B, and he saw the appearance there represented, the true place of the word *Spirit*, and the inverted and erect images below.

Fig. 132.



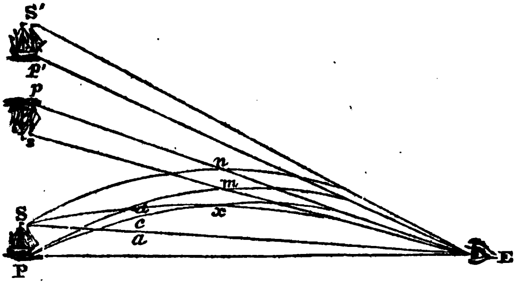
Analogous phenomena may be seen by looking at objects over the surface of a hot poker, or along the surface of a wall or painted board heated by the sun.

The late Mr. H. Blackadder has described some phenomena both of vertical and lateral mirage as seen at King George's Bastion, Leith, which are very instructive. The extensive bulwark, of which this bastion forms the central part, is formed of huge blocks of cut sandstone, and from this to the eastern end the phenomena are best seen. To the east of the tower the bulwark is extended in a straight line to the distance of 500 feet. It is eight feet high towards the land, with a footway about two feet broad, and three feet from the ground. The parapet is three feet wide at top, and is slightly inclined towards the sea.

When the weather is favorable, the top of the parapet resembles a mirror, or rather a sheet of ice; and if in this state another person stands or walks upon it, an observer at a little distance will see an inverted image of the person under him. If, while standing on the footway another person stands on it also, but at some distance, with his face turned towards the sea, his image will appear opposite to him, giving the appearance of two persons talking or saluting each other. If, again, when standing on the footway, and looking in a direction from the tower, another person crosses the eastern extremity of the bulwark, passing through the water-gate, either to or from the sea, there is produced the appearance of two persons moving in opposite directions, constituting what has been termed a lateral mirage: first one is seen moving past, and then the other in an opposite direction, with some interval between them. In looking over the parapet, distant objects are seen variously modified; the mountains (in Fife) being converted into immense bridges; and on going to the eastward extremity of the bulwark, and directing the eye towards the tower, the latter appears curiously modified, part of it being as it were cut off and brought down, so as to form another small and elegant tower in the form of certain sepulchral monuments. At other times it bears an exact resemblance to an ancient altar, the fire of which seems to burn with great intensity.*

(155.) In order to explain as clearly as possible how the erect and inverted image of a ship is produced as in *fig.* 131., let S P (*fig.* 133.) be a ship in the horizon, seen at E by

Fig. 133.



means of rays S E, P E passing in straight lines through a track of air of uniform density lying between the ship and the eye. If the air is more rare at *c* than at *a*, which it may be from the coldness of the sea below *a*, its refractive power will

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[*] *Edinburgh Journal of Science*, No. V. p. 13

be less at *c* than at *a*. In this case, rays S *d*, P *c*, which, under ordinary circumstances, never could have reached the eye at E, will be bent into curve lines P *c*, S *d*; and if the variation of density is such that the uppermost of these rays S *d* crosses the other at any point *x*, then S *d* will be undermost, and will enter the eye E as if it came from the lower end of the object. If E *p*, E *s*, are tangents to these curves or rays, at the point where they enter the eye, the part S of the ship will be seen in the direction E *s*, and the part P in the direction E *p*; that is, the image *s p* will be *inverted*. In like manner, other rays, S *n*, P *m*, may be bent into curves S *n* E, P *m* E, which do not cross one another, so that the tangent E *s'* to the curve or ray S *n* will still be uppermost, and the tangent E *p'* undermost. Hence the observer at E will see an erect image of the ship at *s' p'* above the inverted image *s p*, as in *fig*. 131. It is quite clear that the state of the air may be such as to exhibit only one of these images, and that these appearances may be all seen when the real ship is beneath the horizon.

In one of captain Scoresby's observations we have seen that the ship was drawn out, or magnified, in a vertical direction, while another ship was contracted or diminished in the same direction. If a cause should exist, which is quite possible, which elongated the ship horizontally at the same time that it elongated it vertically, the effect would be similar to that of a convex lens, and the ship would appear magnified, and might be recognized at a distance far beyond the limits of unassisted vision. This very case seems to have occurred. On the 26th July, 1798, at Hastings, at five P. M. Mr. Latham saw the French coast, which is about 40 or 50 miles distant, as distinctly as through the best glasses. The sailors and fishermen could not at first be persuaded of the reality of the appearance; but as the cliffs gradually appeared more elevated, they were so convinced that they pointed out and named to Mr. Latham the different places which they had been accustomed to visit: such as the bay, the windmill at Boulogne, St. Vallery, and other places on the coast of Picardy. All these places appeared to them as if they were sailing at a small distance into the harbor. From the eastern cliff or hill, Mr. Latham saw at once Dungeness, Dover cliffs, and the French coast, all the way from Calais, Boulogne, on to St. Vallery, and, as some of the fishermen affirmed, as far as Dieppe. The day was extremely hot, without a breath of wind, and objects at some distance appeared greatly magnified.

This class of phenomena may be well illustrated, as I have elsewhere* suggested, by holding a mass of heated iron above a considerable thickness of water, placed in a glass trough, with plates of parallel glass. By withdrawing the heated iron, the gradation of density increasing downwards, will be accompanied by a decrease of density from the surface, and through such a medium the phenomena of the mirage may be seen.

(156.) That some of the phenomena ascribed to unusual refraction are owing to unusual reflexion, arising from difference of density, cannot, we think, admit of a doubt. If an observer beyond the earth's atmosphere at S, *fig*. 129., were to look at one composed of strata of different refractive powers as shown in the figure, it is obvious that the light of the sun would be reflected at its passage through the boundary of each stratum, and the same would happen if the variation of refractive power were perfectly gradual. Well described cases of this kind are wanting to enable us to state the laws of the phenomena; but the following fact, as described by Dr. Buchan, is so distinct, as to leave no doubt respecting its origin. "Walking on the cliff," says he, "about a mile to the east of Brighton, on the morning of the 18th of November, 1804, while watching the rising of the sun, I turned my eyes directly towards the sea just as the solar disc emerged from the surface of the water, and saw the face of the cliff on which I was standing represented *precisely opposite to me* at some distance on the ocean. Calling the attention of my companion to this appearance, we soon also discovered *our own figures* standing on the summit of the opposite apparent cliff, as well as the representation of a windmill near at hand. The reflected images were most distinct precisely opposite to where we stood, and the false cliff seemed to fade away, and to draw near to the real one, in proportion as it receded towards the west. This phenomenon lasted about ten minutes, till the sun had risen nearly his own diameter above the sea. The whole then seemed to be elevated into the air, and successively disappeared, like the drawing up of a drop scene in a theatre. The surface of the sea was covered with a dense fog of many yards in height, and which gradually receded before the rays of the sun. The sun's light fell upon the cliff at an incidence of about 73° from the perpendicular."

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* *Edinburgh Encyclopædia*, art. Heat.

CHAP. XXXII.

ON THE RAINBOW.*

(157.) The rainbow is, as every person knows, a luminous arch extending across the region of the sky opposite to the sun. Under very favorable circumstances, two bows are seen, the inner and the outer, or the *primary* and the *secondary*, and within the primary rainbow, and in contact with it, and without the secondary one, there have been seen supernumerary bows.

The primary or inner rainbow, which is commonly seen alone, is part of a circle whose radius is 42°. It consists of seven differently colored bows, viz. *violet*, which is the innermost, *indigo*, *blue*, *green*, *yellow*, *orange*, and *red*, which is the outermost. These colors have the same proportional breadth as the spaces in the prismatic spectrum. This bow is, therefore, only an infinite number of prismatic spectra, arranged in the circumference of a circle; and it would be easy, by a circular arrangement of prisms, or by covering up all the central part of a large lens, to produce a small arch of exactly the same colors. All that we require, therefore, to form a rainbow, is a great number of transparent bodies capable of forming a great number of prismatic spectra from the light of the sun.

As the rainbow is never seen, unless when *rain* is actually falling between the spectator and the sky opposite to the sun, we are led to believe that the transparent bodies required are *drops* of rain which we know to be small spheres. If we look into a globe of glass or water held above the head, and opposite to the sun, we shall actually see a prismatic spectrum reflected from the farther side of the globe. In this spectrum the *violet* rays will be innermost, and the spectrum vertical. If we hold the globe horizontal on a level with the eye, so as to see the sun's light reflected in a horizontal plane, we shall see a horizontal spectrum with the *violet* rays innermost. In like manner, if we hold a globe in a position intermediate between these two, so as to see the sun's light reflected in a plane inclined 45° to the horizon, we shall perceive a spectrum inclined 45° to the horizon with the *violet* innermost. Now, since in a shower of rain there are drops in all positions relative to the eye, the eye will receive spectra inclined at all angles to the horizon, so that when combined they will form the large circular spectrum which constitutes the rainbow.

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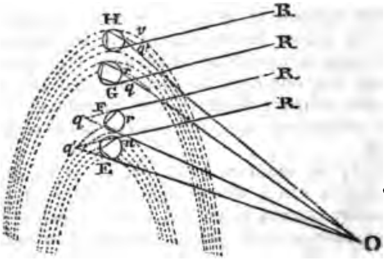
* In the College edition, see Appendix of Am. ed. chap. vii.

To explain this more clearly, let E, F, *fig.* 134., be drops of rain exposed to the sun's rays, incident upon them in the

Fig. 134.



directions R E, R F; out of the whole beam of light which falls upon the drop, those rays which pass through or near the axis of the drop will be refracted to a focus behind it, but those which fall on the upper side of the drop will be refracted, the *red* rays least, and the *violet* most, and will fall upon the back of the drop with an obliquity such that many of them will be reflected, as shown in the figure. These rays will be again refracted, and will meet the eye at O, which will perceive a spectrum or prismatic image of the sun, with the *red* space uppermost, and the *violet* undermost. If the sun, the eye, and the drops E, F, are all in the same vertical plane, the spectrum produced by E, F will form the colors at the very summit of the bow as in the figure. Let us now suppose a drop to be near the horizon, so that the eye, the drop, and the sun, are in a plane inclined to the horizon; a ray of the sun's light will be reflected in the same manner as at E, F, with this difference only, that the plane of reflexion will be inclined to the horizon, and will form part of the bow distant from the summit. Hence, it is manifest, that the drops of rain above the line joining the eye and the upper part of the rainbow, and in the plane passing through the eye and the sun, will form the upper part of the bow; and the drops to the right and left hand of the observer, and without the line joining the eye and the lowest part of the bow, will form the lowest part of the bow on each hand. Not a single drop, therefore, between the eye and the space within the bow is concerned in its production: so that, if a shower were to fall regularly from a cloud, the rainbow would appear before a single drop of rain had reached the ground.

If we compute the inclination of the *red* ray and the *violet* ray to the incident rays R E, R F, we shall find it to be 42° 2′ for the *red*, and 40° 17′ for the *violet*, so that the breadth of the rainbow will be the difference of those numbers, or 1° 45′, or nearly three times and a half the sun's diameter. These results coincide so accurately with observation, as to leave no doubt that the primary rainbow is produced by two refractions and one intermediate reflexion of the rays that fall on the upper sides of the drops of rain.

It is obvious that the *red* and *violet* rays will suffer a second reflexion at the points where they are represented as quitting the drop, but these reflected rays will go up into the sky, and cannot possibly reach the eye at O. But though this is the case with rays that enter the upper side of the drop as at E F, or the side farthest from the eye, yet those which enter it on the under side, or the side nearest the eye, may after two reflexions reach the eye, as shown in the drops H, G, where the rays R, R enter the drops below. The *red* and *violet* rays will be refracted in different directions, and after being twice reflected will be finally refracted to the eye at O; the violet forming the upper part, the red the under part of the spectrum. If we now compute the inclination of these rays to the incident rays R, R, we shall find them to be 50° 57′ for the *red* ray, and 54° 7′ for the *violet* ray; the difference of which or 3° 10′ will be the breadth of the bow, and the distance between the bows will be 8° 55′.* Hence it is clear that a secondary bow will be formed exterior to the primary bow, and with its colors reversed, in consequence of their being produced by two reflexions and two refractions. The breadth of the secondary bow is nearly twice as great as that of the primary one, and its colors must be much fainter, because it consists of light that has suffered two reflexions in place of one.

(158.) Sir Isaac Newton found the semi-diameter of the interior bow to be 42°, its breadth 2° 10′, and its distance from the outer bow 8° 30′; numbers which agree so well with the calculated results as to leave no doubt of the truth of the explanation which has been given. But if any farther evidence were wanted, it may be found in the fact, which I observed in 1812, that *the light of both the rainbows is wholly polarized in planes passing through the eye and the radii of the arch.* This result demonstrates that the bows are formed by reflexion at or near the polarizing angle, from the surface of a transparent body. The production of artificial rainbows by the spray of a waterfall, or by a shower of drops scattered by a mop, or forced out of a syringe, is another proof of the preceding explanation. Lunar rainbows are sometimes seen, but the colors are faint, and scarcely perceptible. In 1814, I saw, at Berne, a *fog-bow*, which resembled a nebulous arch, in which the colors were invisible.

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* No correction for the sun's apparent diameter, is here made.

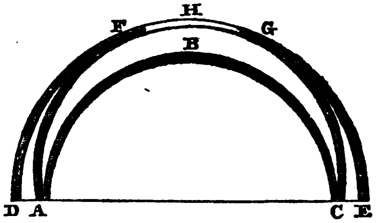
(159.) On the 5th of July, 1828, I observed three supernumerary bows within the primary bow, each consisting of *green* and *red* arches, and in contact with the *violet* arch of the primary bow. On the outside of the outer or secondary bow I saw distinctly a *red* arch, and beyond it a very faint green one, constituting a supernumerary bow, analogous to those within the primary rainbow.

Dr. Halley has shown that the rainbow formed by *three* reflexions within the drops will encircle the sun itself, at the distance of 40° 20′, and that the rainbow formed by *four* reflexions will likewise encircle him at the distance of 45° 33′. The rainbows formed by *five* reflexions will be partly covered by the secondary bow. The light which forms these three bows is obviously too faint to make any impression on our organs, and these rainbows have therefore never been observed.

Many peculiar rainbows have been seen and described. On the 10th August, 1665, a faint rainbow was seen at Chartres, crossing the primary rainbow at its vertex. It was formed by reflexion from the river.

On the 6th August, 1698, Dr. Halley, when walking on the walls of Chester, observed a remarkable rainbow, shown in *fig*. 135., where A B C is the primary bow, D H E the secondary one, and A F H G C the new bow intersecting the secondary bow D H E, and dividing it nearly into three parts. Dr. Halley observed the points F, G to rise, and the arch F G gradually to contract, till at length the two arches F H G and F G coincided, so that the secondary iris for a great space lost its colors, and appeared like a white arch at the top. The new bow, A H C, had its colors in the same order as the primary

Fig. 135.



one A B C, and consequently the reverse of the secondary bow; and on this account the two opposite spectra at G and F counteracted each other, and produced whiteness. The sun at this time shone on the river Dee, which was unruffled, and Dr. Halley found that the bow A H C was only that part of the circle of the primary bow that would have been under the castle bent upwards by reflexion from the river. A third rainbow seen between the two common ones, and not concentric with them, is described in Rozier's Journal, and is doubtless the same phenomenon as that observed by Dr. Halley. Red rainbows, distorted rainbows, and inverted rainbows on the grass, have been seen. The latter are formed by the drops of rain suspended on the spiders' webs in the fields.

CHAP. XXXIII.

ON HALOS, CORONÆ, PARHELIA, AND PARASELENÆ.

(160.) WHEN the sun and moon are seen in a clear sky, they exhibit their luminous discs without any change of color, and without any attendant phenomena. In other conditions of the atmosphere, the two luminaries not only experience a change of color, but are surrounded with a variety of luminous circles of various sizes and forms. When the air is charged with dry exhalations, the sun is sometimes as red as blood. When seen through watery vapors, he is shorn of his beams, but preserves his disc white and colorless; while, in another state of the sky, I have seen the sun of the most brilliant salmon color. When light fleecy clouds pass over the sun and moon, they are often encircled with *one*, *two*, *three*, or even more, colored rings, like those of thin plates; and in cold weather, when particles of ice are floating in the higher regions, the two luminaries are frequently surrounded with the most complicated phenomena, consisting of concentric circles, circles passing through their discs, segments of circles, and mock suns or moons, formed at the points where these circles intersect each other.

The name *halo* is given indiscriminately to these phenomena, whether they are seen round the sun or the moon. They are called *parhelia* when seen round the sun, and *paraselenæ* when seen round the moon.

The small halos seen round the sun and moon in fine weather, when they are partially covered with light fleecy clouds, have been also called *coronæ*. They are very common round the sun, though, from the overpowering brightness of his rays, they are best seen when he is observed by reflexion from the surface of still water. In June, 1692, Sir Isaac Newton observed, by reflexion in a vessel of standing water, three rings of color round the sun, like three little rainbows. The colors of the first or innermost were *blue* next the sun, *red* without, and *white* in the middle between the *blue* and *red.* The colors of the second ring were *purple* and *blue* within, and *pale red* without, and *green* in the middle. The colors of the third ring were *pale blue* within and *pale red* without. The colors and diameters of the rings are more particularly given as follows:—

1st Ring	Blue, white, red	Diameter, 5° to 6°.
2d Ring	Purple, blue, green Pale yellow, red	Diameter, 9⅓°.
3d Ring	Pale blue, pale red	Diameter, 12°.

On the 19th February, 1664, Sir Isaac Newton saw a *halo* round the moon, of two rings, as follows:—

1st Ring	White, bluish green, yellow, red	Diameter, 3°
2d Ring	Blue, green, red	Diameter, 5½°

Sir Isaac considers these rings as formed by the light passing through very small drops of water, in the same manner as the colors of thick plates. On the supposition that the globules of water are the 500th of an inch in diameter, he finds that the diameters of the rings should be as follows:—

1st Red ring	Diameter, 7¼°
2d Red ring	Diameter, 10¼°
3d Red ring	Diameter, 12° 33′

The rings will increase in size as the globules become less, and diminish if the globules become larger.

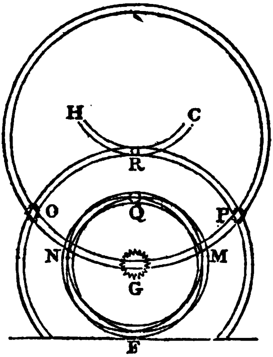
The halos round the sun and moon, which have excited most notice, are those which are about 47° and 94° in diameter. In order to form a correct idea of them, we shall give accurate descriptions of two; one a *parhelion,* and the other a *paraselene.*

The following is the original account of a *parhelion,* seen by Scheiner in 1630:—

(161.) "The diameter of the circle M Q N next to the sun, was about 45°, and that of the circle O R P was about 95° 20′; they were colored like the primary rainbow; but the red was next the sun, and the other colors in the usual order. The breadths of all the arches were equal to one another, and about a third part less than the diameter of the sun, as represented in *fig.* 136.; though I cannot say but the whitish circle

O G P, parallel to the horizon, was rather broader than the rest. The two parhelia M, N were lively enough, but the

Fig. 136.



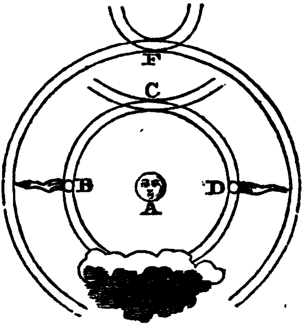
other two at O and P were not so brisk. M and N had a purple redness next the sun, and were white in the opposite parts. O and P were all over white. They all differed in their durations; for P, which shone but seldom and but faintly, vanished first of all, being covered by a collection of pretty thick clouds. The parhelion O continued constant for a great while, though it was but faint. The two lateral parhelia M, N were seen constantly for three hours together. M was in a languishing state, and died first, after several struggles, but N continued an hour after it at least. Though I did not see the last end of it, yet I was sure it was the only one that accompanied the true sun for a long time, having escaped those clouds and vapors which extinguished the rest. However, it vanished at last, upon the fall of some small showers. This phenomenon was observed to last 4½ hours at least, and since it appeared in perfection when I first saw it, I am persuaded its whole duration might be above five hours.

"The parhelia Q, R were situated in a vertical plane passing through the eye at F, and the sun at G, in which vertical the arches H R C, O R P either crossed or touched one another. These parhelia were sometimes brighter, sometimes fainter than the rest, but were not so perfect in their shape and whitish color. They varied their magnitude and color according to the different temperature of the sun's light at G, and the matter that received it at Q and R; and therefore their light and color were almost always fluctuating, and continued, as it were, in a perpetual conflict. I took particular notice that they appeared almost the first and last of the parhelia, excepting that of N.

"The arches which composed the small halo M N next to the sun, seemed to the eye to compose a single circumference, but it was confused, and had unequal breadths; nor did it constantly continue like itself, but was perpetually fluctuating. But in reality it consisted of the arches expressed in the figure, as I accurately observed for this very purpose.* These arches cut each other in a point at Q, and there they formed a parhelion; the parhelia M, N shining from the common intersections of the inner halo, and the whitish circle O N M P."

(162.) Hevelius observed at Dantzic, on the 30th of March, 1660, at one A. M., the paraselene shown in *fig*. 137. The moon A was seen surrounded by an entire whitish circle B C D E, in which there were two mock moons at B and D; one at each side of the moon, consisting of various colors, and shooting out very long and whitish beams by fits. At about two o'clock a larger circle surrounded the lesser, and reached to the horizon. The tops of both these circles were touched by colored arches, like inverted rainbows. The inferior arch at C was a portion of a large circle, and the superior at F a portion of a lesser. This phenomenon lasted nearly three hours. The outward great circle vanished first. Then the larger inverted arch at C, and then the lesser; and last of all the inner

Fig. 137.



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* The four intersecting circles which form this inner halo are described from four centres, one at each angle of a small square.

circle B C D E disappeared. The diameter of this inner circle, and also of the superior arch, was 45°, and that of the exterior circle and inferior arch was 90°.

On another occasion Hevelius observed a large white rectangular cross passing through the disc of the moon, the moon being in the intersection of the cross, and encircled with a halo exactly like the inner one in the preceding figure.

(163.) The frequent occurrence of the halos of 47° and 94° in cold weather, and especially in the northern regions of the globe, led to the belief that they must be formed by crystals of ice and snow floating in the air. Descartes supposed that they were produced by refraction, through flat stars of pellucid ice; and Huygens, who investigated the subject both experimentally and theoretically, has published an elaborate theory of halos, in which he assumes the existence of particles of hail, some of which are globular and others cylindrical, with an opaque nucleus or kernel having a certain proportion to the whole. He supposes these cylinders to be kept in a vertical position, by ascending currents of air or vapor, and to have their axes at all possible inclinations to the horizon, when they are dispersed by the wind or any other causes. He considers these cylinders to have been at first a globular collection of the softest and purest particles of snow, to the bottom of which other particles adhere, the ascending currents preventing them from adhering to the sides; they will, therefore, assume a cylindrical shape. Huygens then supposes that the outer part of the cylinders may be melted by the heat of the sun, a small cylinder remaining unmelted in the centre, and that if the melted part is again frozen, it may have sufficient transparency to refract and reflect the rays of the sun in a regular manner. By means of this apparatus, the existence of which is not impossible, Huygens has given a beautiful solution of almost all the difficulties which have been encountered in explaining the origin of halos.

Sir Isaac Newton regarded the halo of 45° as produced by a different cause from the small prismatic coronæ; and he was of opinion that it arose from refraction "from some sort of hail or snow floating in the air in a horizontal posture, the refracting angle being about 58° or 60°."

When we consider, however, the great variety of crystalline forms which water assumes in freezing; that these crystals really exist in a transparent state in the atmosphere, in the form of crystals of ice, which actually prick the skin like needles; and that simple and compound crystals of snow, of every conceivable variety of shape, are often falling through the atmosphere, and sometimes melting in passing through its lower and warmer strata, we do not require any hypothetical cylinders to account for the principal phenomena of halos.

Mariotte, Young, Cavendish, and others, have agreed in ascribing the halo of 45° or 46° in diameter, to refraction through prisms of ice, with refracting angles of 60° floating in the air, and having their refracting angles in all directions. The crystals of hoar-frost have actually such angles, and if we compute the deviation of the refracted rays of the sun or moon incident upon such a prism, with the index of refraction for ice, taken at 1·31, we shall find it to be 21° 50′, the double of which is 43° 40′. In order to explain the larger halo, Dr. Young supposes that the rays which have been once refracted by the prism may fall on other prisms, and the effect then be doubled by a second refraction, so as to produce a deviation of 90°. This, however, is by no means probable, and Dr. Young has candidly acknowledged the "great apparent probability" of Mr. Cavendish's suggestion, that the external halo may be produced by the refraction of the rectangular terminations of the crystals. With an index of refraction of 1·31, this would give a deviation of 45° 44′, or a diameter of 91° 28′, and the mean of several accurate measures is 91° 40′, a very remarkable coincidence.

The existence of prisms with such rectangular terminations is still hypothetical; but I have removed the difficulty on this point, by observing in the hoar-frost upon stones, leaves, and wood, regular quadrangular crystals of ice, both simple and compound.

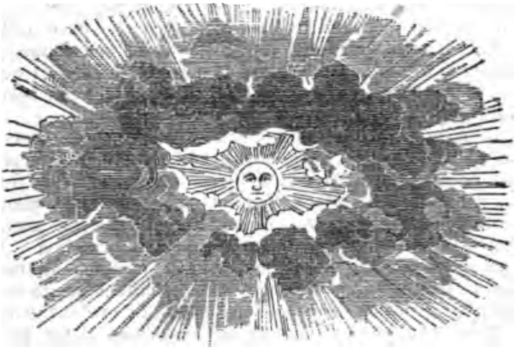
Although halos are generally represented as circles, with the sun or moon in their centres, yet their apparent form is commonly an irregular oval, wider below than above, the sun being nearer their upper than their lower extremity. Dr. Smith has shown that this is an optical deception, arising from the apparent figure of the sky, and he estimates that when the circle touches the horizon, its apparent vertical diameter is divided by the moon, in the proportion of about 2 to 3 or 4; and is to the horizontal diameter drawn through the moon as 4 to 3, nearly.

With the view of ascertaining if any of the halos are formed by reflexion, I have examined them with doubly refracting prisms, and have found that the light which forms them has not suffered reflexion.

The production of halos may be illustrated experimentally by crystallizing various salts upon plates of glass, and looking through the plates at the sun or a candle. When the crystals are granular and properly formed, they will produce the finest effects. A few drops of a saturated solution of alum, for example, spread over a plate of glass so as to crystallize quickly, will cover it with an imperfect crust, consisting of flat octohedral crystals, scarcely visible to the eye. When the observer, with his eye placed close behind the smooth side of the glass plate, looks through it at a luminous body, he will perceive *three fine halos* at different distances, encircling the source of light. The interior halo, which is the whitest of the three, is formed by the refraction of the rays through a pair of faces in the crystals that are least inclined to each other. The second halo, which is *blue* without and *red* within, with all the prismatic colors, is formed by a pair of more inclined faces; and the third halo, which is large and brilliantly colored, from the increased refraction and dispersion, is formed by the most inclined faces. As each crystal of alum has three pairs of each of these included prisms, and as these refracting faces will have every possible direction to the horizon, it is easy to understand how the halos are completed and equally luminous throughout. When the crystals have the property of double refraction, and when their axis is perpendicular to the plates, more beautiful combinations will be produced.

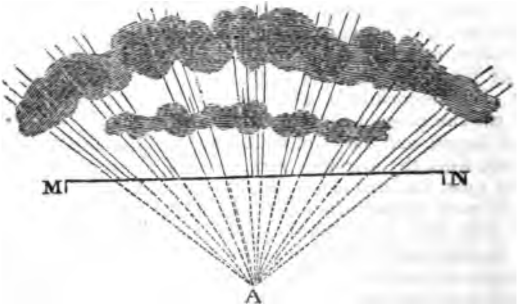
(164.) Among the luminous phenomena of the atmosphere, we may here notice that of converging and diverging solar beams. The phenomenon of *diverging beams*, represented in *fig.* 138., is of frequent occurrence in summer, and when the sun is near the horizon; and arises from a portion of the sun's rays passing through openings in the clouds, while the adjacent portions are obstructed by the clouds. The phenomenon of *converging beams*, which is of much rarer occurrence, is

Fig. 138.



shown in *fig*. 139., where the rays converge to a point A, as far below the horizon M N as the sun is above it. This phenomenon is always seen opposite to the sun, and generally at

Fig. 139.



the same time with the phenomenon of diverging beams, as if another sun, diametrically opposite to the real one, were below the horizon at A, and throwing out his divergent beams. In a phenomenon of this kind which I saw in 1824, the eastern portion of the horizon where it appeared was occupied with a black cloud, which seems to be necessary as a ground, for rendering visible such feeble radiations. A few minutes after the phenomenon was first seen, the converging lines were black, or very dark; an effect which seems to have arisen from the luminous beams having become broad and of unequal intensity, so that the eye took up, as it were, the dark spaces between the beams more readily than the luminous beams themselves.

This phenomenon is entirely one of perspective. Let us suppose beams inclined to one another like the meridians of a globe to diverge from the sun, as these meridians diverge from the north pole of the globe, and let us suppose that planes pass through all these meridians, and through the line joining the observer and the sun, or their common intersection. An eye, therefore, placed in that line, or in the common intersection of all the fifteen planes, will see the fifteen beams converging to a point opposite the sun, just as an eye in the axis of a globe would see all the fifteen meridians of the globe converge to its south pole. If we suppose the axis of a globe or of an armillary sphere to be directed to the centres of the diverging and converging beams, and a plane to pass through the globe parallel to the horizon, it would cut off the meridians so as to exhibit the precise appearances in *fig*. 138. and *fig*. 139.; with this difference only, that there would be fifteen beams in the diverging system in the place of the number shown in *fig*. 139.

CHAP. XXXIV.

ON THE COLORS OF NATURAL BODIES.

(165.) THERE is no branch of the application of optical science which possesses a greater interest than that which proposes to determine the cause of the colors of natural bodies. Sir Isaac Newton was the first who entered into an elaborate investigation of this difficult subject; but though his speculations are marked with the peculiar genius of their author, yet they will not stand a rigorous examination under the lights of modern science.

That the colors of material nature are not the result of any quality inherent in the colored body has been incontrovertibly proved by Sir Isaac. He found that all bodies, of whatever color, exhibit that color only when they are placed in white light. In homogeneous *red* light they appeared *red*, in *violet* light *violet*, and so on; their colors being always best displayed when placed in their own daylight colors. A *red* wafer, for example, appears red in the white light of day, because it reflects *red* light more copiously than any of the other colors. If we place a *red* wafer in *yellow* light, it can no longer appear red, because there is not a particle of red light in the yellow light which it could reflect. It reflects, however, a portion of yellow light, because there is some yellow in the red which it does reflect. If the *red* wafer had reflected nothing but pure homogeneous red light and not reflected white light from its outer surface, which all colored bodies do, it would in that case have appeared absolutely black when placed in yellow light. The colors, therefore, of bodies arise from their property of reflecting or transmitting to the eye certain rays of white light, while they stifle or stop the remaining rays. To this point the Newtonian theory is supported by infallible experiments; but the principal part of the theory, which has for its object to determine the manner in which particular rays are stopped, while others are reflected or transmitted, is not so well founded.

As Sir Isaac has stated the principles of his theory with the greatest clearness, we shall give them in his own words.

"1st, Those superficies of transparent bodies reflect the greatest quantity of light which have the greatest refracting power; that is, which separate media that differ most in their refracting power. And in the confines of equally refracting media there is no reflexion.

"2d, The least parts of almost all natural bodies are in some measure transparent; and the opacity of these bodies arises from the multitude of reflexions caused in their internal parts.

"3d, Between the parts of opaque and colored bodies are many spaces, either empty, or replenished with mediums of other densities; as *water* between the tinging corpuscles wherewith any liquor is impregnated; *air* between the aqueous globules that constitute clouds or mists; and for the most part *spaces*, void of both air and water, but yet perhaps not wholly void of all substance, between the parts of all bodies.

"4th, The parts of bodies and their interstices must not be less than of some definite bigness, to render them opaque and colored.

"5th, The transparent parts of bodies, according to their several sizes, reflect rays of one color, and transmit those of another, on the same grounds that thin plates or bubbles do reflect or transmit these rays; and this I take to be the ground of all their colors."

"6th, The parts of bodies on which their colors depend are denser than the medium which pervades their interstices.

"7th, The bigness of the component parts of natural bodies may be conjectured by their colors."

Upon these principles Sir Isaac has endeavored to explain the phenomena of *transparency*, *black and white opacity*, and *color*. He regards the transparency of *water*, *salt*, *glass*, *stones*, and such like substances, as arising from the smallness of their particles, and the intervals between them; for though he considers them to be as full of pores or intervals between the particles as other bodies are, yet he reckons the particles and their intervals to be too small to cause reflexion at their common surfaces. Hence it follows, from the table in page 93, that the particles of air and their intervals cannot exceed the half of a millionth part of an inch; the particles of water the $\frac{3}{8}$th of a millionth, and those of glass the $\frac{1}{3}$d of a millionth; because at these thicknesses the light reflected is nothing, or the very black of the first order. The opacity of bodies, such as that of white paper, linen, &c., is ascribed by Newton to a greater size of the particles and their intervals, viz. such a size as to reflect the white, which is a mixture of the colors of the different orders. Hence in air they must exceed 77 millionths of an inch, in water 57 millionths, and in glass 50 millionths.

In like manner all the different colors in Newton's table are supposed to be produced when the particles and their intervals have an intermediate size between that which produces transparency and that which produces white opacity. If a film of *mica*, for example, of an uniform *blue* color, is cut into the smallest pieces of the same thickness, every piece will keep its color, and a heap of such pieces will constitute a mass of the same color.

So far the Newtonian theory is plausible; but in attempting to explain *black opacity*, such as that of coal and other bodies absolutely impervious to light, it seems to fail entirely. To produce blackness, "the particles must be less than any of those which exhibit color. For at all greater sizes there is too much light reflected to constitute this color; but if they be supposed a little less than is requisite to reflect the white and very faint blue of the first order, they will reflect so very little light as to appear intensely black." That such bodies will be black when seen by reflexion is evident; but what becomes of all the transmitted light? This question seems to have perplexed Sir Isaac. The answer to it is, "*it may perhaps be variously refracted to and fro* within the body, until it happens to be stifled and lost; by which means it will appear intensely black."

In this theory, therefore, *transparency* and *blackness* are supposed to be produced by the very same constitution of the body; *and a refraction to and fro* is assumed to extinguish the transmitted light in the one case, while in the other such a refraction is entirely excluded.

In the production of colors of every kind, it is assumed that the complementary color, or generally one half of the light, is lost by repeated reflexions. Now, as reflexion only changes the direction of light, we should expect that the light thus scattered would show itself in some form or other; but though many accurate experiments have been made to discover it, it has never yet been seen.

For these and other reasons,*† which it would be out of place here to enumerate, I consider the Newtonian theory of colors as applicable only to a small class of phenomena, while it leaves unexplained the colors of fluids and transparent solids, and all the beautiful hues of the vegetable kingdom. In numerous experiments on the colors of leaves, and on the juices expressed from them, I have never been able to see the complementary color which disappears, and I have almost invariably found that the transmitted and the reflected tint is the same. Whenever there was an appearance of two tints, I have found it to arise from there being two differently colored juices existing in different sides of the leaf. The Newtonian theory is, we doubt not, applicable to the colors of the wings of insects, the feathers of birds, the scales of fishes, the oxidated films on metal and glass, and certain opalescences.

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* See a more detailed examination of the theory in my Life of Sir Isaac Newton.

† For an account of Sir David Brewster's outline of a new theory of the colors of natural bodies, see Note VII. of Am. ed.

The colors of vegetable life and those of various kinds of solids arise, we are persuaded, from a specific attraction which the particles of these bodies exercise over the differently colored rays of light. It is by the light of the sun that the colored juices of plants are elaborated, that the colors of bodies are changed, and that many chemical combinations and decompositions are effected. It is not easy to allow that such effects can be produced by the mere vibration of an ethereal medium; and we are forced, by this class of facts, to reason as if light was material. When a portion of light enters a body, and is never again seen, we are entitled to say that it is detained by some power exerted over the light by the particles of the body. That it is attracted by the particles seems extremely probable, and that it enters into combination with them, and produces various chemical and physical effects, cannot well be doubted; and without knowing the manner in which this combination takes place, we may say that the light is *absorbed*, which is an accurate expression of the fact.

Now, in the case of water, glass, and other transparent bodies, the light which enters their substance has a certain small portion of its particles absorbed, and the greater part of it which escapes from absorption, and is transmitted, comes out colorless, because the particles have absorbed a proportional quantity of all the different rays which compose white light, or, what is the same thing, the body has absorbed white light.

In all *colored solids* and *fluids* in which the transmitted light has a specific color, the particles of the body have absorbed all the rays which constitute the complementary color, detaining sometimes *all* the rays of a certain definite refrangibility, a portion of the rays of other refrangibilities, and allowing other rays to escape entirely from absorption; all the rays thus stopped will form by their union a particular compound color, which will be exactly complementary to the color of the transmitted rays.

In *black* bodies, such as *coal*, &c., all the rays which enter their substance are absorbed; and hence we see the reason why such bodies are more easily heated and inflamed by the action of the luminous rays. The influence exercised by heat and cooling upon the absorptive power of bodies furnishes an additional support to the preceding views.

(166.) Before concluding this chapter, we may mention a few curious facts relative to white opacity, black opacity, and color, as exhibited by some peculiar substances.

1st, Tabasheer, whose refractive power is 1·111, between air and water, is a silicious concretion found in the joints of the bamboo. The finest varieties reflect a delicate azure color, and transmit a straw-yellow tint, which is complementary to the azure. When it is slightly wetted with a wet needle or pin, the *wet spot instantly becomes milk white and opaque*. The application of a greater quantity of water restores its transparency.

2dly, The *cameleon mineral* is a solid substance made by heating the pure oxide of manganese with potash. When it is dissolved in a little warm water, the solution changes its color from *green* to *blue* and *purple*, the last descending in the order of the rings, as if the particles became smaller.

3dly, A mixture of oil of sweet almonds with soap and sulphuric acid is, according to M. Claubry, first *yellow*, then *orange*, *red*, and *violet*. In passing from the *orange* to the *red*, the mixture appears almost *black*.

4thly, If, in place of oil of almonds, in the preceding experiment, we employ the oily liquid obtained from alcohol heated with chlorine, the colors of the mixture will be *pale yellow*, *orange*, *black*, *red*, *violet*, and beautiful *blue*.

5thly, *Tincture of turnsole*, after having been a considerable time shut up in a bottle, has an *orange* color; but when the bottle is opened and the fluid shaken, it becomes in a few minutes *red*, and then *violet-blue*.

6thly, A solution of *hæmatine* in water containing some drops of acètic acid is a *greenish yellow*. When introduced into a tube containing mercury, and heated by surrounding it with a hot iron, it assumes the various colors of *yellow*, *orange*, *red*, and *purple*, and returns gradually to its primitive tint.

7thly, Several of the metallic oxides exhibit a temporary change of color by heat, and resume their original color by cooling. M. Chevreul observed, that when indigo, spread upon paper, is volatilized, its color passes into a very brilliant poppy-red. The yellow phosphate of lead grows green when hot.

8thly, One of the most remarkable facts, however, is that discovered by M. Thenard. He found that phosphorus, purified by repeated distillations, though naturally of a whitish yellow color when allowed to cool slowly, *becomes absolutely black* when thrown melted into cold water. Upon touching some little globules that still remained yellow and liquid when he was repeating this experiment, M. Biot found that they instantly became solid and black.

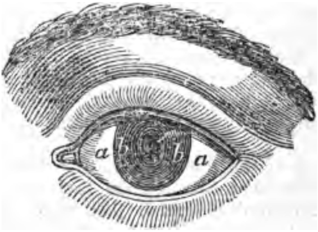
CHAP. XXXV.

ON THE EYE AND VISION.

An account of the structure and functions of the human eye, that masterpiece of divine mechanism, forms an interesting branch of applied optics. This noble organ, by means of which we acquire so large a portion of our knowledge of the material universe, is represented in *figs.* 140. and 141., the former being a front and external view of it, and the latter a section of it through all its humors.

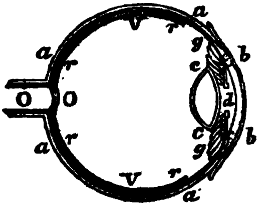
The human eye is of a spherical form, with a slight projection in front. The eyeball or globe of the eye consists of four coats or membranes, which have received the names of the *sclerotic* coat, the *choroid* coat, the *cornea*, and the *retina;* and these coats inclose three humors,—the *aqueous humor*, the *vitreous humor*, and the *crystalline humor*, the last of which has the form of a lens. The sclerotic coat, *a a a a*, or the outermost, is a strong and tough membrane, to which are attached all the muscles which give motion to the eyeball,

Fig. 140.



and it constitutes the white of the eye, *a a*, *fig.* 140. The *cornea*, *b b*, is the clear and transparent coat which forms the front of the eyeball, and is the first optical surface at which the rays of light are refracted. It is firmly united to the *sclerotic* coat, filling up, as it were, a circular aperture in its front. The *cornea* is an exceedingly tough membrane, of equal thickness throughout, and composed of several firmly adhering layers, capable of opposing great resistance to external injury. The *choroid* coat is a delicate membrane lining the inner surface of the *sclerotic*, and covered on its inner surface with a black pigment. Immediately within this pigment, and close to it, lies the *retina*, *r r r*, which is the innermost coat of all. It is a delicate reticulated membrane, formed by the expansion of the optic nerve, O O, which enters the eye at a point about $\frac{1}{10}$ of an inch from the axis on the side next the nose. At the extremity of the axis of the eye, in a line passing through the centre of the *cornea*, and perpendicular to its surface, there is a small hole with a yellow margin, called the *foramen centrale*, which, notwithstanding its name, is not a real opening, but only a transparent spot, free from the soft pulpy matter of which the retina is composed.

Fig. 141.



In looking through the cornea from without, we perceive a flat circular membrane, *e f*, *fig.* 141., or within, *b b*, *fig.* 140., which is grey, blue, or black, and divides the anterior of the eye into two very unequal parts. In the centre of it there is a circular opening, *d*, called the *pupil*, which widens or expands when a small portion of light enters the eye, and closes or contracts when a great quantity of light enters. The two parts into which the iris divides the eye are called the *anterior* and the *posterior* chambers. The anterior chamber, which is anterior to the iris, *e f*, contains the aqueous humor; and the posterior chamber, which is posterior to the iris, contains the crystalline and vitreous humors, the last of which fills a great portion of the eyeball.

The crystalline lens, *c c*, *fig*. 141., is a more solid substance than either the aqueous or the vitreous humor. It is suspended in a transparent bag or capsule by the *ciliary processes*, *g g*, which are attached to every part of the margin or circumference of the capsule. This lens is more convex behind than before; the radius of its anterior surface being 0·30 of an inch, and that of its posterior surface 0·22 of an inch. The lens increases in density from its circumference to its centre, and possesses the doubly refracting structure. It consists of concentric coats, and these are again composed of fibres. The *vitreous* humor, V V, is contained in a capsule, which is supposed to be divided into several compartments.

The total length of the eye from O to *b* is about 0·91 of an inch; the principal focal distance of the lens, *c c*, is 1·73; and the range of the moving eyeball, or the diameter of the field of distinct vision, is 110°. The field of vision is 50° above a horizontal line and 70° below it, or altogether 120° in a vertical plane. It is 60° inwards and 90° outwards, or altogether in a horizontal plane 150°.

I have found the following to be the refractive powers of the different humors of the eye; the ray of light being incident upon them from air:—

Aqueous Humor.	*Crystalline Lens.*			*Vitreous Humor.*
	Surface.	Centre.	Mean.	
1·3366.	1·3767.	1·3990.	1·3839.	1·3394.

But as the rays refracted by the aqueous humor pass into the crystalline, and those from the crystalline into the vitreous humor, the indices of refraction of the separating surface of each of these humors will be:—

From aqueous humor to outer coat of the crystalline	1·0300
From do. to crystalline, using the mean index - -	1·0353
From crystalline outer coat to vitreous - - - - - -	0·9729
From do. to do. using the mean index - - -	0·9679

As the cornea and crystalline lens must act upon the rays of light which fall upon the eye exactly like a convex lens, inverted images of external objects will be formed upon the retina *r r r* in precisely the same manner as if the retina were a piece of white paper in the focus of a single lens placed at *d*. There is this difference, however, between the two cases, that in the eye the spherical aberration is corrected by means of the variation in the density of the crystalline lens, which, having a greater refractive power near the centre of its mass, refracts the central rays to the same point as the rays which pass through it near its circumference *c c*. No provision, however, is made in the human eye for the correction of colo because the deviation of the differently colored rays is too small to produce indistinctness of vision. If we shut up all the pupil excepting a portion of its edge, or look past the finger held near the eye, till the finger almost hides a narrow line of white light, we shall see a distinct prismatic spectrum of this line containing all the different colors; an effect which could not take place if the eye were achromatic.

That an inverted image of external objects is formed on the retina has been often proved, and may be ocularly demonstrated by taking the eye of an ox, and paring away with a sharp instrument the sclerotic coat till it becomes thin enough to see the image through it. Beyond this point optical science cannot carry us. In what manner the retina conveys to the brain the impressions which it receives from the rays of light we know not, and perhaps never shall know.

On the Phenomena and Laws of Vision

(167.) 1. *On the seat of vision.*—The retina, from its delicate structure, and its proximity to the vitreous humor, had always been regarded as the seat of vision, or the surface on which the refracted rays were converged to their foci, for the purpose of conveying the impression to the brain, till M. Mariotte made the curious discovery that the base of the optic nerve, or the circular section of it at O, *fig.* 141., was incapable of conveying to the brain the impression of distinct vision.

He found that when the image of any external object fell upon the base of the optic nerve, it instantly disappeared. In order to prove this, we have only to place upon the wall, at the height of the eye, three wafers, two feet distant from each other. Shutting one eye, stand opposite to the middle wafer, and while looking at the outside wafer on the same hand as the shut eye, retire gradually from the wall till the middle wafer disappears. This will happen at about five times the distance of the wafers, or ten feet from the wall; and when the middle wafer vanishes, the two outer ones will be distinctly seen. If candles are substituted for wafers, the middle candle will not disappear, but it will become a cloudy mass of light. If the wafers are placed upon a colored wall, the spot occupied by the wafer will be covered by the color of the wall, as if the wafer itself had been removed. According to Daniel Bernoulli, the part of the optic nerve insensible to distinct impressions occupies about the seventh part of the diameter of the eye, or about the eighth of an inch.

This unfitness of the base of the optic nerve for giving distinct vision, induced Mariotte to believe that the *choroid* coat, which lies immediately below the retina, performs the functions ascribed to the retina; for where there was no choroid coat there was no distinct vision. The opacity of the choroid coat and the transparency of the retina, which rendered it an unfit ground for the reception of images, were arguments in favor of this opinion. Comparative anatomy furnishes us with another argument, perhaps even more conclusive than any of those urged by Mariotte. In the eye of the *sepia loligo*, or cuttle-fish, an opaque membranous pigment is interposed *between the retina and the vitreous humor;** so that, if the retina is essential to vision, the impressions of the image on this black membrane must be conveyed to the retina by the vibrations of the membrane in front of it. Now, since the human retina is transparent, it will not prevent the images of objects from being formed on the choroid coat; and the vibrations which they excite in this membrane, being communicated to the retina, will be conveyed to the brain. These views are strengthened by another fact of some interest. I have observed in young persons, that the choroid coat (which is generally supposed to be black, and to grow fainter by age,) reflects a brilliant crimson color, like that of dogs and other animals. Hence, if the retina is affected by rays which pass through it, this crimson light which must necessarily be transmitted by it ought to excite the sensation of crimson, which I find not to be the case.

A French writer, M. Lehot, has recently written a work, endeavoring to prove that the seat of vision is in the vitreous humor; and that, in place of seeing a flat picture of the object, we actually see an image of three dimensions, viz. with length, breadth, and thickness. To produce this effect, he supposes that the retina sends out a number of small nervous filaments, which extend into the vitreous humor, and convey to the brain the impressions of all parts of the image. If this theory were true, the eye would not require to adjust itself to different distances; and we besides know for certain, that the eye cannot see with equal distinctness two points of an object at different distances, when it sees one of them perfectly. M. Lehot might indeed reply to the first of these objections, that the nervous filaments may not extend *far enough* into the vitreous humor to render adjustment unnecessary; but if we admit this, we would be admitting an imperfection of workmanship, in so far as the Creator would then be employing two kinds of mechanism to produce an effect which could have been easily produced by either of them separately.

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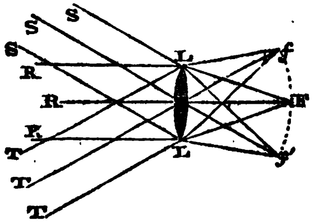
* Dr. Knox, *Edinb. Journal of Science*, No. VI. p. 199.

As difficulties still attach to every opinion respecting the seat of vision, we shall still adhere to the usual expression used by all *optical* writers, viz. that the images of objects are painted on the retina.

(168.) 2. *On the law of visible direction.*—When a ray of light falls upon the retina, and gives us vision of the point of an object from which it proceeds, it becomes an interesting question to determine in what direction the object will be seen, reckoning from the point where it falls upon the retina. In *fig.* 142., let F be a point of the retina on which the image of a point of a distant object is formed by means of the crystalline lens, supposed to be at L L. Now, the rays which form the image of the point at F fall upon the retina in all possible directions from L F to L F, and we know that the point F is seen in the direction F C R. In the same manner, the points $f f'$ are seen somewhere in the directions f' S, f T. These lines F R, f' S, f T, which may be called the *lines of visible direction*, may either be those which pass through the centre C of the lens L L, or, in the case of the eye, through the centre of a lens equivalent to all the refractions employed in producing the image; or it may be the resultant of all the directions within the angles L F L, L f L; or it may be a line perpendicular to the retina at F, $f' f$. In order to determine this point, let us look over the top of a card at the point of the object whose image is at F till the edge of the card is just about to hide it, or, what is the same thing, let us obstruct all the rays that pass through the pupil excepting the upper ones, R L, R C; we shall then find that the point whose image is at F, is seen in the same direction as when it was seen by all the rays L F, C F, L F. If we look beneath the card in a similar manner, so as to see the object by the lower rays, R L F, R C F

Fig. 142.



we shall see it in the same direction. Hence it is manifest that the line of visible direction does not depend on the direction of the ray, but is always perpendicular to the retina. This important truth in the physiology of vision may be proved in another way. If we look at the sun over the top of a card, as before, so as to impress the eye with a permanent spectrum by means of rays L F falling obliquely on the retina, this spectrum will be seen along the axis of vision F C. In like manner, if we press the eyeball at any part where the retina is, we shall see the luminous impression which is produced, in a direction perpendicular to the point of pressure; and if we make the pressure with the head of a pin, so as to press either obliquely or perpendicularly, we shall find that the luminous spot has the same direction.

Now, as the interior eyeball is as nearly as possible a perfect sphere, lines perpendicular to the surface of the retina must all pass through one single point, namely, the centre of its spherical surface. This one point may be called the *centre of visible direction*, because every point of a visible object will be seen in the direction of a line drawn from this centre to the visible point. When we move the eyeball by means of its own muscles through its whole range of 110°, every point of an object within the area of the visible field either of distinct or indistinct vision remains absolutely fixed, and this arises from the immobility of the centre of visible direction, and, consequently, of the lines of visible direction joining that centre and every point in the visible field. Had the centre of visible direction been out of the centre of the eyeball, this perfect stability of vision could not have existed. If we press the eye with the finger, we alter the spherical form of the surface of the retina; we consequently alter the direction of lines perpendicular to it, and also the centre where these lines meet; so that the directions of visible objects should be changed by pressure, as we find them to be.

(169.) 3. *On the cause of erect vision from an inverted image.*—As the refractions which take place at the surface of the *cornea*, and at the surfaces of the crystalline lens, act exactly like those in a convex lens in forming behind it an inverted image of an erect object; and as we know from direct experiment that an inverted image is formed on the retina, it has been long a problem among the learned, to determine how an inverted image produces an erect object. It would be a waste of time to give even an outline of the different opinions which have been entertained on this subject; but there is one so extraordinary as to merit notice. According to this opinion, all infants see objects upside down, and it is only by comparing the erroneous information acquired by vision with the accurate information acquired by touch, that the young learn to see objects in an erect position! To refute such an opinion would be an insult to the intelligent reader. The establishment of the true cause of erect vision necessarily overturns all erroneous hypotheses.

The law of visible direction above explained, and deduced from direct experiment, removes at once every difficulty that besets the subject. The lines of visible direction necessarily cross each other at the centre of visible direction, so that those from the lower part of the image go to the upper part of the object, and those from the upper part of the image to the lower part of the object. Hence, in *fig.* 142. the visible direction of the point f', formed by rays coming from the upper end S of the object, will be f' C S, and the visible direction of the point f, formed by rays coming from the lower end T of the object, will be f C T; so that an inverted image necessarily produces an erect object.

This conclusion may be illustrated in another way. If we hold up against the sun the erect figure of a man, cut out of a piece of black paper, and look at it steadily for a little while; if we then shut both eyes, we shall see an erect spectrum of the man when the figure of the paper is erect, and an inverted spectrum of him when the figure is held in an inverted position. In this case, there are no rays proceeding from the object to the retina after the eye is shut, and therefore the object is seen in the positions above mentioned, in virtue of the lines of visible direction being in all cases perpendicular to the impressed part of the retina.

(170.) 4. *On the law of distinct vision.*—When the eye is directed to any point of a landscape, it sees with perfect distinctness only that point of it which is directly in the axis of the eye, or the image of which falls upon the central hole of the retina. But, though we do not see any point but the one with that distinctness which is necessary to examine it, we still see the other parts of the landscape with sufficient distinctness to enable us to enjoy its general effect. The extreme mobility of the eye, however, and the duration of the impressions made upon the retina, make up for this apparent defect, and enable us to see the landscape as perfectly as if every part of it were seen with equal distinctness.

The indistinctness of vision for all objects situated out of the axis of the eye increases with their distances from that axis; so that we are not entitled to ascribe the distinctness of vision in the axis to the circumstance of the image being formed on the central hole of the retina, where there is no nervous matter; for if this were the case, there would be a precise boundary between distinct and indistinct vision, or the retina would be found to grow thicker and thicker as it receded from the central hole, which is not the case.

In making some experiments on the indistinctness of vision at a distance from the axis of the eye, I was led to observe a very remarkable peculiarity of oblique vision. If we shut one eye, and direct the other to any fixed point, such as the head of a pin, we shall see indistinctly all other objects within the sphere of vision. Let one of these objects thus seen indistinctly be a strip of white paper, or a pen lying upon a green cloth. Then, after a short time, the strip of paper, or the pen, will disappear altogether, as if it were entirely removed, the impression of the green cloth upon the surrounding parts of the eye extending itself over the part of the retina which the image of the pen occupied. In a short time the vanished image will reappear, and again vanish. When both eyes are open, the very same effect takes place, but not so readily as with one eye. If the object seen indistinctly is a black stripe on a white ground, it will vanish in a similar manner. When the object seen obliquely is luminous, such as a candle, it will never vanish entirely, unless its light is much weakened by being placed at a great distance, but it swells and contracts, and is encircled with a nebulous halo; so that the luminous impressions must extend themselves to adjacent parts of the retina which are not influenced by the light itself.

If, when two candles are placed at the distance of about eight or ten feet from the eye, and about a foot from each other, we view the one directly and the other indirectly, the indirect image will swell, as we have already mentioned, and will be surrounded with a bright ring of *yellow* light, while the bright light within the ring will have a *pale blue* color. If the candles are viewed through a prism, the red and green light of the indirect image will vanish, and there will be left only a large mass of yellow terminated with a portion of blue light. In making this experiment, and looking steadily and directly at one of the prismatic images of the candles, I was surprised to find that the red and green rays began to disappear, leaving only yellow and a small portion of blue; and when the eye was kept immovably fixed on the same point of the image, the *yellow light became almost* pure white, so that the prismatic image was converted into an elongated image of white light.

If the strip of white paper which is seen indirectly with both eyes is placed so near the eye as to be seen double, the rays which proceed from it no longer fall upon corresponding points of the retina, and the two images do not vanish instantaneously. But when the one begins to disappear, the other begins soon after it, so that they sometimes appear to be extinguished at the same time.

From these results it appears that oblique or indirect vision is inferior to direct vision, not only in distinctness, but from its inability to preserve a sustained vision of objects; but though thus defective, it possesses a superiority over direct vision in giving us more perfect vision of minute objects, such as small stars, which cannot be seen by direct vision. This curious fact has been noticed by Mr. Herschel and Sir James South, and some of the French astronomers. "A rather singular method," say Messrs. Herschel and South, "of obtaining a view, and even a rough measure, of the angles of stars of the last degree of faintness, has often been resorted to, viz. *to direct the eye to another part of the field.* In this way, a faint star, in the neighborhood of a large one, will often become very conspicuous; so as to bear a certain illumination, which will yet totally disappear, as if suddenly blotted out, when the eye is turned full upon it, and so on, appearing and disappearing alternately as often as you please. The lateral portions of the retina, less fatigued by strong lights, and less exhausted by perpetual attention, are probably more sensible to faint impressions than the central ones; which may serve to account for this phenomenon."

The following explanation of this curious phenomenon seems to me more satisfactory:—A *luminous point* seen by *direct vision*, or a sharp line of light viewed steadily for a considerable time, throws the retina into a state of agitation highly unfavorable to distinct vision. If we look through the teeth of a fine comb held close to the eye, or even through a single aperture of the same narrowness, at a sheet of illuminated white paper, or even at the sky, the paper or the sky will appear to be covered with an infinite number of broken serpentine lines, parallel to the aperture, and in constant motion; and as the aperture is turned round, these parallel undulations will also turn round. These black and white lines are obviously undulations on the retina, which is sensible to the impressions of light in one phase of the undulation, and insensible to it in another phase. An analogous effect is produced by looking stedfastly, and for a considerable time, on the parallel lines which represent the sea in certain maps. These lines will break into portions of serpentine lines, and all the prismatic tints will be seen included between the broken curvilinear portions. A sharp point or line of light is therefore unable to keep up a continued vision of itself upon the retina when seen directly.

Now, in the case of indirect vision, we have already seen that a luminous object does not vanish, but is seen indistinctly, and produces an enlarged image on the retina, beside that which is produced by the defect of convergency in the pencils. Hence, a star seen indirectly, will affect a larger portion of the retina from these two causes, and, losing its sharpness, will be more distinct. It is a curious circumstance, too, that in the experiment with the two candles mentioned above, the candles seen indirectly frequently appear more intensely bright than the candle seen directly.

(171). 5. *On the insensibility of the eye to direct impressions of faint light.*—The insensibility of the retina to indirect impressions of objects ordinarily illuminated, has a singular counterpart in its insensibility to the direct impression of very faint light. If we fix the eye steadily on objects in a dark room that are illuminated with the faintest gleam of light, it will be soon thrown into a state of painful agitation; the objects will appear and disappear according as the retina has recovered or lost its sensibility.

These affections are no doubt the source of many optical deceptions which have been ascribed to a supernatural origin. In a dark night, when objects are feebly illuminated, their disappearance and reappearance must seem very extraordinary to a person whose fear or curiosity calls forth all his powers of observation. This defect of the eye must have been often noticed by the sportsman in attempting to mark, upon the monotonous heaths, the particular spots where moor-game had alighted. Availing himself of the slightest difference of tint in the adjacent heaths, he endeavors to keep his eye steadily upon it as he advances; but whenever the contrast of illumination is feeble, he almost always loses sight of his mark, or if the retina does take it up a second time, it is only to lose it again.*

(172.) 6. *On the duration of impressions of light on the retina.*—Every person must have observed that the effect of light upon the eye continues for some time. During the twinkling of the eye, or the rapid closing of the eyelids for the purpose of diffusing the lubricating fluid over the cornea, we never lose sight of the objects we are viewing. In like manner, when we whirl a burning stick with a rapid motion, its burning extremity will produce a complete circle of light.

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* See the *Edinburgh Journal of Science*, No. VI. p. 288.

although that extremity can only be in one part of the circle at the same instant.

The most instructive experiment, however, on this subject, and one which it requires a good deal of practice to make well, is to look for a short time at the window at the end of a long apartment, and then quickly direct the eye to the dark wall. In general, the ordinary observer will see a picture of the window, in which the dark bars are white and the white panes dark; but the practised observer, who makes the observation with great promptness, will see an accurate representation of the window with dark bars and bright panes; but this representation is instantly succeeded by the complementary picture, in which the bars are bright and the panes dark. M. D'Arcy found that the light of a live coal, moving at the distance of 165 feet, maintained its impression on the retina during the seventh part of a second.*

(173.) 7. *On the cause of single vision with two eyes.*—Although an image of every visible object is formed on the retina of each eye, yet when the two eyes are capable of directing their axes to any given object, it always appears single. There is no doubt that, in one sense, we really see two objects, but these objects appear as one, in consequence of the one occupying exactly the same place as the other. Single vision with two eyes, or with any number of eyes, if we had them, is the necessary consequence of the law of visible direction. By the action of the external muscles of the eyeballs, the axes of each eye can be directed to any point of space at a greater distance than 4 or 6 inches. If we look, for example, at an aperture in a window-shutter, we know that an image of it is formed in each eye; but, as the line of visible direction from any point in the one image meets the line of visible direction from the same point in the other image, each point will be seen as one point, and, consequently, the whole aperture seen by one eye will coincide with or cover the whole aperture seen by the other. If the axes of both eyes are directed to a point beyond the window, or to a point within the room, the aperture will then appear double, because the lines of visible direction from the same points in each image do not meet at the aperture. If the muscles of either of the eyes is unable to direct the two axes of the eyes to the same point, the object will in that case also appear double. This inability of one eye to follow the motions of the other is frequently the cause of squinting, as the eye which is, as it were, left behind necessarily looks in a different direction from the other. The same effect is often produced by the imperfect vision of one eye, in consequence of which the good eye only is used. Hence the imperfect eye will gradually lose the power of following the motions of the other, and will therefore look in a different direction. The disease of squinting may be often easily cured.

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* For a farther illustration, see Note VIII. of Am. ed.

(174.) 8. *On the accommodation of the eye to different distances.*—When the eye sees objects distinctly at a great distance, it is unable, without some change, to see objects distinctly at any less distance. This will be readily seen by looking between the fingers at a distant object. When the distant object is seen distinctly, the fingers will be seen indistinctly; and, if we look at the fingers so as to see them distinctly, the distant object will be quite indistinct. The most distinguished philosophers have maintained different opinions respecting the method by which the eye adjusts itself to different distances. Some have ascribed it to the mere enlargement and diminution of the pupil; some to the elongation of the eye, by which the retina is removed from the crystalline lens; some to the motion of the crystalline lens; and others to a change in the convexity of the lens, on the supposition that it consists of muscular fibres. I have ascertained, by direct experiment, that a variation in the aperture of the pupil, produced artificially, is incapable of producing adjustment, and as an elongation of the eye would alter the curvature of the retina, and consequently the centre of visible direction, and produce a change of place in the image, we consider this hypothesis as quite untenable.

In order to discover the cause of the adjustment, I made a series of experiments, from which the following inferences may be drawn:—

1st, The contraction of the pupil, which necessarily takes place when the eye is adjusted to near objects, does not produce distinct vision by the diminution of the aperture, but by some other action which necessarily accompanies it.

2dly, That the eye adjusts itself to near objects by two actions; one of which is *voluntary*, depending wholly on the will, and the other *involuntary*, depending on the stimulus of light falling on the retina.

3dly, That when the voluntary power of adjustment fails, the adjustment may still be effected by the involuntary stimulus of light.

Reasoning from these inferences, and other results of experiment, it seems difficult to avoid the conclusion that the power of adjustment depends on the mechanism which contracts and dilates the pupil; and as this adjustment is independent of the variation of its aperture, it must be effected by the parts in immediate contact with the base of the iris. By considering the various ways in which the mechanism at the base of the iris may produce the adjustment, it appears to be almost certain that the lens is removed from the retina by the contraction of the pupil.*

(175.) 9. *On the cause of longsightedness and shortsightedness.*—Between the ages of 30 and 50, the eyes of most persons begin to experience a remarkable change, which generally shows itself in a difficulty of reading small type or ill-printed books, particularly by candlelight. This defect of sight, which is called *longsightedness*, because objects are seen best at a distance, arises from a change in the state of the crystalline lens, by which its density and refractive power, as well as its form, are altered. It frequently begins at the margin of the lens, and takes several months to go round it, and it is often accompanied with a partial separation of the laminæ and even of the fibres of the lens. "If the human eye," as I have elsewhere remarked, "is not managed with peculiar care at this period, the change in the condition of the lens often runs into cataract, or terminates in a derangement of fibres, which, though not indicated by white opacity, occasions imperfections of vision that are often mistaken for amaurosis and other diseases. A skilful oculist, who thoroughly understands the structure of the eye, and all its optical functions, would have no difficulty, by means of nice experiments, in detecting the very portion of the lens where this change has taken place; in determining the nature and magnitude of the change which is going on; in applying the proper remedies for stopping its progress; and in ascertaining whether it has advanced to such a state that aid can be obtained from convex or concave lenses. In such cases, lenses are often resorted to before the crystalline lens has suffered a uniform change of figure or of density, and the use of them cannot fail to aggravate the very evils which they are intended to remedy. In diseases of the lens, where the separation of fibres is confined to small spots, and is yet of such magnitude as to give separate colored images of a luminous object, or irregular halos of light, it is often necessary to limit the aperture of the spectacles, so as to allow the vision to be performed by the good part of the crystalline lens."

This defect of the eye, when it is not accompanied with disease, may be completely remedied by a convex lens, which makes up for the flatness and diminished refractive power of the crystalline, and enables the eye to converge the pencils flowing from near objects to distinct foci on the retina.

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* For a fuller account of these experiments, see *Edinburgh Journal of Science*, No. I. p. 77.

Shortsightedness shows itself in an inability to see at a distance; and those who experience this defect bring minute objects very near the eye in order to see them distinctly. The rays from remote objects are in this case converged to foci before they reach the retina, and therefore the picture on the retina is indistinct. This imperfection often appears in early life, and arises from an increase of density in the central parts of the crystalline lens. By using a suitable concave lens the convergency of the rays is delayed, so that a distinct image can be formed on the retina.

CHAP. XXXVI.

ON ACCIDENTAL COLORS AND COLORED SHADOWS.

(176.) WHEN the eye has been strongly impressed with any particular species of colored light, and when in this state it looks at a sheet of white paper, the paper does not appear to it white, or of the color with which the eye was impressed, but of a different color, which is said to be the *accidental color* of the color with which the eye was impressed. If we place, for example, a bright *red wafer* upon a sheet of white paper, and fix the eye steadily upon a mark in the centre of it, then if we turn the eye upon the white paper we shall see a circular spot of *bluish green* light, of the same size as the wafer. This color, which is called the accidental color of *red*, will gradually fade away. The *bluish green* image of the wafer is called an *ocular spectrum*, because it is impressed on the eye, and may be carried about with it for a short time.

If we make the preceding experiment with differently colored wafers, we shall obtain *ocular spectra* whose colors vary with the color of the wafer employed, as in the following table.

Color of the Wafer	Accidental Color, or Color of the Ocular Spectrum.
Red.	Bluish green.
Orange.	Blue.
Yellow.	Indigo.
Green.	Reddish violet.
Blue.	Orange red.
Indigo.	Orange yellow.
Violet.	Yellow green
Black.	White
White	Black

In order to find the accidental color of any color in the spectrum, take half the length of the spectrum in a pair of compasses, and setting one foot in the color whose accidental color is required, the other will fall upon the accidental color. Hence the law of accidental colors derived from observation may be thus stated:—The accidental color of any color in a prismatic spectrum, is that color which in the same spectrum is distant from the first color half the length of the spectrum; or, if we arrange all the colors of any prismatic spectrum in a circle, in their due proportions, the accidental color of any particular color will be the color exactly *opposite* that particular color. Hence the two colors have been called *opposite* colors.

If the primitive color, or that which impresses the eye, is reduced to the same degree of intensity as the accidental color, we shall find that the one is the complement of the other, or what the other wants to make it white light; that is, the primitive and the accidental colors will, when reduced to the same degree of intensity which they have in the spectrum, and when mixed together, make white light. On this account accidental colors have been called *complementary* colors.

With the aid of these facts, the theory of accidental colors will be readily understood. When the eye has been for some time fixed on the *red* wafer, the part of the retina occupied by the red image is strongly excited, or, as it were, deadened by its continued action. The sensibility to red light will therefore be diminished; and, consequently, when the eye is turned from the *red* wafer to the white paper, the deadened portion of the retina will be insensible to the red rays which form part of the white light from the paper, and consequently will see the paper of that color which arises from all the rays in the white light of the paper but the red; that is, of a *bluish green* color, which is therefore the true complementary color of the *red*. When a *black* wafer is placed on a white ground, the circular portion of the retina, on which the black image falls, in place of being deadened, is refreshed, as it were, by the absence of light, while all the surrounding parts of the retina, being excited by the white light of the paper, will be deadened by its continued action. Hence, when the eye is directed to the white paper, it will see a white circle corresponding to the black image on the retina; so that the accidental color of black is white. For the same reason, if a *white* wafer is placed on a *black* ground, and viewed stedfastly for some time, the eye will afterwards see a *black* circular space; so that the accidental color of *white* is *black*.

Such are the phenomena of accidental colors when weak light is employed; but when the eye is impressed powerfully with a bright white light, the phenomena have quite a different character. The first person who made this experiment with any care was Sir Isaac Newton, who sent an account of the results to Mr. Locke, but they were not published till 1829.* Many years before 1691, Sir Isaac, having shut his left eye, directed the right one to the image of the sun reflected from a looking-glass. In order to see the impression which was made, he turned his eye to a dark corner of his room, when he observed a bright spot made by the sun, encircled by rings of colors. This "phantom of light and colors," as he calls it, gradually vanished; but whenever he thought of it, it returned, and became as lively and vivid as at first. He rashly repeated the experiment three times, and his eye was impressed to such a degree, "that whenever I looked upon the clouds, or a book, or a bright object, I saw upon it a round bright spot of light like the sun; and, which is still stranger, though I looked upon the sun with my right eye only, and not with my left, yet my fancy began to make an impression on my left eye as well as upon my right; for if I shut my right eye, or looked upon a book or the clouds with my left eye, I could see the spectrum of the sun almost as plain as with my right eye." The effect of this experiment was such, that Sir Isaac durst neither write nor read, but was obliged to shut himself completely up in a dark chamber for three days together, and by keeping in the dark, and employing his mind about other things, he began, in about three or four days, to recover the use of his eyes. In these experiments, Sir Isaac's attention was more taken up with the metaphysical than with the optical results of them, so that he has not described either the colors which he saw, or the changes which they underwent.

Experiments of a similar kind were made by M. Æpinus. When the sun was near the horizon, he fixed his eye steadily on the solar disc for 15 seconds. Upon shutting his eye he saw an irregular pale *sulphur yellow* image of the sun, encircled with a faint red border. As soon as he opened his eye upon a white ground, the image of the sun was a *brownish red*, and its surrounding border *sky blue*. With his eye again shut, the image of the sun became green with a red border, different from the last. Turning his eye again upon a white ground, the sun's image was more red, and its border a brighter sky blue. When the eye was shut, the green spectrum became a greenish sky blue, and then a fine sky blue, with the border growing a finer red; and when the eye was open, the spectrum became a finer red, and its border a finer blue. M. Æpinus noticed, that when his eye was fixed upon the white ground, the image of the sun frequently disappeared, returned, and disappeared again.

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* In Lord King's Life of Locke.

About the year 1808, I was led to repeat the preceding experiments of Æpinus; but, instead of looking at the sun when of a dingy color, I took advantage of a fine summer's day, when the sun was near the meridian, and I formed upon a white ground a brilliant image of his disc by the concave speculum of a reflecting telescope. Tying up my right eye, I viewed this luminous disc with my left eye through a tube, and when the retina was highly excited, I turned my left eye to a white ground, and observed the following spectra by alternately opening and shutting it:—

Spectra with left eye open.	Spectra with left eye shut.
1. Pink surrounded with green.	Green.
2. Orange mixed with pink.	Blue.
3. Yellowish brown.	Bluish pink.
4. Yellow.	
5. Pure red.	Sky blue
6. Orange.	Indigo.

Upon uncovering my right eye, and turning it to a white ground, I was surprised to observe that it also gave a colored spectrum, exactly the reverse of the first spectrum, which was pink with a green border. The reverse spectrum was a green with a pinkish border. This experiment was repeated three times, and always with the same result; so that it would appear that the impression of the solar image was conveyed by the optic nerve from the left to the right eye. Sir Isaac Newton supposed that it was his fancy that transferred the image from his left to his right eye; but we are disposed to think that in his experiment no transference took place, because the spectrum which he saw with both eyes was the same, whereas in my experiment it was the *reverse* one. We cannot however speak decidedly on this point, as Sir Isaac did not observe that the spectra with the eye shut were the reverse of those seen with the eye open. If a spectrum is strongly formed on one eye, it is a very difficult matter to determine on which eye it is formed, and it would be impossible to do this if the spectrum was the same when the eye was open and shut.

The phenomena of accidental colors are often finely seen when the eye has not been strongly impressed with any particular colored object. It was long ago observed by M. Meusnier, that when the sun shone through a hole a quarter of an inch in diameter in a *red* curtain, the image of the luminous spot was *green*. In like manner, every person must have observed in a brightly painted room, illuminated by the sun, that the parts of any white object on which the colored light does not fall, exhibit the complementary colors. In order to see this class of phenomena, I have found the following method the simplest and the best. Having lighted two candles, hold before one of them a piece of colored glass, suppose bright red, and remove the other candle to such a distance that the two shadows of any body formed upon a piece of white paper may be equally dark. In this case one of the shadows will be *red*, and the other *green*. With blue glass, one of them will be *blue*, and the other *orange yellow;* the one being invariably the accidental color of the other. The very same effect may be produced in daylight by two holes in a window-shutter; the one being covered with a colored glass, and the other transmitting the white light of the sky. Accidental colors may also be seen by looking at the image of a candle, or any white object seen by reflexion from a plate or surface of colored glass sufficiently thin to throw back its color from the second surface. In this case the reflected image will always have the complementary color of the glass. The same effect may be seen in looking at the image of a candle reflected from the water in a blue finger-glass; the image of the candle is yellowish: but the effect is not so decided in this case, as the retina is not sufficiently impressed with the blue light of the glass.

These phenomena are obviously different from those which are produced by colored wafers; because in the present case the accidental color is seen by a portion of the retina which is not affected, or deadened as it were, by the primitive color. A new theory of accidental colors is therefore requisite, to embrace this class of facts.

As in acoustics, where every fundamental sound is actually accompanied with its harmonic sound, so in the impressions of light, the sensation of one color is accompanied by a weaker sensation of its accidental or harmonic color.* When we look at the *red* wafer, we are at the same time, with the same portion of the retina, seeing *green;* but being much fainter, it seems only to dilute the *red*, and make it, as it were, whiter, by the combination of the two sensations. When the eye looks from the wafer to the white paper, the permanent sensation of the accidental color remains, and we see a *green* image. The duration of the primitive impression is only a fraction of a second, as we have already shown; but the duration of the harmonic impression continues for a time proportional to the strength of the impression. In order to apply these views to the second class of facts, we must have recourse to another principle; namely, that when the whole or a great part of the retina has the sensation of any primitive color, a portion of the retina protected from the impression of the color is actually thrown into that state which gives the accidental or harmonic color. By the vibrations probably communicated from the surrounding portions, the influence of the direct or primitive color is not propagated to parts free from its action, excepting in the particular case of oblique vision formerly mentioned. When the eye, therefore, looks at the white spot of solar light seen in the middle of the red light of the curtain, the whole of the retina, except the portion occupied by the image of the white spot, is in the state of seeing every thing *green;* and as the vibrations which constitute this state spread over the portions of the retina upon which no red light falls, it will, of course, see the white circular spot green.

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* The term *harmonic* has been applied to accidental colors; because the primitive and its accidental color harmonise with each other in painting.

(177.) A very remarkable phenomenon of accidental colors, in which the eye is not excited by any primitive color, was observed by Mr. Smith, surgeon in Fochabers. If we hold a narrow strip of white paper vertically, about a foot from the eye, and fix both eyes upon an object at some distance beyond it, then if we allow the light of the sun, or the light of a candle, to act strongly upon the right eye, without affecting the left, which may be easily protected from its influence, the *left* hand strip of paper will be seen of a bright *green* color, and the right hand strip of a *red* color. If the strip of paper is sufficiently broad to make the two images overlap each other, the overlapping parts will be perfectly white, and free from color; which proves that the red and green are complementary. When equally luminous candles are held near each eye, the two strips of paper will be white. If when the candle is held near the right eye, and the strips of paper are seen *red* and *green,* then on bringing the candle suddenly to the left eye, the left hand image of the paper will gradually change to *green,* and the right hand image to *red.*

(178.) A singular affection of the retina, in reference to colors, is shown in the inability of some eyes to distinguish certain colors of the spectrum. The persons who experience this defect have their eyes generally in a sound state, and are capable of performing all the most delicate functions of vision.

Mr. Harris, a shoemaker at Allonby, was unable from his infancy to distinguish the cherries of a cherry-tree from its leaves, in so far as color was concerned. Two of his brothers were equally defective in this respect, and always mistook *orange* for *grass green*, and *light green* for *yellow*. Harris himself could only distinguish black and white. Mr. Scott, who describes his own case in the Philosophical Transactions, mistook *pink* for a pale *blue*, and a full *red* for a full *green*.

All kinds of yellows and blues, except sky blue, he could discern with great nicety. His father, his maternal uncle, one of his sisters, and her two sons, had all the same defect.

A tailor at Plymouth, whose case is described by Mr. Harvey, regarded the solar spectrum as consisting only of *yellow* and *light blue;* and he could distinguish with certainty only *yellow*, *white*, and *green*. He regarded indigo and Prussian blue as black.

Mr. R. Tucker describes the colors of the spectrum as follows:—

Red	mistaken for	Brown.	Blue	sometimes	Pink.
Orange	- - - -	Green.	Indigo	- - -	Purple.
Yellow	sometimes	Orange.	Violet	- - -	Purple.
Green	- - - -	Orange.			

A gentleman in the prime of life, whose case I had occasion to examine, saw only two colors in the spectrum, viz. *yellow* and *blue*. When the middle of the red space was absorbed by a blue glass, he saw the black space, with what he called the yellow, on each side of it. This defect in the perception of color was experienced by the late Mr. Dugald Stewart, who could not perceive any difference in the color of the scarlet fruit of the Siberian crab and that of its leaves. Mr. Dalton is unable to distinguish blue from pink by daylight, and in the solar spectrum the red is scarcely visible, the rest of it appearing to consist of two colors. Mr. Troughton has the same defect, and is capable of fully appreciating only *blue* and *yellow* colors; and when he names colors, the names of blue and yellow correspond to the more and less refrangible rays, all those which belong to the former exciting the sensation of blueness, and those which belong to the latter the sensation of yellowness.

In almost all these cases, the different prismatic colors have the power of exciting the sensation of light, and giving a distinct vision of objects, excepting in the case of Mr. Dalton, who is said to be scarcely able to see the red extremity of the spectrum.

Mr. Dalton has endeavored to explain this peculiarity of vision by supposing that in his own case the vitreous humor is *blue*, and, therefore, absorbs a great portion of the red rays and other least refrangible rays; but this opinion is, we think, not well founded. Mr. Herschel attributes this state of vision to a defect in the sensorium, by which it is rendered incapable of appreciating exactly those differences between rays on which their color depends.*

PART IV.

ON OPTICAL INSTRUMENTS.

ALL the optical instruments now in use have, with the exception of the burning mirrors of Archimedes, been invented by modern philosophers and opticians. The principles upon which most of them have been constructed have already been explained, in the preceding chapters, and we shall therefore confine ourselves, as much as possible, to a general account of their construction and properties.

CHAP. XXXVII.

ON PLANE AND CURVED MIRRORS.

(179.) ONE of the simplest optical instruments is the *single plane* mirror, or looking-glass, which consists of a plate of glass with parallel surfaces, one of which is covered with tin-foil and quicksilver. The glass performs no other part in this kind of plane mirror than that of holding and giving a polished surface to the thin bright film of metal which is extended over it. If the surfaces of the plate of glass are not parallel, we shall see two, three, and four images of all luminous objects seen obliquely; but even when the surfaces are parallel, two images of an object are formed, one reflected from the first surface of glass, and the other from the posterior surface of metal; and the distance of these images will increase with the thickness of the glass. The image reflected from the glass is, however, very faint compared with the other; so that for ordinary purposes a plane glass mirror is sufficiently accurate; but when a plane mirror forms a part of an optical instrument where accuracy of vision is required, it must be made of steel, or silver, or of a mixture of copper and tin; and in this case it is called a *speculum*. The formation of images by mirrors and specula has been fully described in Chap. II.

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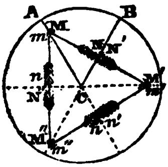
* For the theory recently advanced by Sir David Brewster to explain these cases, see Note IX. of Am. ed.

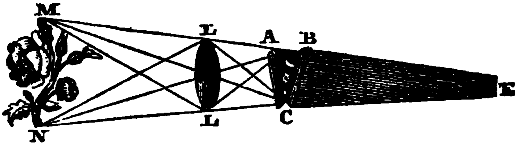
Kaleidoscope.

Fig. 143.



(180.) When two plane mirrors are combined in a particular manner, and placed in a particular position relative to an object, or series of objects, and the eye, they constitute the *kaleidoscope*, or instrument for creating and exhibiting beautiful forms. If A C, B C, for example, be sections of two plane mirrors, and M N an object placed between them or in front of each, the mirror A C will form behind it an image *m n* of the object M N, in the manner shown in *fig.* 16. In like manner, the mirror B C will form an image M′ N′ behind it. But, as we have formerly shown, these images may be considered as new objects, and therefore the mirror A C will form behind it an image, M″ N″, of the object or image M′ N′, and B C will form behind it an image, *m′ n′*, of the object or image *m n*. In like manner it will be found that *m″ n″* will be the image of the object or image M″ N″, formed by B C, and of the object or image *m′ n′*, formed by A C. Hence *m″ n″* will actually consist of two images overlapping each other and forming one, provided the angle A C B is exactly 60°, or the sixth part of a circumference of 360°. In this case all the six images (two of the six forming only one, *m″ n″*,) will, along with the original object, M N, form a perfect equilateral triangle. The object, M N, is drawn perpendicular to the mirror B C, in consequence of which M N and M′ N′ form one straight line; but if M N is moved, all the images will move, and the figure of all the images combined will form another figure of perfect regularity, and exhibiting the most beautiful variations, all of which may be drawn by the methods already described. In reference to the multiplication and arrangement of the images, this is the principle of the kaleïdoscope; but the principle of symmetry, which is essential to the instrument, depends on the position of the object and the eye. This principle will be understood from *fig.* 144., where A C E and B C E represent the two mirrors inclined at an angle A C B, and having C E for their line of junction, or common intersection. If the object is placed at a distance, as at M N, then there is no position of the eye at or above E which will give a symmetrical arrangement of the six images shown in *fig.* 143.; for the corresponding parts of the one will never

Fig. 144



join the corresponding parts of the other. As the object is brought nearer and nearer, the symmetry increases, and is most complete when the object M N is quite close to A B C, the ends of the reflectors. But even here it will not be perfect, unless the eye is placed as near as possible to E, the line of junction of the reflectors. The following, therefore, are the three conditions of symmetry in the kaleidoscope:—

1. That the reflectors should be placed at an angle which is an *even* or an *odd* aliquot part of a circle, when the object is regular and similarly situated with respect to both the mirrors; or an *even* aliquot part of a circle, when the object is irregular.

2. That out of an infinite number of positions for *the object* both within and without the reflectors, there is only one position where perfect symmetry can be obtained, namely, by placing the object in *contact* with the ends of the reflectors, or between them.

3. That out of an infinite number of positions for the *situation of the eye*, there is only one where the symmetry is perfect, namely, as near as possible to the angular point, so that the whole of the circular field can be distinctly seen; and this point is the only one at which the uniformity of the reflected light is greatest.

In order to give variety to the figures formed by the instrument, the objects, consisting of pieces of colored glass, twisted glass of various curvatures, &c., are placed in a narrow cell between two circular pieces of glass, leaving them just room to move about, while this cell is turned round by the hand. The pictures thus presented to the eye are beyond all description splendid and beautiful; an endless variety of symmetrical combinations presenting themselves to view, and never again recurring with the same form and color.

For the purpose of extending the power of the instrument, and introducing into symmetrical pictures external objects, whether animate or inanimate, I applied a convex lens, L L, *fig*. 144., by means of which an inverted image of a distant object, M N, may be formed at the very extremity of the mirrors, and therefore brought into a position of greater symmetry than can be effected in any other way. In this construction the lens is placed in one tube and the reflectors in another; so that by pulling out or pushing in the tube next the eye, the images of objects at any distance can be formed at the place of symmetry. In this way, flowers, trees, animals, pictures, busts, may be introduced into symmetrical combinations. When the distance E B is less than that at which the eye sees objects distinctly, it is necessary to place a convex lens at E, to give distinct vision of the objects in the picture. See my *Treatise on the Kaleidoscope.*

Plane burning Mirrors

(181.) A combination of plane burning mirrors forms a powerful burning instrument; and it is highly probable that it was with such a combination that Archimedes destroyed the ships of Marcellus. Athanasius Kircher, who first proved the efficacy of a union of plane mirrors, went with his pupil Scheiner to Syracuse, to examine the position of the hostile fleet; and they were both satisfied that the ships of Marcellus could not have been more than *thirty* paces distant from Archimedes.

Buffon constructed a burning apparatus upon this principle, which may be easily explained. If we reflect the light of the sun upon one cheek by a small piece of plane looking-glass, we shall experience a sensation of heat less than if the direct light of the sun fell upon it. If with the other hand we reflect the sun's light upon the same cheek with another piece of mirror, the warmth will be increased, and so on, till with five or six pieces we can no longer endure the heat. Buffon combined 168 pieces of mirror, 6 inches by 8, so that he could, by a little mechanism connected with each, cause them to reflect the light of the sun upon one spot. Those pieces of glass were selected which gave the smallest image of the sun at 250 feet.

The following were the effects produced by different numbers of these mirrors:—

No. of Mirrors.	Distance of Object.	Effect produced.
12	20 feet	Small combustibles inflamed.
21	20	Beech plank burned.
40	66	Tarred beech plank inflamed.
45	20	Pewter flask 6lb. weight melted.
98	126	Tarred and sulphured plank set on fire.

No. of Mirrors.	Distance of Object.	Effect produced.
112	138	Plank covered with wool set on fire.
117	20	Some thin pieces of silver melted.
128	150	Tarred fir plank set on fire.
148	150	Beech plank sulphured inflamed violently.
154	150	Tarred plank smoked violently.
154	250	Chips of fir deal sulphured and mixed with charcoal set on fire.
224	40	Plates of silver melted.

As it is difficult to adjust the mirrors while the sun changes his place, M. Peyrard proposes to produce great effects by mounting each mirror in a separate frame, carrying a telescope, by means of which one person can direct the reflected rays to the object which is to be burned. He conceives that with 590 glasses, about 20 inches in diameter, he could reduce a fleet to ashes at the distance of a quarter of a league, and with glasses of double that size at the distance of half a league.

Plane glass mirrors have been combined permanently into a parabolic form, for the purpose of burning objects placed in the focus of the parabola, by the sun's rays; and the same combination has been used, and is still in use, for lighthouse reflectors, the light being placed in the focus of the parabola.

Convex and Concave Mirrors.

(182.) The general properties of convex and concave mirrors have been already described in Chap. II. Convex mirrors are used principally as household ornaments, and are characterized by their property of forming erect and diminished images of all objects placed before them, and these images appear to be situated behind the mirror.

Concave mirrors are distinguished by their property of forming in front of them, and in the air, inverted images of erect objects, or erect images of inverted objects, placed at some distance beyond their principal focus. If a fine transparent cloud of blue smoke is raised, by means of a chafing-dish, around the focus of a large concave mirror, the image of any highly illuminated object will be depicted, in the middle of it, with great beauty. A skull concealed from the observer is sometimes used, to surprise the ignorant; and when a dish of fruit has been depicted in a similar manner, a spectator, stretching out his hand to seize it, is met with the image of a drawn dagger, which has been quickly substituted for the fruit at the other conjugate focus of the mirror

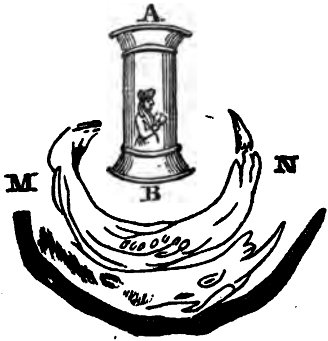
Concave mirrors have been used as lighthouse reflectors, and as burning instruments. When used in lighthouses, they are formed of plates of copper plated with silver, and they are hammered into a parabolic form, and then polished with the hand. A lamp placed in the focus of the parabola will have its divergent light thrown, after reflexion, into something like a parallel beam, which will retain its intensity at a great distance.

When concave mirrors are used for burning, they are generally made spherical, and regularly ground and polished upon a tool, like the specula used in telescopes. The most celebrated of these were made by M. Villele, of Lyons, who executed five large ones. One of the best of them, which consisted of copper and tin, was very nearly four feet in diameter, and its focal length thirty-eight inches. It melted a piece of Pompey's pillar in fifty seconds, a silver sixpence in seven seconds and a half, a halfpenny in sixteen seconds, cast-iron in sixteen seconds, slate in three seconds, and thin tile in four seconds.

Cylindrical Mirrors

(183.) All objects seen by reflexion in a cylindrical mirror are necessarily distorted. If an observer looks into such a mirror with its axis standing vertically, he will see the image of his head of the same length as the original, because the surface of the mirror is a straight line in a vertical direction. The breadth of the face will be greatly contracted in a horizontal direction, because the surface is very convex in that

Fig. 145.



direction, and in intermediate directions the head will have intermediate breadths. If the axis of the mirror is held horizontally, the face will be as broad as life, and exceedingly short. If a picture or portrait M N is laid down horizontally before the mirror A B, *fig*. 145., the reflected image of it will be highly distorted; but the picture may be drawn distorted according to regular laws, so that its image shall have the most correct proportions.

Cylindrical mirrors, which are now very uncommon, used to be made for this purpose, and were accompanied with a series of distorted figures, which, when seen by the eye, have neither shape nor meaning, but when laid down before a cylindrical mirror, the reflected image of them has the most perfect proportions. This effect is shown in *fig*. 145., where M N is a distorted figure, whose image in the mirror A B has the appearance of a regular portrait.

CHAP. XXXVIII.

ON SINGLE AND COMPOUND LENSES.

SPECTACLES and reading glasses are among the simplest and most useful of optical instruments. In order to enable a person who has imperfect vision to see small objects distinctly, when they are not far from the eye, such as small manuscript, or small type, a convex lens of very short focus must be used both by those who are long and short sighted.

When a short-sighted person, who cannot see well at a distance, wishes to have distinct vision at any particular distance, he must use a *concave* lens, whose focal length will be found thus,—Multiply the distance at which he sees objects most distinctly by the distance at which he wishes to see them distinctly with a concave lens, and divide this product by the difference of the above distances.

A long-sighted person, who cannot see near objects distinctly, must use a *convex lens*, whose focal length is found by the preceding rule.

In choosing spectacles, however, the best way is to select, out of a number, those which are found to answer best the purposes for which they are particularly intended.

Dr. Wollaston introduced a new kind of spectacles, called *periscopic*, from their property of giving a wider field of distinct vision than the common ones. The lenses used for this purpose, as shown at H and I, *fig*. 19., are meniscuses, in which the convexity predominates, for long-sighted persons, and concavo-convex lenses, in which the concavity predominates, for short-sighted persons. Periscopic spectacles decidedly give more imperfect vision than common spectacles, because they increase both the aberration of figure and of color; but they may be of use in a crowded city, in warning us of the oblique approach of objects.

Burning and Illuminating Lenses.

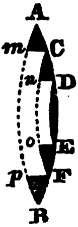
(184.) Convex lenses possess peculiar advantages for concentrating the sun's rays, and for conveying to an immense distance a condensed and parallel beam of light. M. Buffon found that a convex lens, with a long focal length, was preferable to one of a short focal length for fusing metals by the concentration of the sun's rays. A lens, for example, 32 inches in diameter and 6 inches in focal length, with the diameter of its focus 8 lines, melted copper in less than a minute; while a small lens 32 lines in diameter, with a focal length of 6 lines, and its focus $\frac{2}{3}$ of a line, was scarcely capable of heating copper.

The most perfect burning lens ever constructed was executed by Mr. Parker, of Fleet Street, at an expense of 700*l.* It was made of flint glass, was three feet in diameter, and weighed 212 pounds. It was $3\frac{1}{4}$ inches thick at the centre; the focal distance was 6 feet 8 inches, and the diameter of the image of the sun in its focus one inch. The rays refracted by the lens were received on a second lens, in whose focus the objects to be fused were placed. This second lens had an exposed diameter of 13 inches; its central thickness was $1\frac{5}{8}$ of an inch; the length of its focus was 29 inches. The diameter of the focal image was $\frac{3}{8}$ of an inch. Its weight was 21 pounds. The combined focal length of the two lenses was 5 feet 3 inches, and the diameter of the focal image $\frac{1}{2}$ an inch. By means of this powerful burning lens, platinum, gold, silver, copper, tin, quartz, agate, jasper, flint, topaz, garnet, asbestos, &c. were melted in a few seconds.

Various causes have prevented philosophers from constructing burning lenses of greater magnitude than that made by Mr. Parker. The impossibility of procuring pure flint glass tolerably free from veins and impurities for a large solid lens; the trouble and expense of casting it into a lenticular form without flaws and impurities; the great increase of central thickness which becomes necessary by increasing the diameter of the lens; the enormous obstruction that is thus opposed to the transmission of the solar rays, and the increased aberration which dissipates the rays at the focal point, are insuperable obstacles to the construction of solid lenses of any considerable size.

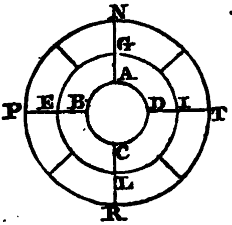
(185.) In order to improve a solid lens formed of one piece of glass, whose section is A *m p* B E D A, Buffon proposed to cut out all the glass left white in the figure, viz. the portions between *m p*, *fig*. 146., and *n o*, and between *n o* and the left hand surface of D E. A lens thus constructed would be incomparably superior to the solid one A *m p* B E D A; but such a process we conceive to be impracticable on a large scale, from the extreme difficulty of polishing the surfaces A *m*, B *p*, C *n*, F *o*, and the left hand surface of D E; and even if it were practicable, the greatest imperfections in the glass might happen to occur in the parts which are left.

Fig. 146.



In order to remove these imperfections, and to construct lenses of any size, I proposed, in 1811, to build them up of separate zones or rings, each of which rings was again to be composed of separate segments, as shown in the front view of the lens in *fig*. 147. This lens is composed of one central lens, A B C D, corresponding with its section D E in *fig*. 146., of a middle ring G E L I corresponding to C D E F in *fig*. 146., and consisting of *five* segments; and another ring, N P R T, corresponding to A C F B, and consisting of *eight* segments.

Fig. 147.



The preceding construction obviously puts it in our power to execute these compound lenses, to which I have given the name of *polyzonal lenses*, of pure flint glass free from veins; but it possesses another great advantage, namely, that of enabling us to correct, very nearly, the spherical aberration, by making the foci of each zone coincide.

One of these lenses was constructed, under my direction, for the Commissioners of Northern Lighthouses, by Messrs. W. and P. Gilbert. It was made of pure flint glass, was three feet in diameter, and consisted of many zones and segments. Lenses of this kind have been made in France of crown glass, and have been introduced into the principal French lighthouses; a purpose to which they are infinitely better adapted than the best constructed parabolic reflectors made of metal.

A polyzonal lens of at least *four* feet in diameter will be speedily executed as a burning-glass, and will, no doubt, be the most powerful ever made. The means of executing it have been, to a considerable degree, supplied by the scientific liberality of Mr. Swinton and Mr. Calder, and other gentlemen of Calcutta.

CHAP. XXXIX.

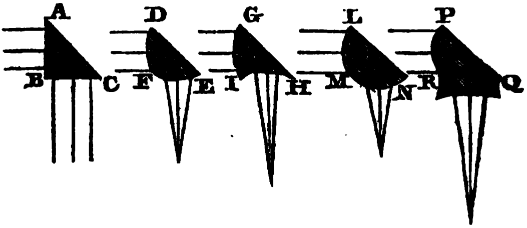
ON SIMPLE AND COMPOUND PRISMS.

Prismatic Lenses.

(186.) THE general properties of the prism in refracting and decomposing light have already been explained; but its application as an optical instrument, or as an important part of optical instruments, remains to be described.

A rectangular prism, A B C, *fig.* 148., was first applied by Sir Isaac Newton as a plane mirror for reflecting to a side the rays which form the image in reflecting telescopes. The

Fig. 148.



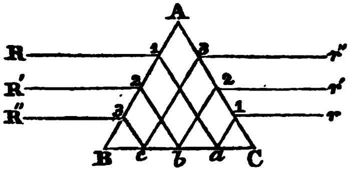
angles, B A C, B C A, being each 45°, and B a right angle, rays falling on the face A B will be reflected by the back surface B C as if it were a plane metallic mirror; for whatever be the refraction which they suffer at their entrance into the face A B, they will suffer an equal and opposite one at the face B C. The great value of such a mirror is, that as the incident rays fall upon A C at an angle greater than that at which total reflexion commences, *they will all suffer total reflexion*, and not a ray will be lost; whereas in the best metallic speculum nearly half the light is lost. A portion of light, however, is lost by reflexion at the two surfaces A B, B C, and a small portion by the absorption of the glass itself. Sir Isaac Newton also proposed the *convex prism*, shown at

D E F, the faces D F, F E being ground convex. An analogous prism, called the *meniscus prism*, and shown at G H I, has been used by M. Chevalier, of Paris, for the camera obscura. It differs only from Newton's in the second face, I H, being concave in place of convex.

On account of the difficult execution of these prisms, I have proposed to use a hemispherical lens, L M N, the two convex surfaces of which are ground at the same time. When a longer focus is required, a concave lens, R Q, of a longer focus than the hemisphere P R Q, may be placed or cemented on its lower surface, and if the concave lens is formed out of a substance of a different dispersive power, it may be made to correct the color of the convex lens.

A single prism is used with peculiar advantage for inverting pencils of light, or for obtaining an erect image from pencils that would give an inverted one. This effect is shown in *fig.* 149., where A B C is a rectangular prism, and R R′ R″ a parallel pencil of light, which, after being refracted at the points 1, 2, 3, of the face A B, and reflected at the points *a*, *b*, *c*, of the base B C, will be again refracted at the points 1, 2, 3, of the face A C, and move on in parallel lines, 3 *r*″, 2 *r*′, 1 *r*; the ray R I, that was uppermost, being now undermost, as at 1 *r*.

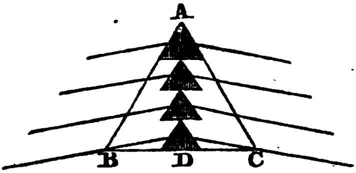
Fig. 149.



Compound and Variable Prisms.

(187.) The great difficulty of obtaining glass sufficiently pure for a prism of any size, has rendered it extremely difficult to procure good ones; and they have therefore not been introduced as they would otherwise have been into optical instruments. The principle upon which polyzonal lenses are constructed is equally applicable to prisms. A prism constructed like A D, *fig.* 150., if properly executed, would have exactly the same properties as A B C, and would be incomparably superior to it, from the light passing through such a small thickness of glass. It would obviously be difficult to execute such a prism as A D out of a single piece of glass, though it is quite practicable; but there is no difficulty in combining six small prisms all cut out of one prismatic rod, and therefore

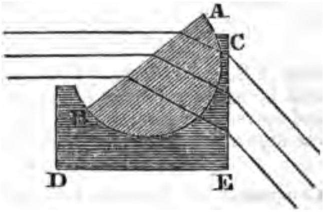
Fig. 150.



necessarily similar. The summit of the rod should have a flat narrow face parallel to its base, which would be easily done if the prismatic rod were cut out of a plate of thick parallel glass. The separate prisms being cemented to one another, as in the figure, will form a compound prism, which will be superior to the common prism for all purposes in which it acts solely by refraction.

(188.) A compound prism of a different kind, and having a variable angle, was proposed by Boscovich, as shown in *fig*. 151., where A B C is a hemispherical convex lens, moving in a concave lens, D E C, of the same curvature. By turning

Fig. 151.



one of the lenses round upon the other, the inclination of the faces A B, D E, or A B, C E, may be made to vary from 0° to above 90°.

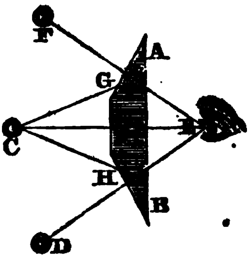
(189.) As this apparatus is both troublesome to execute and difficult to use, I have employed an entirely different principle for the construction of a variable prism, and have used it to a great extent in numerous experiments on the dispersive powers of bodies. If we produce a vertical line of light by nearly closing the window-shutters, and view the line with a flint glass prism whose refracting angle is 60°, the edge of the refracting angle being held vertical, or parallel to the line of light, the luminous line will be seen as a brightly colored spectrum, and any small portion of it will resemble almost exactly the solar spectrum. If we now turn the prism in the plane of one of its refracting faces, so that the inclination of the edge to the line of light increases gradually from 0° up to 90° when it is perpendicular to the line of light, the spectrum will gradually grow less and less colored, exactly as if it were formed by a prism of a less and less refracting angle, till at an inclination of 90° not a trace of color is left. By this simple process, therefore, namely, by using a line of light instead of a circular disc, we have produced the very same effect as if the refracting angle of the prism had been varied from 90° down to 0°.

(190.) Let it now be required to determine the relative dispersive powers of flint glass and crown glass. Place the crown glass prism so as to produce the largest spectrum from the line of white light, and let the refracting angle of the prism be 40°. Then place the flint glass prism between it and the eye, and turn it round, as before described, till it corrects the color produced by the crown glass prism, or till the line of light is perfectly colorless. The inclination of the edge of the flint glass prism to the line of light being known, we can easily find, by a simple formula the angle of a prism of flint glass which corrects the color of a prism of crown glass with a refracting angle of 40°. See my *Treatise on New Philosophical Instruments*, p. 291.

Multiplying Glass.

(191.) This lens is more amusing than useful, and is intended to give a number of images of the same object. Though it has the circular form of a lens, it is nothing more than a

Fig. 152



number of prisms formed by grinding various flat faces on the convex surface of a plano-convex glass, as shown in *fig.* 152, where A B is the section of a multiplying glass in which only three of the planes are seen. A direct image of the object C will be seen through the face G H, by the eye at E; another image will be seen at D, by the refraction of the face H B, and a third at F, by the refraction of the face A G, an image being seen through every plane face that is cut upon the lens. The image at C will be colorless, and all those formed by planes inclined to A B will be colored in proportion to the angles which the planes form with A B.

Natural multiplying glasses may be found among transparent minerals which are crossed with veins oppositely crystallized, even though they are ground into plates with parallel faces. In some specimens of Iceland spar more than a hundred finely colored images may be seen at once. The theory of such multiplying glasses has already been explained in Chap. XXIX.

CHAP. XL.

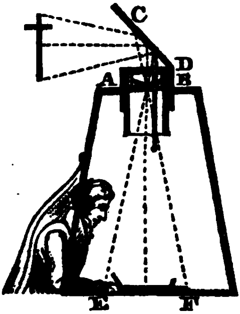
ON THE CAMERA OBSCURA, MAGIC LANTERN, AND CAMERA LUCIDA.

(192.) THE *camera obscura*, or *dark chamber*, is the name of an amusing and useful optical instrument, invented by the celebrated Baptista Porta. In its original state it is nothing more than a dark room with an opening in the window-shutter, in which is placed a convex lens of one or more feet focal length. If a sheet of white paper is held perpendicularly behind the lens, and passing through its focus, there will be painted upon it an accurate picture of all the objects seen from the window, in which the trees and clouds will appear to move in the wind, and all living objects to display the same movements and gestures which they exhibit to the eye. The perfect resemblance of this picture to nature astonishes and delights every person, however often they may have seen it. The image is of course inverted, but if we look over the top of the paper it will be seen as if it were erect. The ground on which the picture is received should be hollow, and part of a sphere whose radius is the focal distance of the convex lens. It is customary, therefore, to make it of the whitest plaster of Paris, with as smooth and accurate a surface as possible.

In order to exhibit the picture to several spectators at once, and to enable any person to copy it, it is desirable that the image should be formed upon a horizontal table. This may be done by means of a metallic mirror, placed at an angle of 45° to the refracted rays, which will reflect the picture upon the white ground lying horizontally; or, as in the portable camera obscura, it may be reflected upwards by the mirror, and received on the lower side of a plate of ground glass, with its rough side uppermost, upon which the picture may be copied with a fine sharp-pointed pencil.

A very convenient portable camera obscura for drawing landscapes or other objects is shown in *fig.* 153., where A B is a meniscus lens, with its concave side uppermost, and the radius of its convex surface being to the radius of its concave surface as 5 to 8, and C D a plane metallic speculum inclined at an angle of 45° to the horizon, so as to reflect the landscape downwards through the lens A B. The draughtsman introduces his head through an opening in one side, and his hand with the pencil through another opening, made in such a manner as to allow no light to fall upon the picture which is exhibited on the paper at E F. The tube containing the mirror and lens can be turned round by a rod within, and the inclination of the mirror changed, so as to introduce objects in any part of the horizon.

Fig. 153.



When the camera is intended for public exhibition, it consists of the same parts similarly arranged; but they are in this case placed on the top of a building, and the rotation of the mirror, and its motion in a vertical plane, are effected by turning two rods within the reach of the spectator, so that he can introduce any object into the picture from all points of the compass and at all distances. The picture is received on a table, whose surface is made of stucco, and of the same radius as the lens, and this surface is made to rise and fall to accommodate it to the change of focus produced by objects at different distances. A camera obscura which throws the image down upon a horizontal surface may be made without any mirror, by using any of the lenticular prisms D E F, G H I, M L N, when the objects are extremely near, and P R Q, *fig.* 148. The convex surfaces of these prisms converge the rays which are reflected to their focus by the flat faces D E, G H, L N, and P Q; these lenticular prisms may be formed by cementing plano-convex or concave lenses on the faces A B, B C of the rectangular prism A B C, or the convex lens may be placed near to A B.

If we wish to form an erect image on a vertical plane, the prism A B C, *fig.* 148., may be placed in front of the convex lens, or immediately behind it. The same effect might be produced by *three* reflexions from *three* mirrors or specula.

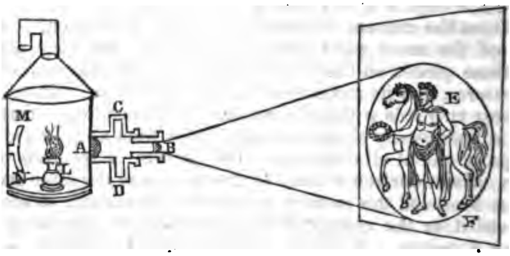
I have found that a peculiarly brilliant effect is given to the images formed in the camera obscura when they are received upon the silvered back of a looking-glass, smoothed by grinding it with a flat and soft hone. In the portable camera obscura I find that a film of skimmed milk, dried upon a plate of glass, is superior to ground glass for the reception of images.

A modification of the camera obscura, called the megascope, is intended for taking magnified drawings of small objects placed near the lens. In this case, the distance of the image behind the lens is greater than the distance of the object before it. By altering the distance of the object, the size of the image may be reduced or enlarged. The hemispherical lens L M N, *fig.* 148., is particularly adapted for the megascope.

Magic Lantern.

(193.) The magic lantern, an invention of Kircher, is shown in *fig.* 154., where L is a lamp with a powerful Argand burner, placed in a dark lantern. On one side of the lantern is a concave mirror M N, the vertex of which is opposite to the centre of the flame, which is placed in its focus. In the opposite side of the lantern is fixed a tube A B, containing a hemispherical illuminating lens A, and a convex lens B; between A and B the diameter of the tube is increased for the

Fig. 154.



purpose of allowing sliders to be introduced through the slit C D. These sliders contain 4 or 5 pictures, each painted and highly colored with transparent varnishes, and, by sliding them through C D, any of the subjects may be introduced into the axis of the tube and between the two lenses A, B. The light of the lamp L, increased by the light reflected from the mirror falling upon the lens A, is concentrated by it upon the picture in the slider; and this picture, being in one of the conjugate foci of the lens B, an enlarged image of it will be painted on a white cloth, or on a screen of white paper, E, standing or suspended perpendicularly. The distance of the lens B from the object or the slider may be increased or diminished by pulling out or pushing in the tube B, so that a distinct picture of the object may be formed of any size and at any distance from B, within moderate limits. If the screen E F is made of fine semi-transparent silver paper, or fine muslin properly prepared, the image may be distinctly seen by a spectator on the other side of the screen.

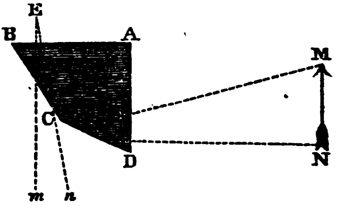
(194.) The *phantasmagoria* is nothing more than a magic lantern, in which the images are received on a transparent screen, which is fixed in view of the spectator. The magic lantern, mounted upon wheels, is made to recede from or approach to the screen; the consequence of which is, that the picture on the screen expands to a gigantic size, or contracts into an invisible object or mere luminous spot. The lens B is made to recede from the slider in C D when the lantern approaches the screen, and to approach to it when the lantern recedes from the screen, in order that the picture upon the screen may always be distinct. This may be accomplished, according to Dr. Young, by jointed rods or levers, connected with the screen, which pull out or push in the tube B; but we are of opinion that the required effect may be much more elegantly and efficaciously produced by the simplest piece of mechanism connected with the wheels.

Camera Lucida.

(195.) This instrument, invented by Dr. Wollaston in 1807, has come into very general use for drawing landscapes, delineating objects of natural history, and copying and reducing drawings.

Dr. Wollaston's form of the instrument is shown in *fig.* 155., where A B C D is a glass prism, the angle B A D being 90°, A D C 67½°, and D C B 135°. The rays proceeding from any object, M N, after being reflected by the faces D C, C B to the eye, E, placed above the angle B, the observer will see an image *m n* of the object M N projected upon a piece of paper at *m n*. If the eye is now brought down close to the

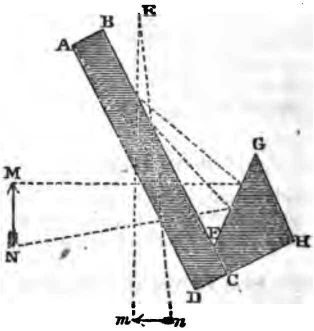
Fig. 155.



angle B, so that it at the same time sees into the prism with one-half of the pupil, and past the angle B with the other half, it will obtain distinct vision of the image *m n*, and also see the paper and the point of the pencil. The draughtsman has, therefore, only to trace the outline of the image upon the paper, the image being seen with half of the pupil, and the paper and pencil with the other half.

Many persons have acquired the art of using this instrument with great facility, while others have entirely failed. In examining the causes of this failure, professor Amici, of Modena, succeeded in removing them, and has proposed various forms of the instrument free from the defects of Dr. Wollaston's.* The one which M. Amici thinks the best is shown in

Fig. 156.



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* An account of these various forms will be found in the *Edinburgh Journal of Science*, No. V. p. 157.

fig. 156., where A B C D is a piece of thick parallel glass, F G H C a metallic mirror, whose face, F G, is highly polished, and inclined 45° to B C. Rays from an object, M N, after passing through the glass A B C D, are reflected from F G, and afterwards from the face B C of the glass plate to the eye at E, by which the object, M N, is seen at *m n*, where the paper is placed. The pencil and the paper are readily seen through the plane glass A B C D. In order to make the two faces of the glass, A D, B C, perfectly parallel, M. Amici forms a triangular prism of glass, and cuts it through the middle; he then joins the two prisms or halves, A D C, C A B, so as to form a parallel plate, and by slightly turning round the prisms, he can easily find the position in which the two faces are perfectly parallel.

CHAP. XLI.

ON MICROSCOPES.

A MICROSCOPE is an optical instrument for magnifying and examining minute objects. Jansen and Drebell are supposed to have separately invented the single microscope, and Fontana and Galileo seem to have been the first who constructed the instrument in its compound form.

Single Microscope.

(196.) The single microscope is nothing more than a lens or sphere of any transparent substance, in the focus of which minute objects are placed. The rays which issue from each point of the object are refracted by the lens into parallel rays, which, entering the eye placed immediately behind the lens, afford distinct vision of the object. The magnifying power of all such microscopes is equal to the distance at which we could examine the object most distinctly, divided by the focal length of the lens or sphere. If this distance is 5 inches, which it does not exceed in good eyes when they examine minute objects, then the magnifying power of each lens will be as follows:—

Focal length in inches.	Linear magnifying power.	Superficial magnifying power.
5	1	1
1	5	25
$\frac{1}{10}$	50	2500
$\frac{1}{100}$	500	250000

The *linear* magnifying power is the number of times an object is magnified in length, and the *superficial* magnifying power is the number of times that it is magnified in surface. If the object is a small square, then a lens of one inch focus will magnify the side of the square 5 times, and its area or surface 25 times.

The best single microscopes are minute lenses ground and polished on a concave tool; but as the perfect execution of these requires considerable skill, small spheres have been often constructed as substitutes. Dr. Hooke executed these spheres in the following manner: having drawn out a thin strip of window-glass into threads by the flame of a lamp, he held one of these threads with its extremity in or near the flame, till it ran into a globule. The globule was then cut off and placed above a small aperture, so that none of the rays which it transmitted passed through the part where it was joined to the thread of glass.' He sometimes ground off the end of the thread, and polished that part of the sphere. Father di Torre of Naples improved these globules by placing them in small cavities in a piece of calcined tripoli, and remelting them with the blowpipe; the consequence of which was, that they assumed a perfectly spherical form. Mr. Butterfield executed similar spheres by taking upon the wetted point of a needle some finely pounded glass, and melting it by a spirit lamp into a globule. If the part next the needle was not melted, the globule was removed from the needle and taken up with the wetted needle on its round side, and again presented to the flame till it was a perfect sphere. M. Sivright, of Meggetland, has made lenses by putting pieces of glass in small round apertures between the 10th and 20th of an inch, made in platinum leaf. They were then melted by the blowpipe, so that the lenses were made and set at the same time.

Mr. Stephen Gray made globules for microscopes by inserting drops of water in small apertures. I have made them in the same way with oils and varnishes; but the finest of all single microscopes may be executed by forming minute plano-convex lenses upon glass with different fluids. I have also formed excellent microscopes by using the spherical crystalline lenses of minnows and other small fish, and taking care that the axis of the lens is the axis of vision, or that the observer looks through the lens in the same manner that the fish did.*

The most perfect single microscopes ever executed of solid substances are those made of the gems, such as *garnet*, *ruby*

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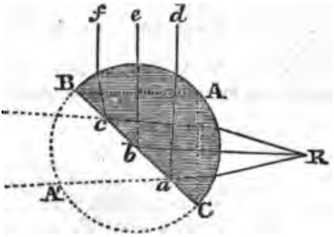
* See *Edinburgh Journal of Science*, No. III. p. 98.

sapphire, and *diamond*. The advantages of such lenses I first pointed out in my *Treatise on Philosophical Instruments;* and two lenses, one of ruby and another of garnet, were executed for me by Mr. Peter Hill, optician in Edinburgh. These lenses performed admirably, in consequence of their producing, with surfaces of inferior curvature, the same magnifying power as a glass lens; and the distinctness of the image was increased by their absorbing the extreme blue rays of the spectrum. Mr. Pritchard, of London, has carried this branch of the art to the highest perfection, and has executed lenses of sapphire and diamond of great power and perfection of workmanship.

When the diamond can be procured perfectly homogeneous and free from double refraction, it may be wrought into a lens of the highest excellence; but the sapphire, which has double refraction, is less fitted for this purpose. Garnet is decidedly the best material for single lenses, as it has no double refraction, and may be procured, with a little attention, perfectly pure and homogeneous. I have now in my possession two garnet microscopes, executed by Mr. Adie, which far surpass every solid lens I have seen. Their focal length is between the 30th and the 50th of an inch. Mr. Veitch, of Inchbonny, has likewise executed some admirable garnet lenses out of a Greenland specimen of that mineral given to me by Sir Charles Giesecké.

(197.) A single microscope, which occurred to me some years ago, is shown in *fig*. 157., and consists in a new method of *using a hemispherical lens so as to obtain from it* twice *the magnifying power which it possesses when used in the common way*. If A B C is a hemispherical lens, rays issuing from any object, R, will be refracted at the first surface A C, and, after total reflexion at the plane surface B C, will be again refracted at the second surface A B, and emerge in par-

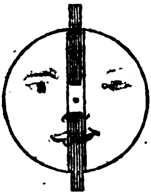
Fig. 157.



allel directions *d e f*, exactly in the same manner as if they had not been reflected at the points *a*, *b*, *c*, but had passed through the other half B A′ C of a perfect sphere A B A′ C. The object at R will therefore be magnified in the same manner, and will be seen with the same distinctness as if it had been seen through a sphere of glass A B A′ C. We obtain, consequently, by this contrivance, all the advantages of a spherical lens, which we believe never has been executed by grinding. The periscopic principle, which will presently be mentioned, may be communicated to this *catoptric* lens, as it may be called, by merely grinding off the angles B C, or rough grinding an annular space on the plane surface B C. The confusion arising from the oblique refractions will thus be prevented, and the pencils from every part of the object will fall symmetrically upon the lens, and be symmetrically refracted.

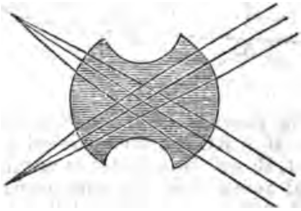
Before I had thought of this lens, Dr. Wollaston had proposed a method of improving lenses, which is shown in *fig.* 158. He introduced between two plano-convex lenses of equal size and radius, a plate of metal with a circular aperture equal to $\frac{1}{5}$th of the focal length, and when the aperture was well centered, he found that the visible field was 20° in diameter. In this compound lens the oblique pencils pass, like the central ones, at right angles to the surface. If we compare this lens with the *catoptric* one above described, we shall see that the effect which is produced in the one case with two spherical and two plane surfaces, all ground separately, is produced in the other case by one spherical and one plane surface.

Fig. 158.



(198.) The idea of Dr. Wollaston may, however, be improved in other ways, by filling up the central aperture with

Fig. 159.



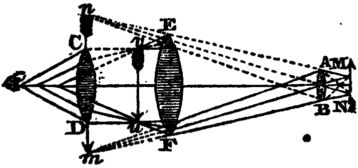
a cement of the same refractive power as the lenses, or, what is far better, by taking a sphere of glass and grinding away the equatorial parts, so as to limit the central aperture, as shown in *fig.* 159.; a construction which, when executed in garnet, and used in homogeneous light, we conceive to be the most perfect of all lenses, either for single microscopes, or for the object lenses of compound ones.

When a single microscope is used for opaque objects, the lens is placed within a concave silver speculum, which concentrates parallel or converging rays upon the face of the object next the eye.

Compound Microscopes.

(199.) When a microscope consists of two or more lenses or specula, one of which forms an enlarged image of objects, while the rest magnify that image, it is called a *compound microscope.* The lenses, and the progress of the rays through them in such an instrument, are shown in *fig.* 160., where A B is the object glass, and C D the eye glass. An object, M N, placed a little farther from A B than its principal focus, will have an enlarged image of itself formed at *m n* in an inverted position. If this enlarged image is in the focus of another lens, C D, placed nearer the eye than in the figure, it will be again magnified, as if *m n* were an object. The magnifying effect of the lens A B is found by dividing the distance of the image *m n* from the lens A B by the distance of the object from the same lens; and the magnifying effect of the eye glass C D is found by the rule for single microscopes; and these two numbers being multiplied together, will be the magnifying power of the compound microscope. Thus, if M A is $\frac{1}{4}$th of an inch, A *n*, 5 inches, and C *n* $\frac{1}{2}$ an inch, (*m n* being supposed in the focus of C D,) the effect of the lens A B will be 20, and that of C D 10, and the whole power 200. A larger lens than any of the other two, called the field glass, and shown at E F, is generally placed between A B and the image

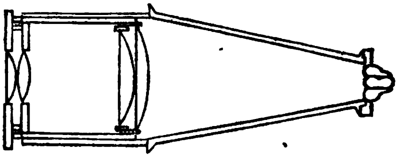
Fig. 160.



m n, for the purpose of enlarging the field of view. It has the effect of diminishing the magnifying power of the instrument by forming a smaller image at *v u*, which is magnified by C D.

The ingenuity of philosophers and of artists has been nearly exhausted in devising the best forms of object glasses and of eye glasses for the compound microscope. Mr. Coddington has recommended four lenses to be employed in the eye piece of compound microscopes, as shown in *fig.* 161.; and along with these he uses, as an object glass, the sphere excavated at the equator, as in *fig.* 159., for the purpose of reducing the aberration and dispersion. "With a sphere," says he, "properly cut away at the centre, so as to reduce the aberration and dispersion to insensible quantities, which may be done most completely and most easily, as I have found in practice, the whole image is perfectly distinct, whatever extent of it be taken; and the radius of curvature of it is no less than the focal length, so that the one difficulty is entirely removed, and the other at least diminished to one-half. Besides all this, another advantage appears in practice to attend this construction, which I did not anticipate, and for which I cannot now at all account. I have stated that when a pencil of rays is admitted into the eye, which, having passed without deviation through a lens, is bent by the eye, the vision is never free from the colored fringes produced by excentrical dispersion. Now, with the sphere I certainly do not perceive this defect, and I therefore conceive that if it were possible to make the spherical glass on a very minute scale, it would be the most perfect simple microscope, except, perhaps, Dr. Wollaston's doublet. * * * Now, the sphere has this advantage, that it is more peculiarly fitted for the object glass of a compound instrument, since it gives a perfectly distinct image of any required extent, and that, when combined with a proper eye piece, it may without difficulty be employed for opaque objects."* The difficulty of making the spherical glass on a

Fig. 161.



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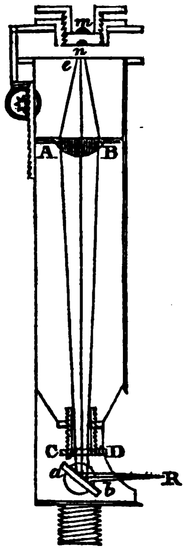
* *Cambridge Transactions*, 1830.

very minute scale, which Mr. Coddington here mentions, and which is by no means insurmountable, is, I conceive, entirely removed by substituting a hemisphere, as shown in *fig.* 157., and contracting the aperture in the manner there mentioned.

Fig. 162.



Dr. Wollaston's microscopic doublet shown in *fig.* 162., consists of two plano-convex lenses *m*, *n*, with their plane sides turned towards the object. Their focal lengths are as *one* to *three*, and their distance from $1\frac{4}{10}$ to $1\frac{1}{2}$ inch, the least convex being next the eye. The tube is about six inches long, having at its lower end, C D, a circular perforation about $\frac{3}{15}$ of an inch in diameter; through which light radiating from R is reflected by a plane mirror *a b* below it. At the upper end of the tube is a plano-convex lens A B, about $\frac{3}{4}$ of an inch focus, with its plane side next the observer, the object of which is to form a distinct image of the circular perforation, at *e*, at the distance of about $\frac{8}{10}$ of an inch from A B. With this instrument, Dr. Wollaston saw the finest striæ and serratures upon the scales of the *lepisma* and *podura*, and upon the scales of a gnat's wing.

(200.) Double and triple achromatic lenses have been recently much used for the object glasses of microscopes, and two or three of them have been combined; but though they perform well they are very expensive, and by no means superior to other instruments that are properly constructed.* The power of using homogeneous light, indeed, renders them in a great measure unnecessary, especially as we can employ either of Mr. Herschel's double lenses shown in *figs.* 43. and 44., which are entirely free from spherical aberration. One of these, *fig.* 44., has been executed $\frac{1}{5}$ of an inch focus, with an aperture of $\frac{1}{15}$ of an inch; and Mr. Pritchard, to whom it belongs, informs us that it brings out all the test objects, and exhibits opaque ones with facility.

In applying the compound microscope to the examination of objects of natural history, I have recommended the immersion of the object in a fluid, for the purpose of expanding it and giving its minute parts their proper position and appearance. In order to render this method perfect, it is proper to immerse the anterior surface of the object glass in the same fluid which holds the object; and if we use a fluid of greater dispersive power than the object glass, and accommodate the interior surface to the difference of their dispersive powers, the object glass may be made perfectly achromatic. The superiority of such an instrument in viewing animalculæ and the molecules of bodies noticed by Mr. Brown, does not require to be pointed out.

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* See *Edinburgh Journal of Science*, No. VIII. new series p. 244

On Reflecting Microscopes.

(201.) The simplest of all reflecting microscopes is a concave mirror, in which the face of the observer is always magnified when its focus is more remote than the observer. When the mirror is very concave, a small object *m n*, *fig*. 14., will have a magnified picture of it formed at M N; and when this picture is viewed by the eye, we have a single reflecting microscope, which magnifies as many times as the distance A *n* of the object from the mirror is contained in the distance A M of the image.

But if, instead of viewing M N with the naked eye, we magnify it with a lens, we convert the simple reflecting microscope into a compound reflecting microscope, composed of a mirror and a lens. This microscope was first proposed by Sir Isaac Newton; and after being long in disuse has been revived in an improved form by Professor Amici of Modena. He made use of a concave ellipsoidal reflector, whose focal distance was $2\frac{4}{10}$ inches. The image is formed in the other focus of the ellipse, and this image is magnified by a single or double eye piece, eight inches from the reflector. As it is impracticable to illuminate the object *m n* when situated as in *fig*. 14., professor Amici placed it without the tube or below the line B N, and introduced it into the speculum A B by reflexion from a small plane speculum placed between *m n* and A B, and having its diameter about half that of A B.

Dr. Goring, to whom microscopes of all kinds owe so many improvements, has greatly improved this instrument. He uses a small plane speculum less than $\frac{1}{3}$ of the diameter of the concave speculum, and employs the following specula of very short focal distances:—

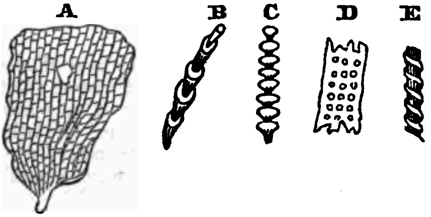
Focal distance in inches.	Aperture in inches.
1·5	0·6
1·0	0·3
0·6	0·3
0·3	0·3

That ingenious artist Mr. Cuthbert, who executed these improvements, has more recently, under Dr. Goring's direction, finished truly elliptical specula, whose aperture is equal to their focal length. This he has done with specula having *half an inch* focus and half an inch aperture, and *three tenths* of an inch focus and *three tenths* of an inch aperture. Dr. Goring assures us that this microscope exhibited a set of longitudinal lines on the scales of the *podura* in addition to the two sets of diagonal ones previously discovered, and two sets of diagonal lines on the scales of the cabbage butterfly in addition to the longitudinal ones with the cross stripe, hitherto observed.*

On Test Objects.

(202.) Dr. Goring has the merit of having introduced the use of test objects, or objects whose texture or markings required a certain excellence in the microscope to be well seen. A few of these are shown in *fig.* 163. as given by Mr. Pritchard. A is the wing of the *menelaus*, B and C the hair of the bat, and D and E the hair of the mouse. The most difficult of all the test objects are those in the scales of the *podura* and the cabbage butterfly mentioned above.

Fig. 163



Rules for microscopic Observations.

(203.) 1. The eye should be protected from all extraneous light, and should not receive any of the light which proceeds from the illuminating centre, excepting what is transmitted through or reflected from the object.

2. Delicate observations should not be made when the fluid which lubricates the cornea is in a viscid state.

3. The best position for microscopical observations is when

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* See *Edinburgh Journal of Science*, No. IV. new series, p. 321.

the observer is lying horizontally on his back. This arises from the perfect stability of his head, and from the equality of the lubricating film of fluid which covers the cornea. The worst of all positions is that in which we look downwards vertically.

4. If we stand straight up and look horizontally, parallel markings or lines will be seen most perfectly when their direction is vertical; viz. the direction in which the lubricating fluid descends over the cornea.

5. Every part of the object should be excluded, except that which is under immediate observation.

6. The light which illuminates the object should have a very small diameter. In the day-time it should be a single hole in the window-shutter of a darkened room, and at night an aperture placed before an Argand lamp.

7. In all cases, particularly when high powers are used, the natural diameter of the illuminating light should be diminished, and its intensity increased, by optical contrivances.

8. In every case of microscopical observations, homogeneous yellow light, procured from a monochromatic lamp, should be employed. Homogeneous red light may be obtained by colored glasses.*

Solar Microscope.

(204.) The solar microscope is nothing more than a magic lantern, the light of the sun being used instead of that of a lamp. The tube A B, *fig.* 154., is inserted in a hole in the window-shutter, and the sun's light reflected into it by a long plane piece of looking-glass, which the observer can turn round to keep the light in the tube as the sun moves through the heavens.

Living objects, or objects of natural history, are put upon a glass slider, or stuck on the point of a needle, and introduced into the opening C D, so as to be illuminated by the sun's rays concentrated by the lens A. An enlarged and brilliant image of the object will then be formed on the screen E F.

Those who wish to see the various external forms of microscopes of all kinds, and the different modes of putting them up, are referred to the article MICROSCOPE, in the *Edinburgh Encyclopædia*, vol. xiv. p. 215—233. In the latest work on the microscope, viz. Dr. Goring and Mr. Pritchard's "Microscopical Illustrations," London, 1830, the reader will find much valuable and interesting information.

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* See the article MICROSCOPE, *Edinburgh Encyclopædia*, vol. xiv. p. 228.

CHAP. XLII.

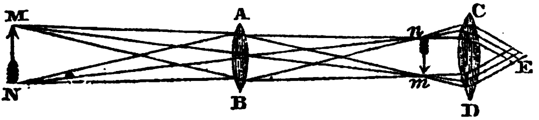
ON REFRACTING AND REFLECTING TELESCOPES.

Astronomical Telescope.

(205.) THAT the telescope was invented in the thirteenth century, and perfectly known to Roger Bacon, and that it was used in England by Leonard and Thomas Digges before the time of Jansen or Galileo, can scarcely admit of a doubt. The principle of the refracting telescope, and the method of computing its magnifying power, have been already explained. We shall therefore proceed to describe the different forms which it successively assumed.

The *astronomical telescope* is represented in *fig*. 164. It consists of two convex lenses A B, C D, the former of which is called the *object glass*, from being next the object M N, and the latter the *eye glass*, from its being next the eye E. The object glass is a lens with a long focal distance; and the eye glass is one of a short focal distance. An inverted image *m n* of any distant object M N is formed in the focus of the object glass A B; and this image is magnified by the eye glass C D, in whose anterior focus it is placed. By tracing the rays through the two lenses, it will be seen that they enter the eye E parallel. If the object M N is near the observer, the image *m n* will be found at a greater distance from A B; and the eye glass C D must be drawn out from A B to obtain distinct vision of the image *m n*. Hence it is usual to fix the object glass A B at the end of a tube longer than its focal distance, and to place the eye glass C D in a small tube, called the eye tube, which will slide out of, and into, the larger tube, for the purpose of adjusting it to objects at different distances. The magnifying power of this telescope is equal to the focal length of the object glass divided by the focal length of the eye glass.

Fig. 164.



Telescopes of this construction were made by Campani Divini and Huygens, of the enormous length of 120 and 136 feet; and it was with instruments 12 and 24 feet long that Huygens discovered the ring and the fourth satellite of Saturn.

In order to use object glasses of such great focal lengths without the encumbrance of tubes, Huygens placed the object glass in a short tube at the top of a very long pole, so that the tube could be turned in every possible direction upon a ball and socket by means of a string, and brought into the same line with another short tube containing the eye glass, which he held in his hand.

As these telescopes were liable to all the imperfections arising from the aberration of refrangibility and that of spherical figure, they could not show objects distinctly when the aperture of the object glass was great; and on this account their magnifying power was limited. Huygens found that the following were the proper proportions:—

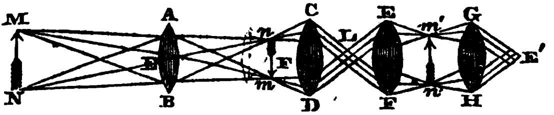
Focal length of the object glass.	Aperture of the object glass.	Focal length of the eye glass.	Magnifying power.
1 ft.	0·545 inches.	0·605	20
3	0·94	1·04	33½
5	1·21	1·33	44
10	1·71	1·88	62
50	3·84	4·20	140
100	5·40	5·95	197
120	5·90	6·52	216

In the astronomical telescope, the object, M N, is always seen inverted.

Terrestrial Telescope.

(206.) In order to accommodate this telescope to land objects which require to be seen erect, the instrument is constructed as in *fig.* 165., which is the same as the preceding one, with the addition of two lenses E F, G H, which have the same focal length as C D, and are placed at distances equal to double their common focal length. If the focal lengths are not equal, the distance of any two of them must be equal to the sum of their focal lengths. In this telescope the progress of the rays is exactly the same as in the astronomical one, as far as L, where the two pencils of parallel rays C L, D L cross in the anterior focus L of the second eye glass E F. These rays falling on E F form in its principal focus an erect image, $m' n$

Fig. 165.

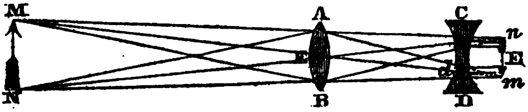


which is seen erect by the third eye glass G H, as the rays diverging from m' and n' in the focus of G H enter the eye in parallel pencils at E′. The magnifying power of this telescope is the same as that of the former when the eye glasses are equal.

Galilean Telescope.

(207.) This telescope, which is the one used by Galileo, differs in nothing from the astronomical telescope, excepting in a concave eye glass C D, *fig*. 166. being substituted for the convex one. The concave lens C D is placed between the image $m\,n$ and the object glass, so that the image is in the principal focus of the concave lens. The pencils of rays A B n, A B m fall upon C D, converging to its principal focus, and will therefore be refracted into parallel lines, which will enter the eye at E, and give distinct vision of the object. The magnifying power of this telescope is found by the same rule as that for the astronomical telescope: it gives a smaller and less agreeable field of view than the astronomical telescope, but it has the advantage of showing the object erect, and of giving more distinct vision of it. and of shortening the instrument.

Fig. 166.



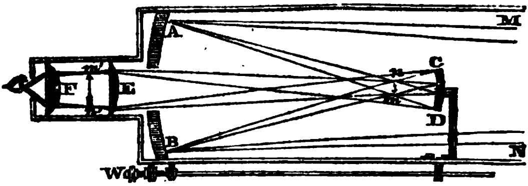
Gregorian Reflecting Telescope.

(208.) Father Zucchius seems to have been the first person who magnified objects by means of a lens and a concave speculum; but there is no evidence that he constructed a reflecting telescope with a small speculum.

James Gregory was the first who described the construction of this instrument, but he does not seem to have executed one; and the honor of doing this with his own hands was reserved for Sir Isaac Newton.

The Gregorian telescope is shown in *fig*. 167., where A B is a concave metallic speculum with a hole in its centre. For very remote objects the curve of the speculum should be a parabola. For nearer ones it should be an ellipse in whose farther focus is the object, and in whose nearer focus is the image; and in both these cases the speculum would be free from spherical aberration. But, as these curves cannot be communicated with certainty to specula, opticians are satisfied with giving to them a correct spherical figure. In front of the large speculum is placed a small concave one, C D, which can be moved nearer to and farther from the large speculum by means of the screw W at the side of the tube. This speculum should have its curvature elliptical, though it is generally made spherical. An eye-piece consisting of two convex lenses, E, F, placed at a distance equal to half the sum of their focal lengths, is screwed into the tube immediately behind the great speculum A B, and permanently fixed in that position. If rays M A, N B, issuing nearly parallel from the extremities M and N of a distant object, fall upon the speculum A B, they will form an inverted image of it at *m n*, as more distinctly shown in *fig.* 14.

Fig. 167.



If this image *m n* is farther from the small speculum C D than its principal focus, an inverted image of it, *m' n'*, or an erect image of the real object, since *m n* is itself an inverted one, will be formed somewhere between E and F, the rays passing through the opening in the speculum. This image *m' n'* might have been viewed and magnified by a convex eye glass at F, but it is preferable to receive the converging rays upon a lens E called the field glass, which hastens their convergence, and forms the image of *m n* in the focus of the lens F, by which they are magnified; or, what is the same thing, the pencils diverging from the image *m' n'* are refracted by F, so as to enter the eye parallel, and give distinct vision of the image. If the object M N is brought nearer the speculum A B, the image of it, *m n*, will recede from A B and approach to C D; and, consequently, the other image *m' n'* in the conjugate focus of C D will recede from its place *m' n'*, and cease to be seen distinctly. In order to restore it to its place *m' n'*, we have only to turn the screw W, so as to remove C D farther from A B, and consequently farther from *m n*, which will cause the image *m' n'* to appear perfectly distinct as before. The magnifying power of this telescope may be found by the following rule:—

Multiply the focal distance of the great speculum by the distance of the small mirror from the image next the eye, as formed in the anterior focus of the convex eye glass, and multiply also the focal distance of the small speculum by the focal distance of the eye glass. The quotient arising from dividing the former product by the latter will be the magnifying power.

This rule supposes the eye-piece to consist of a single lens.

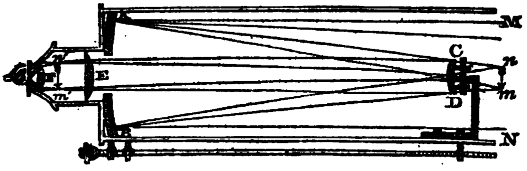
The following table, showing the focal lengths, apertures, powers, and prices of some of Short's telescopes, will exhibit the great superiority of reflecting telescopes to refracting ones:—

Focal lengths in feet.	Aperture in inches.	Magnifying powers.	Price in guineas.
1	3·0	35 to 100	14
2	4·5	90 300	35
3	6·3	100 400	75
4	7·6	120 500	100
7	12·2	200 800	300
12	18·0	300 1200	800

Cassegrainian Telescope.

(209.) The *Cassegrainian telescope*, proposed by M. Cassegrain, a Frenchman, differs from the Gregorian only in having its small speculum C D, *fig.* 168., convex instead of concave. The speculum is therefore placed before the image *m n* of the object M N, and an image of M N will be formed at *m' n'* between E and F as in the Gregorian instrument. The advantage of this form is, that the telescope is shorter than the Gregorian by more than twice the focal length of the small speculum; and it is generally admitted that it gives more light, and a distincter image, in consequence of the convex speculum correcting the aberration of the concave one.

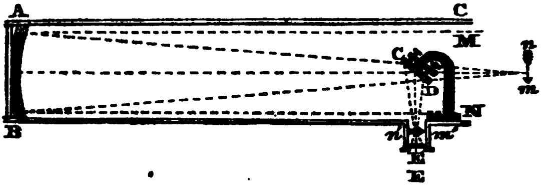
Fig. 168.



Newtonian Telescope.

(210.) The Newtonian telescope, which may be regarded as an improvement upon the Gregorian one, is represented in *fig.* 169., where A B is a concave speculum, and *m n* the inverted image which it forms of the object from which the rays M, N proceed. As it is impossible to introduce the eye into the tube to view this image without obstructing the light which comes from the object, a small plane speculum C D, inclined 45° to the axis of the large speculum, and of an oval form, its axes being to one another as 7 to 5, is placed between the speculum and the image *m n*, in order to reflect it to a side at *m' n'*, so that we can magnify it with an eye glass E, which causes the rays to enter the eye parallel. The small mirror is fixed upon a slender arm, connected with a slide, by which the mirror may be made to approach to or recede from the large speculum A B, according as the image *m n* approaches to or recedes from it. This adjustment might also be effected by moving the eye lens E to or from the small speculum. The magnifying power of this telescope is equal to the focal length of the great speculum divided by that of the eye glass.

Fig. 169.



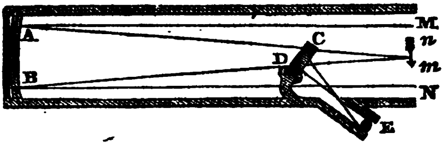
As about half of the light is lost in metallic reflexions, Sir Isaac Newton proposed to substitute, in place of the metallic speculum, a rectangular prism A B C, *fig.* 148., in which the light suffers total reflexion. For this purpose, however, the glass requires to be perfectly colorless and free from veins, and hence such a prism has rarely been used. Sir Isaac also proposed to make the two faces of the prism convex, as D E F, *fig.* 148., and by placing it between the image *m n* and the object, he not only erected the image, but was enabled to vary the magnifying power of the telescope. The original telescope, constructed by Sir Isaac's own hands, is preserved in the library of the Royal Society.

The following table shows the dimensions of Newtonian telescopes, which we have computed by taking a fine telescope made by Hawksbee as a standard:—

Focal length of great speculum.	Aperture of speculum.	Focal length of eye glass.	Magnifying power.
1 ft.	2·28 inches.	0·129 inches.	93
2	3·79	0·152	158
3	5·14	0·168	214
4	6·36	0·181	265
6	8·64	0·200	360
12	14·50	0·238	604
24	24·41	0·283	1017

(211.) On account of the great loss of light in metallic reflexions, which, according to the accurate experiments of Mr. R. Potter, amounts to 45 rays in every 100, at an incidence of 45°,* and the imperfections of reflexion, which even with perfect surfaces make the rays stray *five* or *six* times more than the same imperfections in refracting surfaces, I have proposed to construct the Newtonian telescope, as shown in *fig*. 170., where A B is the concave speculum, *m n* the image of the object M N, and C D an achromatic prism, which refracts the image *m n* into an oblique position, so that it can be viewed by the eye at E through a magnifying lens. Nothing more is required by the prism than to turn the rays as much aside as will enable the observer to see the image without obstructing the rays from the object M N. As the prisms of crown and flint glass which compose the achromatic prism may be cemented by a substance of intermediate refractive power, no more light will be lost than what is reflected at the two surfaces.

Fig. 170.



In place of setting the small speculum, C D, of the Newtonian telescope, *fig*. 169., at 45°, to the incident rays, I have proposed to place it much more obliquely, so as to reflect the image *m n*, *fig*. 170., out of the way of the observer, and no farther. This would of course require a plane speculum, C D, of much greater length; but the greater obliquity of the reflexion would more than compensate for this inconvenience. It might be advisable, indeed, to use a small speculum of dark glass, of a high refractive power, which at great incidences reflects as much light as metals, and which is capable of being brought to a much finer surface. The fine surfaces of some crystals, such as ruby silver, oxide of tin, or diamond, might be used.

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* *Edinburgh Journal of Science*, No. VI., new series, p. 283.

A Newtonian reflector, *without an eye glass*, may be made by using a reflecting glass prism, with one or both of its surfaces concave, when the prism is placed between the image *m n* and the great speculum, so as to reflect the rays parallel to the eye. The magnifying power will be equal to the focal length of the great speculum, divided by the radius of the concave surface of the prism if both the surfaces are concave, and of equal concavity, or by twice the radius, if only one surface is concave.

Sir William Herschel's Telescope.

(212.) The fine Gregorian telescopes executed by Short were so superior to any other reflectors, that the Newtonian form of the instrument fell into disuse. It was revived, however, by Sir W. Herschel, whose labors form the most brilliant epoch in optical science. With an ardor never before exhibited, he constructed no fewer than 200 seven feet Newtonian reflectors, 150 ten feet, and 80 twenty feet in focal length. But his zeal did not stop here. Under the munificent patronage of George III., he began, in 1785, to construct a telescope *forty feet* long, and on the 27th of August, 1789, the day on which it was completed, he discovered with it the sixth satellite of Saturn.

The great speculum had a diameter of 49½ inches, but its concave surface was only 48 inches. Its thickness was about 3½ inches, and its weight when cast was 2118 lbs. Its focal length was forty feet, and the length of the sheet iron tube which contained it was 39 feet 6 inches, and its breadth 4 feet 10 inches. By using small convex lenses, Dr. Herschel was enabled to apply a power of 6450 to the fixed stars, but a very much lower power was in general used.

In this telescope the observer sat at the mouth of the tube, and observed by what is called the *front view*, with his back to the object, without using a plane speculum, the eye lens being applied directly to magnify the image formed by the great speculum. In order to prevent the head, &c. from obstructing too much of the incident light, the image was formed out of the axis of the speculum, and must, therefore, have been slightly distorted.

As the frame of this instrument was exposed to the weather, it had greatly decayed. It was, therefore, taken down, and another telescope, of 20 feet focus, with a speculum 18 inches in diameter, was erected in its place, in 1822, by J. F. W. Herschel, Esq., with which many important observations have been made.

Mr. Ramage's Telescope.

(213.) Mr. Ramage, of Aberdeen, has constructed various Newtonian telescopes, of great lengths and high powers. The largest instrument at present in use in this country, and we believe in Europe, was constructed by him, and erected at the Royal Observatory of Greenwich in 1820. The great speculum has a focal length of 25 feet, and a diameter of 15 inches. The image is formed out of the axis of the speculum, which is inclined so as to throw it just to the side of the tube, where the observer can view it without obstructing the incident rays. The tube is a 12-sided prism of deal, and when the instrument is not in use it is lowered into a box, and covered with canvas. The apparatus for moving and directing the telescope is extremely simple, and displays much ingenuity.

CHAP. XLIII.

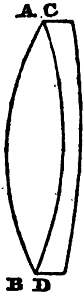
ON ACHROMATIC TELESCOPES.

(214.) THE principle of the achromatic telescope has been briefly explained in Chap. VII., and we have there shown how a convex lens, combined with a concave lens of a longer focus, and having a higher refractive and dispersive power, may produce refraction without color, and consequently form an image free from the primary prismatic colors. It has been demonstrated mathematically, and the reader may convince himself of its truth by actually tracing the rays through the lenses, that a convex and a concave lens will form an achromatic combination, or will give a colorless image, when their focal lengths are in the same proportion as their dispersive powers. That is, if the dispersive powers of crown and flint glass are as 0·60 to 1, or 6 to 10; then an achromatic object glass could be formed by combining a convex crown glass lens of 6, or 60, or

600 inches with a concave flint glass lens of 10, or 100, or 1000 inches in focal length.

But though such a combination would form an image free from color, it would not be free from spherical aberration, which can only be removed by giving a proper proportion to the curvatures of the first and last surface, or the two outer surfaces of the compound lens. Mr. Herschel has found that a double object glass will be nearly free from aberration, provided the radius of the exterior surface of the crown lens be 6·72, and of the flint 14·20, the focal length of the combination being 10·00, and the radii of the interior surfaces being computed from these data by the formulæ given in elementary works on optics, so as to make the focal lengths of the two glasses in the direct ratio of their dispersive powers. This combination is shown in *fig.* 171., where A B is the convex lens of crown glass, placed on the outside towards the object, and C D the concavo-convex lens of flint glass placed towards the eye. The two inside surfaces that come in contact are so nearly of the same curvature that they may be ground on the same tool, and united together by a cement to prevent the loss of light at the two surfaces.

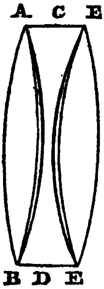
Fig. 171.



In the double achromatic object glasses constructed previous to the publication of Mr. Herschel's investigations, the surface of the concave lens next the eye was, we believe, always concave.

Triple achromatic object glasses consist of three lenses A B, C D, E F, *fig.* 172., A B and E F being convex lenses of crown glass, and C D a double concave lens of flint glass.

Fig. 172.



The object of using three lenses was to obtain a better correction of the spherical aberration; but the greater complexity of their construction, the greater risk of imperfect centering, or of the axes of the three lenses not being in the same straight line, together with the loss of light at six surfaces, have been considered as more than compensating their advantages; and they have accordingly fallen into disuse.

The following were the radii of two triple achromatic object glasses, as constructed by Dollond:—

	FIRST OBJECT GLASS.	SECOND OBJECT GLASS.
A B, or first Crown Lens.		
Radii of first surface,	28 inches	28
——— second surface,	40	35·5
C D, or Flint Lens.		
Radii of first surface,	20·9	21·1
——— second surface,	28	25·75
E F, or second Crown Lens.		
Radii of first surface,	28·4	28
——— second surface,	28·4	28
Focal length of the compound lens,	46 inches	46·3

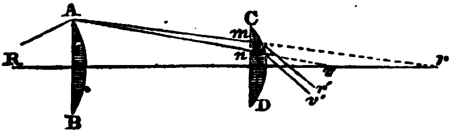
In consequence of the great difficulty of obtaining flint glass free from veins and imperfections, the largest achromatic object glasses constructed in England did not greatly exceed 4 or 5 inches in diameter. The neglect into which this important branch of our national manufactures was allowed to fall by the ignorance and supineness of the British government, stimulated foreigners to rival us in the manufacture of achromatic telescopes. M. Guinand of Brenetz, in Switzerland, and M. Fraunhofer, of Munich, successively devoted their minds to the subject of making large lenses of flint glass, and both of them succeeded. Before his death, M. Fraunhofer executed two telescopes with achromatic object glasses of $9\frac{9}{12}$ inches, and 12 inches in diameter; and he informed me that he would undertake to execute one 18 inches in diameter. The first of these object glasses was for the magnificent achromatic telescope ordered by the emperor of Russia, for the observatory at Dorpat. The object glass was a double one, and its focal length was 25 feet; it was mounted on a metallic stand which weighed 5000 Russian pounds. The telescope could be moved by the slightest force in any direction, all the movable parts being balanced by counter weights. It had four eye glasses, the lowest of which magnified 175, and the highest 700 times. Its price was 1300*l*., but it was liberally given at prime cost, or 950*l*. The object glass, 12 inches in diameter, was made for the king of Bavaria, at the price of 2720*l*.; but as it was not perfectly complete at the time of Fraunhofer's death, we do not know that it is at present in use. In the hands of that able observer, Professor Struve, the telescope of Dorpat has already made many important discoveries in astronomy.

A French optician, we believe, M. Lerebours, has more recently executed two achromatic object glasses of glass made by Guinand. One of them is nearly 12 inches in diameter, and another above 13 inches. The first of these object glasses was mounted as a telescope at the Royal Observatory of Paris; and the French government had expended 500*l.* in the purchase of a stand for it, but had not the liberality to purchase the object glass, itself. Sir James South, our liberal and active countryman, saw the value of the two object glasses, and acquired them for his observatory at Kensington.

ON ACHROMATIC EYEPIECES.

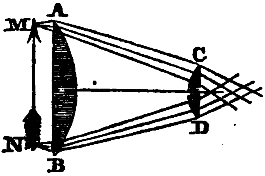
(215.) Achromatic eyepieces when one lens only is wanted, may be composed of two or three lenses exactly on the same principles as object glasses. Such eyepieces, however, are never used, because the color can be corrected in a superior manner, by a proper arrangement of single lenses of the same kind of glass. This arrangement is shown in *fig.* 173., where A B and C D are two plano-convex lenses, A B being the one next the object glass, and C D the one next the eye, a ray of white light R A, proceeding from the achromatic object glass, will be refracted by A B at A, so that the red ray A r crosses the axis at r, and the violet ray A v at v. But these rays being intercepted by the second lens C D at the points m, n, at different distances from the axis, will suffer different degrees of refraction. The red ray $m\,r$ suffering a greater refraction than the violet one $n\,v$, notwithstanding its inferior refrangibility, so that the two rays will emerge parallel from the lens C D (and therefore be colorless) as shown at $m\,r'$, $m\,v'$.

Fig. 173.



When these two lenses are made of crown glass, they must be placed at a distance equal to half the sum of their focal lengths, or, what is more accurate, their distance must be equal to half the sum of the focal distance, of the eye glass C D, and the distance at which the field glass A B would form an image of the object glass of the telescope. This eyepiece is called the *negative eyepiece.* The stop or diaphragm must be placed half-way between the two lenses. The focal length of an equivalent lens, or one that has the same magnifying power as the eyepiece, is equal to twice the product of the focal lengths of the two lenses divided by the sum of the same numbers.

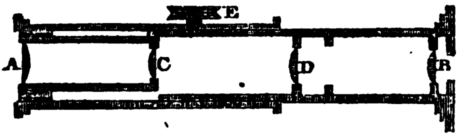
Fig. 174.



An eyepiece nearly achromatic, called *Ramsden's Eyepiece*, and much used in transit instruments and telescopes with micrometers, is shown in *fig*. 174., where A B, C D, are two plano-convex lenses with their convex sides inwards. They have the same focal length, and are placed at a distance from each other, equal to two-thirds of the focal length of either. The focal length of an equivalent lens is equal to three-fourths the focal length of either lens. The use of this eyepiece is to give a flat field, or a distinct view of a system of wires placed at M N. This eyepiece is not quite achromatic, and it might be rendered more so by increasing the distance of the lenses; but as this would require the wires at M N to be brought nearer A B, any particles of dust or imperfections in the lens A B would be seen magnified by the lens C D.

The erecting achromatic eyepiece now in universal use in all achromatic telescopes for land objects is shown in *fig*. 175. It consists of four lenses, A, C, D, B, placed as in the figure.

Fig. 175.



Mr. Coddington has shown, that if the focal lengths, reckoning from A, are as the numbers 3, 4, 4 and 3, and the distances between them on the same scale 4, 6, and 5·2, the radii, reckoning from the outer surface of A, should be thus:—

A	First surface	27	nearly plano-convex.
	Second surface	1	
C	First surface	9	a meniscus.
	Second surface	4	
D	First surface	1	nearly plano-convex.
	Second surface	21	
B	First surface	1	Double convex.
	Second surface	24	

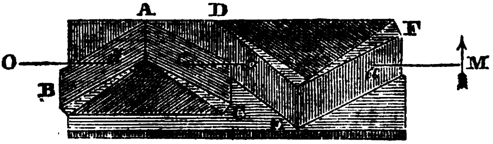
The magnifying power of this eyepiece, as usually made, differs little from what would be produced by using the first or fourth lens alone. I have shown, that the magnifying power of this eyepiece may be increased or diminished by varying the distance between C and D, which even in common eyepieces of this kind may be done, as A and C are placed in one tube A C, and D and B in another tube D B, so that the latter can be drawn out of the general tube. In *fig.* 175., I have shown the eyepiece constructed in this way, and capable of having its two parts separated by a screw nut E, and rack. This contrivance for obtaining a variable magnifying power, and consequently of separating optically a pair of wires fixed before the eye glass, I communicated to Mr. Carey in 1805, and had one of the instruments constructed by Mr. Adie in 1806. It is fully described in my *Treatise on Philosophical Instruments*, and has been more recently brought out as a new invention by Dr. Kitchener, under the name of the *Pancratic Eye Tube.*

Prism Telescope.

(216.) In 1812, I showed that colorless refraction may be produced by combining two prisms of the same substance, and the experiments which led to this result were published in my *Treatise on New Philosophical Instruments* in 1813. The practical purposes to which this singular principle seemed to be applicable were the construction of an achromatic telescope with lenses of the same glass, and the construction of a *Teinoscope*, for extending or altering the lineal proportions of objects.

If we take a prism, and hold its refracting edge downwards and horizontal, so as to see through it one of the panes of glass in a window, there will be found a position, namely, that in which the rays enter the prism and emerge from it at equal angles, as in *fig.* 20., where the square pane of glass is of its natural size. If we turn the refracting edge towards the window, the pane will be extended or magnified in its length or vertical direction, while its breadth remains the same. If we now take the same prism and hold its refracting edge vertically, we shall find, by the same process, that the pane of glass is extended or magnified in breadth. If two such prisms, therefore, are combined in these positions, so as to magnify the same *both* in *length* and *breadth*, we have a telescope composed of two prisms, but unfortunately the objects are all highly fringed with the prismatic colors. We may correct these colors in three ways: 1st, We may make the prisms of a kind of glass which obstructs all the rays but those of one homogeneous color; or, we may use a piece of the same glass to absorb the other rays when two common glass prisms are used: 2d, We may use achromatic prisms in place of common prisms: or, 3d, What is best of all for common purposes, we may place other two prisms exactly similar, but in reverse positions, or they may be placed as shown in *fig.* 176., which represents the prism telescope; A B and A C being two prisms of the same kind of glass, and of the same refracting angles, with their planes of refraction vertical, and E D, E F, other two perfectly similar prisms, similarly placed, but with their planes of refraction horizontal. A ray of light, M *a*, from an object, M, enters the first prism, E F, at *a*, emerges from the second prism, E D, at *b*, enters the third prism, A C, at *c*, emerges from the fourth prism, A B, at *d*, and enters the eye at O. The object, M, is extended or magnified horizontally by each of the two prisms, E F, E D, and vertically by each of the two prisms, A B, A C; objects are magnified by looking through the prisms.

Fig. 176.



This instrument was made in Scotland by the writer of this Treatise, under the name of a *Teinoscope*, and also by Dr. Blair, before it was proposed or executed by Professor Amici of Modena. Dr. Blair's model is now before me, being composed of four prisms of plate glass with refracting angles of about 15°. It was presented to me two years ago by his son; but as no account of it was ever published, Mr. Blair could not determine the date of its construction.

In constructing this instrument, the perfect equality of the four prisms is not necessary. It will be sufficient if A B and D E are equal, and A C and E F, as the color of the one prism can be made to correct that of the other by a change in its position. For the same reason it is not necessary that they be all made of the same kind of glass.

Achromatic Opera Glasses with Single Lenses.

(217.) M. d'Alembert has long ago shown that an achromatic telescope may be constructed with a single object glass and a single eye glass of different refractive and dispersive powers. To effect this, the eye glass must be concave, and be made of glass of a much higher dispersive power than that of which the object glass is made; but the proposal was quite Utopian at the time it was suggested, as substances with a sufficient difference of dispersive power were not then known. Even now, the principle can be applied only to opera glasses.

If we use an object glass of very low dispersive power, the refraction of the violet rays may be corrected by a concave eye lens of a high dispersive power, as will be seen by the following table.

Object glass made of	Eye glass made of	Magnifying power.
Crown glass	Flint glass	1¼
Water	Oil of cassia	2
Rock crystal	Flint glass	2
Rock crystal	Oil of aniseseed	3
Crown glass	Oil of cassia	3
Rock crystal	Oil of cassia	6

Although all the rays are made to enter the eye parallel in these combinations, yet the correction of color is not satisfactory.

Mr. Barlow's Achromatic Telescope.

(218.) In the year 1813 I discovered the remarkable dispersive power of sulphuret of carbon, having found that it "exceeds all fluid bodies in refractive power, surpassing even flint glass, topaz, and tourmaline; and that in dispersive power it exceeds every fluid substance except oil of cassia, holding an intermediate place between phosphorus and balsam of tolu. * * * Although oil of cassia surpasses the sulphuret of carbon in its power of dispersion, yet, from the yellow color with which it is tinged, it is greatly inferior to the latter as an optical fluid, unless in cases where a very thin concave lens is required. The extreme volatility of the sulphuret is undoubtedly a disadvantage; but as this volatility may be restrained, we have no hesitation in *considering the sulphuret of carbon as a fluid of great value in optical researches, and which may be of incalculable service in the construction of optical instruments.*"* This anticipation has been realized by Mr.

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* On the Optical Properties of Sulphuret of Carbon, in *Edinburgh Trans.* vol. viii. p. 285. Feb. 7. 1814.

Barlow, who has employed sulphuret of carbon as a substitute for flint glass, in correcting the dispersion of the convex lens. It had been proposed, and the experiment even tried, to place the concave lens between the convex one and its focus, for the purpose of correcting the dispersion of the convex lens, with a lens of less diameter, but Mr. Barlow has the merit of having first carried this into effect.

The telescope which he has made on this principle, consists of a single object lens of plate glass, 7·8 inches in clear aperture, with a focal length of 78 inches. At the distance of 40 inches from this lens was placed a concave lens of sulphuret of carbon, with a focal length of 59·8 inches, so that parallel rays falling on the convex plate lens, and converging to its focus, would, when refracted by the fluid concave lens, have their focus at the distance of 104 inches from the fluid lens, and 144 inches, or 12 feet, from the plate glass lens. The fluid is contained between two meniscus cheeks, and a glass ring, so that the radius of the concave fluid lens is 144 inches towards the eye, and 56·4 towards the object lens. The fluid is put in at a high temperature, and the contraction which it experiences in cooling is said to keep every thing perfectly tight. No decomposition of the fluid has yet been observed. The great secondary spectrum which I found to exist in sulphuret of carbon is approximately corrected by the distance of the fluid lens from the object glass; but we are persuaded that it is not free from secondary color. Mr. Coddington remarks, that the general course of an oblique pencil is bent outward by the fluid lens, and the violet rays more than the red, so as to produce indistinctness; but we are not aware that this defect was observed in the instrument. The tube of the telescope is 11 feet, and the eyepieces one foot. "The telescope," says Mr. Barlow, "bears a power of 700 on the closest double stars in South's and Herschel's catalogue, although the field is not then so bright as I could desire. Venus is beautifully white and well defined with a power of 120, but shows some color with 360. Saturn, with the 120 power, is a very brilliant object, the double ring and belts being well and satisfactorily defined, and with the 360 power it is still very fine." Mr. Barlow remarks, also, that the telescope is not so competent to the opening of the close stars, as it is powerful in bringing to light the more minute luminous points.

Achromatic Solar Telescopes with single Lenses.

(219.) An achromatic telescope for viewing the sun or any highly luminous object may be constructed by using a single object glass of plate glass; and by making any one of the eye glasses out of a piece of glass which transmits only *homogeneous* light: or the same thing may be effected by a piece of plane glass of the same color; but this introduces the errors of other two surfaces. In such a construction it would be preferable to absorb all the rays but the red; and there are various substances by which this may be readily effected. The object glass of this telescope, though thus rendered monochromatic, will still be liable to spherical aberration. But if the radii of the lens are properly adjusted, the excess of solar light will permit us to diminish the aperture, so as to render the spherical aberration almost imperceptible. Such a telescope, when made of a great length, would, we are persuaded, be equal to any instrument that has yet been directed to the sun. If we could obtain a solid or a fluid which would absorb all the other rays of the spectrum but the *yellow*, with as little loss as there is in red glasses, a telescope of the preceding construction would answer for day objects, and for all the purposes of astronomy. If the art of giving lenses a hyperbolic form shall be brought to perfection, which we have no doubt will yet be done, the spherical aberration would disappear; and a telescope upon this principle would be the most perfect of all instruments.

Even by using red light only, a great improvement might be effected in the common telescopes for day objects and for astronomical purposes. If the red rays, for example, form $\frac{1}{10}$th of white light, we have only to increase the area of the aperture 10 times to make up completely for this defect of light. The spherical aberration is, no doubt, greatly increased also: but if we consider that, when compared to the aberration of color, it is only as 1 to 1200, we can afford to increase it in order to gain so great an advantage. Common telescopes, indeed, may be considerably improved by applying colored glasses, which absorb only the *extreme* rays of the spectrum, even though they do not produce an achromatic or homogeneous image.

These observations are made for the benefit of those who cannot afford expensive instruments, but who may yet wish to devote themselves to astronomical observations, with the ordinary instruments which they may happen to possess.

On the Improvement of imperfectly achromatic Telescopes.

(220.) There are many achromatic telescopes of considerable size, in which the flint lens either over corrects or under corrects the colors of the crown glass lens. This defect may be easily removed by altering slightly the curvature of one or other of the lenses. But all achromatic telescopes whatever, when made of crown and flint glass, exhibit the secondary colors, viz. the *wine-colored* and the *green* fringes. These colors are not very strong; and in many, if not in all cases, we may destroy them by *absorption* through glasses that will not weaken greatly the intensity of the light. The glasses requisite for this purpose must be found by actual experiment; as the secondary tints, though generally of the colors we have mentioned, are variously composed, according to the nature of the glass of which the two lenses are made.

June 3d 1841.
[illegible]

APPENDIX OF THE AUTHOR,

CONTAINING

TABLES OF REFRACTIVE AND DISPERSIVE POWERS, &c.

OF DIFFERENT MEDIA.

TABLE I.

(Referred to from Page 30.)

Table of the Refractive Powers of Solid and Fluid Bodies.

	Index of Refraction.		Index of Refraction.
Realgar artificial	2·549	Amber	1·547
Octohedrite	2·500	Plate glass, from 1·514 to	1·542
Diamond	2·439	Crown glass, from 1·525 to	1·534
Nitrite of lead	2·322	Oil of cloves	1·535
Blende	2·260	Balsam capivi	1·528
Phosphorus	2·224	Gum arabic	1·502
Sulphur melted	2·148	Oil of beech nut	1·500
Zircon	1·961	Castor oil	1·490
Glass—lead 2 parts, flint 1 part	1·830	Cajeput oil	1·483
Garnet	1·815	Oil of turpentine	1·475
Ruby	1·779	Oil of olives	1·470
Glass—lead 3 parts, flint 1 part	2·028	Alum	1·457
Sapphire	1·794	Fluor Spar	1·434
Spinelle	1·764	Sulphuric acid	1·434
Cinnamon stone	1·759	Nitric acid	1·410
Sulphuret of carbon	1·768	Muriatic acid	1·410
Oil of cassia	1·641	Alcohol	1·372
Balsam of Tolu	1·628	Cryolite	1·349
Guaiacum	1·619	Water	1·336
Oil of aniseseed	1·601	Ice	1·309
Quartz	1·548	Fluids in minerals 1·294 to	1·131
Rock salt	1·557	Tabasheer	1·111
Sugar melted	1·554	Ether expanded to thrice its volume	1·057
Canada balsam	1·549	Air	1·000294

Table of the Refractive Powers of Gases.

	Index of Refraction.		Index of Refraction.
Vapor of sulphuret of carbon	1·001530	Carbonic acid	1·000449
Phosgene gas	1·001159	Carburetted hydrogen	1·000443
Cyanogen	1·000834	Ammonia	1·000385
Chlorine	1·000772	Carbonic oxide	1·000340
Olefiant gas	1·000678	Nitrous gas	1·000303
Sulphurous acid	1·000665	Azote	1·000300
Sulphuretted hydrogen	1·000644	Atmospheric air	1·000294
Nitrous oxide	1·000503	Oxygen	1·000272
Hydrocyanic acid	1·000451	Hydrogen	1·000138
Muriatic acid	1·000449	Vacuum	1·000000

TABLE II.

(Referred to from Page 31.)

Table of the Absolute Refractive Powers of Bodies.

	Index of Refraction.
Tabasheer	0·0976
Cryolite	0·2742
Fluor spar	0·3426
Oxygen	0·3799
Sulphate of baryta	0·3829
Sulphurous acid gas	0·4455
Nitrous gas	0·4491
Air	0·4528
Carbonic acid	0·4537
Azote	0·4734
Chlorine	0·4813
Nitrous oxide	0·5078
Phosgene	0·5188
Selenite	0·5386
Carbonic oxide	0·5387
Quartz	0·5415
Glass	0·5436
Muriatic acid	0·5514
Sulphuric acid	0·6124
Calcareous spar	0·6424
Alum	0·6570
Borax	0·6716
Nitre	0·7079
Rain water	0·7845
Flint glass	0·7986
Cyanogen	0·8021
Sulphuretted hydrogen	0·8419
Vapor of sulphuret of carbon	0·8743
Ammonia	1·0032
Alcohol rectified	1·0121
Camphor	1·2551
Olive oil	1·2607
Amber	1·3654
Octohedrite	1·3816
Sulphuret of carbon	1·4200
Diamond	1·4566
Realgar	1·6666
Ambergris	1·7000
Oil of cassia*	1·7634
Sulphur	2·2000
Phosphorus	2·8857
Hydrogen	3·0953

No. I.

(Referred to from Page 72.)

In order to convey to the reader some idea of the variety of dispersive powers which exist in solid and fluid bodies, I have given the following table, selected from a much larger one, founded on observations which I made in 1811 and 1812.†

The first column contains the difference of the indices of refraction for the extreme red and violet rays, or the part of the whole refraction to which the dispersion is equal; and the second column contains the dispersive power.

Table of the Dispersive Powers of Bodies.

	Dispersive power.	Diff. of Indices of Refraction for extreme Rays
Oil of cassia	0·139	0·089
Sulphur after fusion	0·130	0·149
Phosphorus	0·128	0·156
Sulphuret of carbon	0·115	0·077
Balsam of Tolu	0·103	0·065
Balsam of Peru	0·093	0·058

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* See *Edinburgh Journal of Science*, No. XX. p. 308.

† See my *Treatise on New Philosophical Instruments*, p. 315.

	Dispersive power.	Diff. of Indices of Refraction for extreme Rays.
Barbadoes aloes	0·085	0·058
Oil of bitter almonds	0·079	0·048
Oil of aniseseed	0·077	0·044
Acetate of lead melted	0·069	0·040
Balsam of Styrax	0·067	0·039
Guaiacum	0·066	0·041
Oil of cumin	0·065	0·033
Oil of tobacco	0·064	0·035
Gum ammoniac	0·063	0·037
Oil of Barbadoes tar	0·062	0·032
Oil of cloves	0·062	0·033
Oil of sassafras	0·060	0·032
Rosin	0·057	0·032
Oil of sweet fennel seeds	0·055	0·028
Oil of spearmint	0·054	0·026
Rock salt	0·053	0·029
Caoutchouc	0·052	0·028
Oil of pimento	0·052	0.020
Flint glass	0·052	0·026
Oil of angelica	0·051	0·025
Oil of thyme	0·050	0·024
Oil of caraway seeds	0·049	0.024
Flint glass	0·048	0·029
Gum thus	0·048	0·028
Oil of juniper	0·047	0·022
Nitric acid	0.045	0·019
Canada balsam	0·045	0·021
Cajeput oil	0·044	0·021
Oil of rhodium	0·044	0·022
Oil of poppy	0·044	0·022
Zircon, greatest ref.	0·044	0·045
Muriatic acid	0·043	0·016
Gum copal	0·043	0·024
Nut oil	0·043	0·022
Oil of turpentine	0·042	0·020
Feldspar	0·042	0·022
Balsam capivi	0·041	0·021
Amber	0·041	0·023
Calcareous spar — greatest	0·040	0·027
Oil of rape-seed	0·040	0·019
Diamond	0·038	0·056
Oil of olives	0·038	0·018
Gum mastic	0·038	0·022
Oil of rue	0·037	0·016
Beryl	0·037	0·022
Ether	0·037	0·012
Selenite	0·037	0·020
Alum	0·036	0·017
Castor oil	0·036	0·018
Crown glass, green	0·036	0·020
Gum arabic	0·036	0·018
Water	0·035	0·012
Citric acid	0·035	0·019
Glass of Borax	0·034	0·018

	Dispersive power.	Diff. of Indices of Refraction for extreme Rays.
Garnet	0·034	0·018
Chrysolite	0·033	0·022
Crown glass	0·033	0·018
Oil of wine	0·032	0·012
Glass of phosphorus	0·031	0·017
Plate glass	0·032	0·017
Sulphuric acid	0·031	0·014
Tartaric acid	0·030	0·016
Nitre, least ref.	0·030	0·009
Borax	0·030	0·014
Alcohol	0·029	0·011
Sulphate of baryta	0·029	0·011
Rock crystal	0·026	0·014
Borax glass (1 bor. 2 silex)	0·026	0·014
Blue sapphire	0·026	0·021
Bluish topaz	0·025	0·016
Chrysoberyl	0·025	0·019
Blue topaz	0·024	0·016
Sulphate of strontia	0·024	0·015
Prussic acid	0·027	0·008
Fluor spar	0·022	0·010
Cryolite	0·022	0·007

No. II.

(Referred to from Page 73.)

The following table contains the results of several experiments which I made in the manner described in pp. 72, 73. The bodies at the top of the table have the least action upon green light, and those at the bottom of it the greatest. The relative position of some of the substances is empirical; but, by referring to the original experiments in my *Treatise on New Philosophical Instruments*, p. 354., it will be seen whether or not the relative action of any two bodies upon green light has been determined.

Table of Transparent Bodies, in the order in which they exercise the least action upon Green Light.

OIL OF CASSIA.
Sulphur.
Sulphuret of carbon.
Balsam of Tolu.
Oil of bitter almonds.
Oil of aniseseed.
Oil of cumin.
Oil of sassafras.
Oil of sweet fennel seeds.
Oil of cloves.
Canada balsam.
Oil of turpentine.
Oil of poppy.
Oil of spearmint.
Oil of caraway seeds.
Oil of nutmeg.
Oil of peppermint.
Oil of castor.
Gum copal.
Diamond.
Nitrate of potash.
Nut oil.
Balsam of capivi.
Oil of rhodium.
FLINT GLASS.
Zircon.

Table of Transparent Bodies, &c.—continued.

Oil of olives.
Calcareous spar.
Rock salt.
Gum juniper.
Oil of almonds.
CROWN GLASS.
Gum arabic.
Alcohol.
Ether.
Glass of borax.
Selenite.
Beryl.
Topaz.
Fluor spar.
Citric acid.
Acetic acid.
Muriatic acid.
Nitric acid.
Rock crystal.
Ice.
WATER.
Phosphorous acid.
SULPHURIC ACID.

No. III.

(Referred to from Page 80.)

Table of the Indices of Refraction of several Glasses and Fluids.

Refracting Media.	Spec. Grav.	Indices of Refraction for the Seven Rays in the Spectrum marked in *fig.* 86. with the following Letters.						
		B Red ray.	C Red ray.	D Orange.	E Green.	F Blue.	G Indigo.	H Violet.
Water . . .	1·000	1·330935	1·331712	1·333577	1·335851	1·337818	1·341293	1·344177
Solution of Potash	1·416	1·399629	1·400515	1·402805	1·405632	1·408082	1·412579	1·416368
Oil of Turpentine	0·885	1·470496	1·471530	1·474434	1·478353	1·481736	1·488198	1·493874
Crown Glass	2·535	1·525832	1·526849	1·529587	1·533005	1·536052	1·541657	1·546566
Crown Glass	2·756	1·554774	1·555933	1·559075	1·563150	1·566741	1·573535	1·579470
Flint Glass	3·723	1·627749	1·629681	1·635036	1·642024	1·648260	1·660285	1·671062
Flint Glass	3·512	1·602042	1·603800	1·608494	1·614532	1·620042	1·630772	1·640373

NOTES

BY

THE AMERICAN EDITOR.

No. I.

(Referred to from article 32.)

If the remark of Mr. Herschel be admitted, the consequence may be drawn in relation to all the simple gases, except oxygen, that their absolute refractive powers will be expressed by the square of the index of refraction, diminished by unity: for in them, the specific gravity is directly proportional to the weight of the atom. The same remark applies to the vapors of simple bodies, and to many compound gases.

If the specific gravity, and weight of the atom, of hydrogen be called unity, the specific gravity of nitrogen, chlorine, &c. will be expressed by the weight of the atom of each: hence the square of the index of refraction diminished by unity, will be, by the process directed in article 32, multiplied and divided by the same quantity.

The inflammable substance hydrogen, instead of presenting a high intrinsic refractive power, would occupy a low place on the scale, while chlorine would rank high upon it. This consequence was observed by Mr. Herschel himself.

No. II.

(Referred to from article 66, page 67.)

This remark in relation to the absorptive power of water, though true for moderate thicknesses, in relation to the colored rays of the spectrum, appears, by a recent discovery of Signor Melloni, not to be true in regard to the heating rays. An account of the interesting experiments which have established this fact, will be more in place, in connexion with the article which treats of the heating power of the spectrum.

No. III.

(Referred to from article 66, page 70.)

This interesting analysis of the solar spectrum, by Sir David Brewster, will, probably, have its value to the reader increased by a brief statement of the experiments from which the results, given in pages 69 and 70, were deduced. The matter of this note is taken from a paper, by Sir David Brewster, in the Edinburgh Journal of Science for October, 1831.*

1. The first position is that, "*red*, *yellow*, and *blue* light exist at every point of the solar spectrum." The eye gives evidence of the existence of *red* light, in the red, orange, and violet spaces, which, together, constitute more than half the length of the spectrum. If the blue and

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* And *Edinburgh Trans.* Vol. XII. Part. I.

indigo spaces be transmitted through olive oil, the light becomes of a violet tint, rendering evident the red, previously existing in the blue and indigo, by absorbing rays which had neutralized it. In the yellow and green spaces the existence of red is proved by showing that white light may be detected in them.

Yellow light is recognized by the eye in rather more than one-fifth of the length of the spectrum, namely in the orange, yellow, and green spaces. It may be proved to be present in the blue and indigo spaces by many experiments, among which is the one already described, in which a violet tint is developed, by passing the spaces through olive oil: the tint absorbed by the oil cannot be red, because violet, reddish blue, is made to appear by the transmission; it cannot be blue, for blue taken from blue will not leave violet; it is, then, yellow, which mixed with the red and blue had formed white light, at this part of the spectrum. Farther, the spectrum examined through a deep blue glass, shows green in the blue space, and through a transparent wafer of gelatine, produces a whitish band in the same space. Yellow is shown to exist in the red space by examining it through a prism of port wine, the refracting angle of which is 90°, and the whole of the red space assumes a yellowish tint by the absorption of the blue rays, by certain thicknesses of pitch, balsam of Peru, &c. In the violet space, owing to the extreme facility with which that color is absorbed, and the extreme faintness of the rays, yellow light has not yet been detected.

Blue light is perceptible to the eye through more than two-thirds of the length of the spectrum, that is in the green, blue, indigo, and violet spaces. The absorptive powers of pitch, balsam of Peru &c., show green light extending considerably within the red space; and the blue is farther proved to be spread throughout that space by the yellow tinge which it assumes, when viewed through the media already alluded to; a tint which could only result from the absorption of blue rays.

2. *White* light exists, at every point of the spectrum, and may be insulated by absorbing the excess of the colored rays at any point.

By a particular thickness of smalt-blue glass, the yellow space, the brightest of the spectrum, becomes greenish white, and, with a different blue, reddish white. A mixture of red ink and sulphate of copper reduces the yellow space to nearly a white, the tint being slightly red when the ink is in excess, and green when there is too much of the solution of sulphate of copper. By particular methods, not described, Sir David Brewster states that he has succeeded in insulating white light in both the orange and green spaces.

The curious property possessed by this white light of not being decomposable by refraction, is a powerful support of the new theory of the spectrum.

By a principle of absorption applied to the heating rays of the solar spectrum, and which will be described in a subsequent note, the existence of a spectrum of heating rays, exceeding in length the three colored spectra, is proved, bringing a new analogy to bear upon this question.

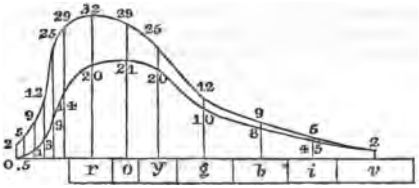
No. IV.

(Referred to from article 71, page 82.)

Comparative experiments on the heat in different parts of the solar spectrum, require the most delicate instruments. The new branch of science, thermo-magnetism, furnished Signor Melloni with a much more sensible means of measuring temperature than the common thermometer, namely, by the magnetic currents developed by heat, in a battery composed of bars of bismuth and antimony.

By the aid of this instrument, he found that the heat accompanying the violet space of the spectrum, from a crown glass prism, was not at all absorbed by pure water, while a small portion of the heat in the indigo was absorbed, a greater portion of that in the blue, and so on through the colored spaces and into the invisible heating rays beyond the spectrum, the extreme rays of which were entirely absorbed.

The relative degrees of heat in the spectrum formed by a crown glass prism, which, however, it must be recollected, has absorbed the rays unequally, is represented by the annexed diagram, in which the ordinates, 2 *v*, 5 *i*, &c., of the upper curve represent, nearly, the relative temperatures; the lengths of the ordinates, and of course the relative degrees of heat, being expressed by the numbers written above them: thus the amount of heat in the middle of the space *v*, the *violet* space, compared with that in the middle of the *blue* space *b*, is as 2 to 9; with the middle of the *red* space *r*, as 2 to 32. The points marked 25, 12, &c. beyond the colored space *r*, correspond to the bands, in the spectrum, having, respectively, the same temperatures as the middle of the yellow, of the green, of the blue, &c. By passing the spectrum through a thickness of less than a twelfth of an inch of water, contained between plates of thin glass with parallel surfaces and free from defects, the heating powers of the several rays became as represented in the lower curve; none of the heat accompanying the violet rays having been absorbed, a little of that accompanying the blue, and so on increasing as the refrangibility diminished, until in the band, 2, 0, having the same temperature as the violet, all the heating rays were absorbed. Different media stopped the heating rays in different degrees, those of higher refractive powers permitting them to pass more readily than those of lower powers.



Although this subject cannot be considered as fully developed, we are able to understand by it, why Seebeck found the point of greatest heat to vary, according to the material of the prism used to form the spectrum, being in the red when a prism of crown glass was used, and in the yellow when water was the refracting material. Melloni found the greatest heat in the *orange* after passing the spectrum formed by the crown glass prism through water, as appears by the diagram, in which the greatest heat in the red and yellow are 20, and in the orange is 21. Seebeck found the point of greatest heat in the yellow, when the prism contained water; a sufficiently near coincidence with the observation of Melloni, if we consider that in the experiments of Seebeck the rays were exposed to absorption by the glass forming the hollow prism in which the water was contained, and in those of Signor

Melloni to the absorption by the crown glass prism first, and then to that by two plates of glass and the water contained between them.

The power of transmitting the heating rays without absorption being greater as the refractive power is greater, according to the law before referred to, we should expect that in flint glass the greatest heat would lie farthest from the violet end of the spectrum; in plate and crown glass, that it should be at a less distance from that end; in sulphuric acid and oil of turpentine still less; in alcohol and water yet nearer to the violet end: and these deductions we find, by consulting the table on page 82, to be correct. Minute differences, which could not be detected by the instruments used by Seebeck and others, in the points of greatest heat as given by that table, will probably hereafter appear.

The experiments discussed in this note authorize the addition of a fourth spectrum to the three colored spectra represented in *fig.* 51., namely, a heating spectrum containing rays which are less refrangible than the extreme red rays of the spectrum.

No. V.

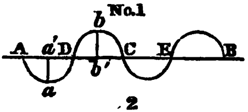
(Referred to from page 116.)

The undulatory hypothesis represents in so simple a manner the phenomena to which Dr. Young applied his principle of interference, that I have been induced to refer to it here, with a view to a general explanation of the hypothesis. The reader will be better satisfied if he take up the subject, as briefly referred to in the 84th article of the text, before entering upon the account to be given in this note. As stated in that article, the hypothesis of undulations supposes all space, the planetary spaces as well as the interstices between the particles of bodies, to be occupied by an elastic medium, or ether, which is put in a state of vibratory motion by luminous bodies, and in which impulses are propagated according to the same mechanical laws, as the impulses which, communicated to air, produce sound.

If we suppose a luminous point surrounded by this elastic medium, the particles immediately about the point have a vibratory motion impressed upon them, or a motion to and fro; this they communicate to the adjacent particles, and thus a wave is formed, which spreads about the point as a centre, just as the waves formed by a stone, thrown into still water, spread around the point at which the stone struck the surface. As these waves would communicate to a floating body which they might meet, an impulse in a direction radiating from the point where they originated, so luminous waves striking the retina, give the sensation of light in a similar direction.

In the annexed diagram, let A B, No. 1, represent one of the directions in which the impulse given by a luminous body, is propagated; we shall find, according to the hypothesis, along that line particles of the elastic medium, or ether, having all rates of motion, from rest, or when the motion is nothing, to the greatest rapidity of the vibration; and in the two opposite directions, from A towards B, and from B towards A. For example, let the particles at A, D, and C, be at rest, then if from A to D we find particles moving towards B, their velocities, or rates of motion, will be found increasing between A and a', mid-way from A to D, and then decreasing between a' and D; between D and C the vibration will be in the contrary direction, namely, from B towards A; and the velocities after increasing to *b′*, will diminish to C. The distance between A and C includes all velocities from nothing to the greatest, and in the two opposite directions A B and B A; the same would be true of C B, if made equal to A C; this distance A C, or C B, is called the length of a wave, and it is this which in red light is 256 ten millionths, and in violet light 174 ten millionths, of an inch (see page 119, text.) To represent to the eye the velocities of the different particles of ether between A and D, the curve A *a* D is described, in which the ordinates, or lines in the same direction as *a′ a*, represent these velocities, as, for example, *a′ a* the velocity at *a′*; the particles between D and C vibrating in a contrary direction, the curve D *b* C, representing their velocities, is traced on the side of the line A B opposite to A *a* D. In the same way the velocities between C and E, and between E and B, are shown by similar curves, on opposite sides of A B. Let No. 2. represent a system of waves in which the lengths M P and P O are equal to A D and D C, and let the motion between M and P coincide in direction with that between A and D, the curves of velocities M *m* P and A *a* D coinciding, and the motion between P and O with that between D and C, the curves P *p* C and D *b* C coinciding; then it is plain that the motion of the particles between O Q, C E, and Q N, E B, &c. will be the same, or that the undulations will coincide throughout; one undulation will, therefore, add to the effect of the other, and the light will be the united light produced by the two undulations. The same will be true whether the point M coincides with A, with C, or with B, &c.; that is, whether the lengths of the paths of the two rays A B and M N are exactly equal, or differ by one, two, or more undulations. If the rays, instead of moving in the same direction, meet under a small angle, the remarks will still apply; and the first result, stated on page 115, text, is in accordance with the hypothesis, under consideration, namely, that bright spots, illuminated by the sum of the two lights, will be formed when the differences in the lengths of the paths of the rays, are *d*, (A C,) 2 *d*, (A B,) 3 *d*, &c.

No. 1



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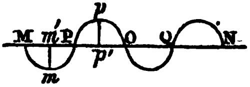

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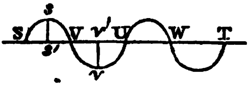


Next let the curves described in No. 3 represent the velocities in another system of undulations, S V and V U being equal to A D and B C in No. 1, the particles of the ether between S and V, as shown by the curve, being in a state of vibration from T towards S, those between V and U from S towards T, and so on. If the ray S T in No. 3 were brought to coincide with A B in No. 1, the point S being placed at A, the particles between S and V in No. 3 moving in opposite directions from those between A and D, and those between V and U in opposite directions from those between D and C, and their velocities being supposed equal, their motions would destroy each other, and the wave would be destroyed, or darkness would result. The path S T differs from A B by the distance S V, or half an undulation. The same would be the result if No. 3 began one undulation to the left hand of S, or two or more undulations, that is if the path S T, No. 3, differed from A B, No. 1, by one and a half, two and a half, &c. undulations. As the same consequences would follow, if the rays S T and A B met under a small angle, we infer (page 115, text) that when the difference in the lengths of the paths, of the two pencils of rays, is ½ *d* (A D), 1½ *d* (A E), 2½ *d*, &c., "instead of adding to one another's intensity, they destroy each other and produce a dark spot."

No. VI.

(Referred to from page 120.)

Professor Hare has observed, in relation to the translucency of gold leaf;—"Gold leaf transmits a greenish light, but it is questionable, if it be truly translucent. Placed on glass, and viewed by transmitted light, it appears like a retina. It is erroneously spoken of as a continuous superficies." The nature of the process by which gold is reduced to leaves, strengthens this conclusion.

On examining gold leaf by the solar microscope, I find in it innumerable rents, and also various gradations of thickness; the rents have their edges colored, a blue fringe appearing on one side, and a reddish brown on the opposite side; the thickest parts transmit no light, and through the very thin parts a delicate *green* light is transmitted. The surface thus exhibited is very beautiful, The rents are visible to the naked eye, when the leaf is very strongly illuminated.

No. VII.

(Referred to from page 237.)

The following classification of colored bodies is alluded to in the text, as given by Sir David Brewster, in the life of Newton. The colors of each of the classes require, in his view, to be explained upon different principles.

1. "Transparent colored fluids, transparent colored gems, transparent colored glasses, colored powders, and the colors of the leaves and flowers of plants."

The colors of these bodies are derived from the absorption of particular colored rays: thus, water at great depths appears red by transmitted light, owing to the absorption of the blue and yellow rays which with the red constituted white light; certain thicknesses of smalt-blue glass appear intensely blue by transmitted light, while at greater thicknesses the glass appears red. In the case of opaque and colored bodies, as in the leaves of plants, we are to suppose certain rays to be absorbed by the indefinitely thin film through which the rays reflected from any surface may be supposed to pass, the complementary tint being reflected; as all the incident light is not reflected, the transmitted tint will be complementary to the colors absorbed, and thus the body will appear of the same color by both reflected and transmitted light. Colored powders would seem not always to belong to this class, since many of them change their hues with a change in the size of the particles.

2. "Oxidations on metals, colors of Labrador feldspar, colors of precious and hydrophanous opal and other opalescences, the colors of the feathers of birds, of the wings of insects, and of the scales of fishes."

To these the Newtonian theory is strictly applicable.

3. "Superficial colors, as those of mother-of-pearl and striated surfaces."

4. "Opalescences and colors in composite crystals having double refraction."

5. "Colors from the absorption of common and polarized light, by doubly refracting crystals."

6. "Colors at the surfaces of media of different dispersive powers."

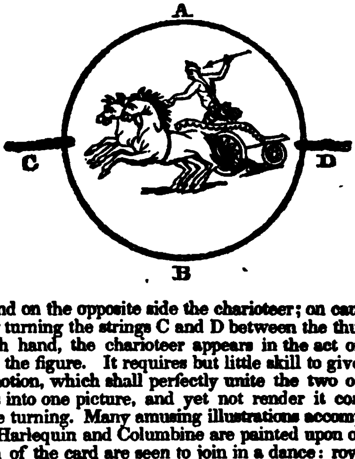
7. "Colors at the surfaces of media in which the reflecting forces extend to different distances, or follow different laws."

No. VIII.

(Referred to from page 251.)

The philosophic toy called by its inventor, Dr. Paris, the *thaumatrope*, or wonder-turner, illustrates very perfectly the fact of the duration of impressions on the retina.

On one side of the card, represented in the diagram, is drawn a chariot and horses, and on the opposite side the charioteer; on causing the card to revolve by turning the strings C and D between the thumb and forefinger of each hand, the charioteer appears in the act of driving the chariot, as in the figure. It requires but little skill to give to the card, exactly the motion, which shall perfectly unite the two objects on the opposite sides into one picture, and yet not render it confused by the rapidity of the turning. Many amusing illustrations accompany the toy; for example, Harlequin and Columbine are painted upon opposite sides, and by a turn of the card are seen to join in a dance: royalty, stripped of its robes, occupies one side of the card, and the robes the opposite, the robes are donned by a turn: a potter seated at his wheel moulding the unformed clay, occupies one side of the card, and an urn is grasped by an arm on the reverse; on turning, the urn appears grasped by the potter's arm, the foot of the vase being yet unfinished.



No. IX.

(Referred to from page 261.)

Referring to his new analysis of the solar spectrum, Sir David Brewster advances the following hypothesis to account for certain of the cases just detailed.[*]

"By means of this analysis we are now able to explain the phenomenon observed by those who are insensible to particular colors. (Edin. Journ. Sc. No. XIX. old series. No. IX, new series.) The eyes of such persons are blind to *red* light; and when we abstract all the *red* rays from a spectrum constituted as already[†] described, there will be left two colors, *blue* and *yellow*, the only colors which are recognized by those who have this defect of vision. To such eyes, light is always seen in the red space; but this arises from the eye being sensible to the *yellow* and *blue* rays, which are mixed with the red light.

Hence blue light will be seen in the place of the *violet*, and a greenish *yellow* will appear in the *orange* and *red* spaces, or, which is the same thing, the spectrum will consist only of the *yellow* and the *blue* spectra. The physiological fact, and the optical principle, are therefore in perfect accordance; and while the latter gives a precise explanation of the former, the former yields to the latter a new and an unexpected support."

The details of the cases referred to, fully sustain this conclusion. There are other circumstances connected with them, and with others described in the text, page 260, not unworthy of notice.

In the second of the cases described in the Journal of Science, No. XIX, although the individual never failed to detect a full blue or a full yellow, he seems to have had very imperfect ideas of those colors when presented in a state of mixture; green, as such, he did not know, and when blue was diluted with yellow, forming what to a good eye would appear yellowish green, the blue tint escaped him, and the mixture appeared yellow. In like manner, his discrimination of yellow, when mixed with blue, was very defective; he called *grass green* yellow, and yet *yellowish green* appeared to be "yellow with a good deal of blue in it." This remark may serve to explain why the same white seen at different times, appeared to him to vary in its tint, at one time being white, at another "white with a dash of yellow and blue," at another "white with yellow and blue in it." When requested to arrange colors so as to produce the strongest contrasts, he divided them into two classes, to one of which he gave the name of *blue*, and to the other *yellow*. In these contrasts he invariably placed white among the blues, and was never perplexed, as in the preceding examinations, when tasking himself as to the precise shades. That white should be classed by him as blue, appears consistent with the other observations, for being blind to red light the tint of white should be that which appears when red is removed from the spectrum or a bluish green, which tint he saw as a *blue*.

In examining other cases we shall find reason to be satisfied that this blindness extends to light of other colors than red, and that in those cases also there is a want of discrimination between shades in mixtures of the colors to which the eye is sensible. The Plymouth tailor, whose case is described by Mr. Harvey (see page 260, text), seems not to have been entirely blind to red light, and to have been in a measure blind to blue; thus the prismatic spectrum appeared to consist entirely of *yellow*, and *light blue;* the red, orange, and yellow spaces appearing as if red had been withdrawn from them, while the full blue, the indigo and violet were *light* blue, and dark blue and indigo stuffs appeared to be *black*, and crimson was either *blue* or *black*. A dark green he regarded as *brown*, by which, since he was blind to red light, he must have meant a shade of black; and light green as orange, by which, for the reason just stated, he meant a variety of yellow. The blue in both these mixtures escaped his perception.

---

[*] *Edinburgh Trans.* Vol. XII. Part. I., or *Edin. Journal of Science*, No. X. new series.

[†] See text, p. 69.

An extreme case seems to have occurred in the vision of Mr. Harris, of Allonby, who, according to the statement of Mr. Huddart, could only distinguish *black* from *white*, or was entirely blind to colors.

It is much to be desired, for the elucidation of this curious subject, that more well examined cases were on record. The colors of the spectrum afford rigid tests, not to be found in colored stuffs; and by such tests only, the minutiæ of peculiarities of vision can be satisfactorily determined.

APPENDIX

TO

SIR DAVID BREWSTER'S

TREATISE ON OPTICS;

INTENDED TO ADAPT THE WORK TO USE, AS A TEXT-BOOK
IN THE COLLEGES OF THE UNITED STATES.

BY A. D. BACHE,
PROFESSOR OF NAT. PHILOS. AND CHEMISTRY IN THE
UNIVERSITY OF PENNSYLVANIA.

ADVERTISEMENT.

THE object of the following Appendix is to place in the hands of the students of our Colleges, a text-book which will furnish them with some of the analytical methods of the most recent writers, upon the elements of Optics.

The work of Sir David Brewster is from the pen of a master, and presents, in a popular form, the results which have flowed from experiment and from theory, applied to the investigation of the different branches of Optics. The Appendix merely aims at supplying to the student the mode of determining the results given in the text, more particularly in what relates to Reflexion and Refraction.

It may not be amiss to state, that I do not present, to the notice of instructers, an untried course, but that most of the propositions in the following pages have entered into the mathematical portion of the course, taught to the Senior Class of the University of Pennsylvania.

The works in which the full development of these subjects may be found, and which have been consulted in the composition of the Appendix, are Coddington's Optics, Coddington on Reflexion and Refraction, Lloyd's Treatise on Light and Vision. The more advanced student will find the subject treated by the most general methods in Herschel's Treatise on Light.

A. D. BACHE.

PHILADELPHIA, *March*, 1833.

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THE AMERICAN EDITOR.

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APPENDIX.

REFLEXION.

CHAP. I.

REFLEXION BY SPHERICAL AND PLANE MIRRORS*

(1.) In considering the cases of reflexion from spherical or plane surfaces, two divisions will be made: in the first, the axis of the incident pencil will be supposed perpendicular to the surface of the mirror, in the second, oblique to it; in the first case the pencil is termed *direct*, in the second, *oblique*.

(2.) Prop. I. *To determine the form given by reflexion to a small direct pencil of light, proceeding from a point in the axis of a mirror.*

Case 1. In *fig.* 8., p. 18 of the text, let A represent the radiant point of a pencil of *diverging* rays, F its focus, and C the centre of a *concave* mirror. Call $AD = u$, $FD = v$, and $CD = r$. The angle of reflexion FMC being equal to the angle of incidence AMC, the line CM bisects the vertical angle of the triangle AMF: whence (Legendre's Geom., Book III., Art. 201., or Euclid, VI. 3.)

$$AC : AM :: FC : FM, \text{ or}$$

$$\frac{AC}{AM} = \frac{FC}{FM}$$

The pencil being, by supposition, very small, the point M is very near to D, hence for the approximate value of AM we may take AD, and for that of FM, FD. The equation just found becomes

$$\frac{AC}{AD} = \frac{FC}{FD}.$$

By the notation adopted above $AC = AD - CD = u - r$, and $FC = CD - FD = r - v$. We have therefore

$$\frac{u-r}{u} = \frac{r-v}{v}, \text{ or}$$

$$1 - \frac{r}{u} = \frac{r}{v} - 1, \text{ whence}$$

---

* Throughout the Appendix, the student is supposed to be acquainted with the corresponding chapters in the body of the work.

dividing by r and transposing

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r} \;\ldots\ldots (a)$$

(3.) CASE 2. We next proceed to the case in which a *converging* pencil falls upon a *concave* mirror. *Fig.* 10., *p.* 20 of the text.

AM, *AN* representing the two extreme rays of the pencil, converging to *A'*, the imaginary radiant point, *MF* and *NF* are the reflected rays. The radius *CM* therefore bisects the angle *AMF*, the outward angle of the triangle *A'MF*, and (Euc. VI. A.)

$$\frac{A'C}{A'M} = \frac{FC}{FM},$$

but for *A'M* and *FM* we may substitute *A'D* and *FD*, their approximate values, whence

$$\frac{A'C}{A'D} = \frac{FC}{FD}.$$

Using the notation before employed, $A'D = u$, $FD = v$, and $DC = r$, whence $A'C = u + r$, and $FC = r - v$; these values substituted in the equation just found give,

$$\frac{u+r}{u} = \frac{r-v}{v}, \text{ or,}$$

$$1 + \frac{r}{u} = \frac{r}{v} - 1 \text{ and}$$

$$-\frac{1}{u} + \frac{1}{v} = \frac{2}{r} \;\ldots\ldots (b)$$

(4.) Comparing equation (b) with (a) we find that it differs from it only in the sign of $\frac{1}{u}$ which is positive in (a) and negative in (b); both these cases may, therefore, be represented by the same equation, if we agree to give the positive sign to the distance of the radiant point for diverging rays, negative for converging rays; that is, if we consider the distance (u) positive, when the radiant point is in front of the mirror, negative when it is behind the mirror.

If, then, $\pm u$ represents the distance of the radiant point from the mirror, $\frac{1}{\pm u}$ will denote the degree of divergency or convergency of the incident rays. In like manner, $\frac{1}{v}$ will represent the convergency of the reflected rays. From equations (a) and (b)

$$\pm\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

where r is a constant quantity; a result which may be thus expressed: *the divergency (or convergency) of the incident rays together with the convergency of the reflected rays is a constant quantity for the same mirror.* The curvature of the mirror is measured by $\frac{1}{r}$, the reciprocal of the radius.

(5.) Case 3. *Diverging* rays falling upon a *convex* mirror. *Fig.* 12., p. 21, text.

The line CM, bisects the outward angle of the triangle AMF, whence (Euc. VI. A.)

$$\frac{AC}{AM} = \frac{FC}{FM}, \text{ or}$$

substituting their approximate values for AM and FM,

$$\frac{AC}{AD} = \frac{FC}{FD},$$

and by the notation adopted in the cases already considered,

$$\frac{u+r}{u} = \frac{r-v}{v}, \text{ whence}$$

$$\frac{1}{u} - \frac{1}{v} = -\frac{2}{r} \quad \ldots\ldots \text{(c)}$$

On comparing this equation, in which we have made $\frac{1}{u}$ positive as it corresponds to a real radiant, with (a), we perceive that the signs of both $\frac{1}{v}$ and $\frac{2}{r}$ are different. From the figure we observe that v corresponds to an imaginary focus, and that the radius is now behind the mirror. Equation (a) may, then, be used to represent this case if the sign of the radius be changed; the resulting negative value of the focal distance corresponds to a focus behind the mirror.

(6.) Case 4. *Converging* rays falling upon a *convex* mirror. The formula for this case may be deduced from *fig.* 12., if BM and BN be made to represent the incident, and MA, NA the reflected rays. We should have by proceeding as in the last case,

$$\frac{FC}{FM} = \frac{AC}{AM}, \text{ or}$$

$$\frac{FC}{FD} = \frac{AC}{AD},$$

and since $FD = u$, F being the imaginary radiant point, and $AD = v$, A being the focus; $FC = r - u$, and $AC = r + v$ whence

$$\frac{r-u}{u}=\frac{r+v}{v}, \text{ or}$$

$$-\frac{1}{u}+\frac{1}{v}=-\frac{2}{r} \dots\dots (d)$$

the first term being made negative to correspond to the case of converging rays. This formula differs from (c) in the sign of $\frac{1}{v}$, which was negative in (c), and in that of $\frac{1}{u}$. The figure shows that v in this latter case corresponds to a real focus, while in the former $-v$ denoted the distance to an imaginary focus. The change of sign in $\frac{1}{u}$ conforms to the remarks made in article (4.)

(7.) Comparing the four equations (a) (b) (c) and (d), we perceive that the formula

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{r} \dots\dots (1)$$

may be made to include them all, by attributing to u, and r, respectively, the positive sign when the radiant point or the centre is in front of the mirror, the negative sign when either of these points is behind the mirror, and by considering the positive value of v as corresponding to a focus in front of the mirror, its negative value to one behind it.

(8.) We might have commenced by giving to the student this conventional mode of considering the quantities used in the analysis, and then have deduced the general equation by reference to a single diagram: we have preferred in the outset to show him that the variations in the algebraic signs are not arbitrary, but required by the geometrical relations of the quantities.

From the formula

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{r} \dots\dots (1)$$

we deduce the general rule, that the *sum of the vergencies* of the incident and reflected pencils is a constant quantity.*

(9.) Having obtained an equation (1) expressing the relation between the distances of the radiant point and focus of a small pencil, by means of the radius of the mirror, we shall proceed to interpret it in its application to different kinds of mirrors, and under different circumstances of the incident pencil.

---

* This convenient term, expressing, as the case may be, either divergency or convergency, is proposed and used by Lloyd in his Treatise on Light and Vision.

Prop. II. *To determine the form given to a small pencil of rays by reflexion from a plane mirror.*

In the plane mirror the radius is infinite, or $\frac{2}{r} = o$, whence from (1),

$$\frac{1}{u} + \frac{1}{v} = o \; \ldots\ldots (2), \text{ or,}$$

$$v = -u.$$

The focus and radiant point are at equal distances from the mirror, but on opposite sides of it.

(10.) If *parallel* rays fall upon the mirror $u = \infty$, and $v = -\infty$, or the reflected rays are parallel. This corresponds to the case represented in *fig.* 4., p. 15, text.

(11.) If the rays *diverge* before reflexion, the formula

$$\frac{1}{v} = -\frac{1}{u},$$

shows that they will be equally divergent after reflexion; and

$$v = -u$$

that the focus is as far behind the mirror as the radiant point is in front of it. *Fig.* 5., p. 15, text.

(12.) For *converging* rays (*fig.* 6., p. 16, text,) u takes the negative sign, and (2) becomes

$$-\frac{1}{u} + \frac{1}{v} = o \; \ldots\ldots (3), \text{ whence,}$$

$$\frac{1}{v} = \frac{1}{u}$$

$$v = u.$$

The sign of v being positive the focus is real, its distance v in front of the mirror is equal to the distance of the imaginary radiant point behind it.

(13.) **Prop. III.** *Reflexion of a small pencil of light by a concave mirror.*

The formula which applies to this case is,

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r} \; \ldots\ldots (1).$$

By transposition

$$\frac{1}{v} = \frac{2}{r} - \frac{1}{u} = \frac{2u - r}{ur}, \text{ or,}$$

$$v = \frac{ur}{2u - r}.$$

This value of v gives the rule on page 19 of the text.

(14.) When the rays are *parallel* $\frac{1}{u} = 0$. And

$$\frac{1}{v} = \frac{2}{r}, \text{ or,}$$

$$v = \frac{r}{2}.$$

The sign of v being positive shows that the focus is in front of the mirror, and its value $\frac{r}{2}$, that the focal distance is half the radius. This is represented in *fig.* 7., p. 17, of the text, where $FD = \frac{1}{2} CD$.

The focus of parallel rays is called the *principal* focus. As it serves as a point of comparison for the foci of other rays, we shall represent its distance by a symbol, f. Placing for $\frac{2}{r}$ its value $\frac{1}{f}$, formula (1) becomes

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \ldots\ldots (4).$$

(15.) The next case to be considered is that of *diverging* rays. In this, u is positive and the formula is

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \ldots\ldots (4), \text{ whence,}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

As long as $u > f$, or the point A in *fig.* 8., p. 17, is farther than the principal focus from D, we have $\frac{1}{u} < \frac{1}{f}$, and $\frac{1}{f} - \frac{1}{u}$ is a positive quantity, or the rays converge after reflexion. Since $\frac{1}{f} - \frac{1}{u} < \frac{1}{f}$, we have $\frac{1}{v} < \frac{1}{f}$, and $v > f$, or the focus is farther from the mirror than the principal focus.

If we suppose, besides, that A is beyond the centre C, we have $u > r$, and $\frac{1}{u} < \frac{1}{r}$, whence $\frac{1}{f} - \frac{1}{u}$, or $\frac{2}{r} - \frac{1}{u} > \frac{2}{r} - \frac{1}{r}$, or $> \frac{1}{r}$, and $\frac{1}{v} > \frac{1}{r}$, or $v < r$, the focal distance is less than the radius.

We infer, then, that rays diverging from a point beyond the centre of the mirror, are reflected to a point between the principal focus and the centre.

As $\frac{1}{u}$ diminishes in value, $\frac{1}{v}$ evidently must increase, or, as u increases, v diminishes, and vice versa: that is, as the radiant point recedes from the mirror the focus approaches it, and vice versa. When the radiant point becomes infinitely distant, the rays are parallel, and their focus is the principal focus.

We have seen that as long as the radiant point is farther from the mirror than its centre (that is, $u > r$) the focus cannot coincide with that centre (or $v < r$) which it approaches. If $u = r$, or the radiant point coincides with the centre, then

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{r} = \frac{2}{r} - \frac{1}{r} = \frac{1}{r}\text{, or,}$$

$$v = r,$$

and the rays are reflected to the centre.

Let the radiant point now pass the centre towards the principal focus, that is, let $u < r$, and at the same time $u > f$, or $\frac{1}{u} > \frac{1}{r}$ and $\frac{1}{u} < \frac{1}{f}$. Since $\frac{1}{u} < \frac{1}{f}$, $\frac{1}{f} - \frac{1}{u}$ is still a positive quantity; the rays are, therefore, still brought to a focus: but since $\frac{1}{u} > \frac{1}{r}$, $\frac{1}{f} - \frac{1}{u}$ or $\frac{2}{r} - \frac{1}{u} < \frac{1}{r}$, whence $\frac{1}{v} < \frac{1}{r}$ and $v > r$; that is, the focus lies beyond the centre.

When the radiant point coincides with the principal focus $u = f$ whence, $\frac{1}{u} = \frac{1}{f}$, and $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = 0$, or $v = \infty$: the rays are rendered parallel by reflexion.

If we suppose the radiant point to approach still nearer to the mirror, so that $u < f$, we have $\frac{1}{u} > \frac{1}{f}$, whence $\frac{1}{f} - \frac{1}{u}$ is negative; that is, $\frac{1}{v}$ has a negative sign, or the focus is imaginary, the rays diverging after reflexion. The divergency of the reflected rays is less than that of the incident rays, for $\frac{1}{v} = -\left(\frac{1}{u} - \frac{1}{f}\right)$, and $\frac{1}{u} - \frac{1}{f} < \frac{1}{u}$, the divergency before reflexion.

(16.) For the case of *converging* rays falling upon a *concave* mirror (*fig.* 9., p. 19, text,) we make, in formula (4), u negative; whence,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \dots\dots (5), \text{ or,}$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}.$$

The signs of the quantities on the right-hand side of this equation being both positive, $\frac{1}{v}$ is always positive. Converging rays are, therefore, always brought to a focus. Moreover, since $\frac{1}{u} + \frac{1}{f} > \frac{1}{u}$, $\frac{1}{v} > \frac{1}{u}$, or the convergency is greater after, than before, reflexion. Since $\frac{1}{u} + \frac{1}{f} > \frac{1}{f}$, $\frac{1}{v} > \frac{1}{f}$ and $v < f$, whence the focus of converging rays is nearer the mirror than the principal focus.

Substituting, in equation (5) for $\frac{1}{f}$, its value $\frac{2}{r}$, and transposing $\frac{1}{u}$, we have

$$\frac{1}{v} = \frac{2}{r} + \frac{1}{u},$$

whence the value of v is

$$v = \frac{ur}{2u + r},$$

agreeing with the rule on p. 20 of the text.

(17.) Prop. III. *Reflexion of a small pencil of rays by a convex mirror.*

In this case, r, the radius of the mirror, takes the negative sign, and equation (1) becomes

$$\frac{1}{u} + \frac{1}{v} = -\frac{2}{r} \dots\dots (6).$$

(18.) For *parallel rays* (*fig.* 11., p. 21, text,) $\frac{1}{u} = 0$, whence $\frac{1}{v} = -\frac{2}{r}$, or $v = -\frac{r}{2}$.

The focus is behind the mirror, and at the distance, from the vertex, of half the radius.

If we call the principal focal distance f, the formula for the reflexion by a convex mirror becomes

$$\frac{1}{u} + \frac{1}{v} = -\frac{1}{f} \dots\dots (7).$$

(19.) For *diverging rays* (*fig.* 12., p. 21,) we have

$$\frac{1}{v} = -\frac{1}{f} - \frac{1}{u} = -\left(\frac{1}{f} + \frac{1}{u}\right),$$

and therefore $\frac{1}{v}$ is always negative; such rays diverge after reflexion, and since $\frac{1}{f} + \frac{1}{u} > \frac{1}{u}$, or $\frac{1}{v} > \frac{1}{u}$, their divergency is increased by the reflexion.

(20.) *Figure* 12. will, as has already been stated, represent the case of *converging* rays falling upon a *convex* mirror, if we suppose *BM*, and *BN* to represent the incident rays, meeting in the imaginary radiant point *F*. To express this case analytically, *u* must be made negative, and the formula is

$$\frac{1}{v} - \frac{1}{u} = -\frac{1}{f} \;\ldots\ldots (8), \text{ or,}$$

$$\frac{1}{v} = \frac{1}{u} - \frac{1}{f}.$$

The position of the focus, as shown by this equation, passes through variations, corresponding to those in the case of diverging rays falling upon a concave mirror (Art. 15.).

(21.) It is sometimes convenient to refer the distance of the radiant point and focus, to the centre of the mirror, instead of to the vertex. Formula (1) may be readily transformed into one which shall refer to the centre.

Prop. IV. *To determine the relation of the distance of the focus and radiant point, of a small pencil of rays, from the centre of a mirror.*

By *fig.* 8., p. 18, text, it appears that $AD = AC + CD$, and $FD = CD - CF$. Calling $AC = u'$ and $FC = v'$, CD, as before, $= r$, $AD = u$, and $FD = v$; we have $u = u' + r$, and $v = r - v'$.

Substituting these values of u and v in the equation

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r} \;\ldots\ldots (1), \text{ it becomes}$$

$$\frac{1}{u'+r} + \frac{1}{r-v'} = \frac{2}{r},$$

$$\frac{r}{u'+r} + \frac{r}{r-v'} = 2,$$

$$\frac{r}{r-v'} - 1 = 1 - \frac{r}{u'+r}.$$

Bringing to a common denominator the quantities on both sides of the equation, and reducing, we have,

$$\frac{v'}{r - v'} = \frac{u'}{u' + r},$$

$$\frac{r - v'}{v'} = \frac{u' + r}{u'},$$

$$\frac{r}{v'} - 1 = 1 + \frac{r}{u'}, \text{ or,}$$

$$\frac{1}{v'} - \frac{1}{u'} = \frac{2}{r} \quad \ldots\ldots (9).$$

(22.) This equation is the same in form with the one found when the distances were estimated from the vertex, except that the sign of u' is negative, the distances v' and u' being now reckoned in opposite directions.

Equation (9) might have been deduced directly from the relation of the lines *AM*, *FM*, *AC*, *CF*, *fig.* 8., in the triangle *AMF*. The solution of the question by that method, would have been more simple.

(23.) If we substitute for $\frac{2}{r}$, in equation (9), its value $\frac{1}{f}$, we have

$$\frac{1}{v'} - \frac{1}{u'} = \frac{1}{f} \quad \ldots\ldots (10).$$

From this equation, may be deduced the relation expressed in the following proposition.

(24.) Prop. V. *The distance of the principal focus of a mirror from the centre, is a mean proportional between the distances of the radiant point and focus of any small pencil, from the principal focus.*

Equation (10) gives,

$$\frac{1}{v'} = \frac{1}{f} + \frac{1}{u'} = \frac{f + u'}{fu'}, \text{ or,}$$

$$v' = \frac{fu'}{f + u'}, \text{ whence,}$$

$$f - v' = f - \frac{fu'}{f + u'} = \frac{f^2}{f + u'}, \text{ or,}$$

$$f - v' : f :: f : f + u',$$

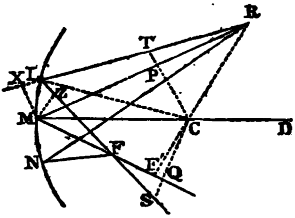
in which proportion $f - v'$ represents *FO* (*fig.* 8., p. 18, text,) or *CO* — *CF*, and $u' + f$, *AO* or *AC* + *CO*.

(25.) The subject of reflexion at curved surfaces in general, will be treated briefly in a subsequent chapter. The properties of the surfaces formed by the revolution of the parabola and ellipse about their axes, are readily understood, from very simple geometrical considerations, and gain nothing by being presented analytically. They will, however, be referred to in another part of this Appendix.

(26.) We pass next to the reflexion of a small *oblique* pencil by a spherical mirror, on which subject two propositions will be given; in the first will be considered the case in which the axis of the pencil does not cross that of the mirror, and the pencil falls upon the mirror near to the vertex, and in the second, the case in which the axis of the pencil crosses that of the mirror.

(27.) **Prop. VI.** ***A small pencil having its radiant point out of the axis of a mirror, meets the surface near the vertex, required the form of the reflected pencil.***

Fig. A.



Let *LMN* represent a section of the mirror, made by a plane passing through the lines *MR* and *MC*, or through the axis of the pencil and the centre of the mirror. *RL* is an extreme ray of the pencil incident very near to *M*, *LF* is the corresponding reflected ray, meeting the reflected ray *MF*, which corresponds to the axis of the pencil, in the point *F*. *F* is the focus of the pencil *LRN*, in the plane of the section *RMC*.

(28.) To determine the focus of rays which meet the mirror in a plane perpendicular to *RMC*; suppose a plane to pass through *RM* at right angles to that of the figure, this plane will cut from the pencil, *RLN*, two rays, which reflected will meet the axis, *MF*, of the reflected pencil, in the focus required. If, now, a plane be passed through one of the incident rays, just described, and the corresponding reflected ray, it will pass through the centre of the mirror; the line *RC*, joining the radiant point and centre, will be, therefore, its intersection with the plane *RMC* containing the axis of the incident and of the reflected pencil; and the point, *F′*, in which *RC* produced meets *MF*, will be the point in which the supposed plane meets *MF*, that is, the focus of the reflected pencil.

The focus, of the reflected pencil, in any plane between *RMC* and the one at right angles to it,* will be found between *F* and *F′*

---

*Coddington terms the former of these planes the *primary plane*, the latter, the *secondary plane*.

(29.) To determine, analytically, the position of the point F; draw from the vertex M, MX and MZ perpendicular, respectively, to RL and LF; also from C, CP and CQ perpendicular, respectively, to RM and MF. As the arc LM is very small, it may be considered a straight line perpendicular to the radius CL; whence the incident and reflected rays making equal angles with CL, also make equal angles with LM, and the triangles LMX and LMZ are similar. But they have the side LM common, hence they are equal, and MX is equal to MZ. The two triangles CPM and CQM being also equal, CP is equal to CQ. And since CT and CS may be considered as perpendicular to RL and LF, they may be taken as equal. Whence $PT = QS$. By the similar triangles RPT and RMX,

$$RP : RM :: PT : MX;$$

and by the similar triangles FMZ and FQS,

$$QS : MZ :: FQ : FM;$$

whence, since $PT = QS$, and $MX = MZ$,

$$RP : RM :: FQ : FM \ldots\ldots (e).$$

Let $RM = u$, $MF = v$, $CM = r$, and the angle $RMC = \phi$. Then,

$$RP = RM - MP = RM - CM \,.\, \cos RMC, \text{ or,}$$

$$RP = u - r \cos \phi; \text{ and}$$

$$FQ = MQ - MF = MC \,.\, \cos CMF - MF, \text{ or,}$$

$$FQ = r. \cos \phi - v.$$

By substituting, in the proportion (e) above, for RP, and FQ, the values just found, and for RM, and FM, u, and v, we have

$$u - r. \cos \phi : u :: r. \cos \phi - v : v, \text{ or,}$$

$$\frac{u - r. \cos \phi}{u} = \frac{r. \cos \phi - v.}{v}. \text{ Dividing by } r,$$

$$\frac{1}{r} - \frac{\cos \phi}{u} = \frac{\cos \phi}{v} - \frac{1}{r}, \text{ and}$$

$$\frac{\cos \phi}{v} + \frac{\cos \phi}{u} = \frac{2}{r} \ldots\ldots (11),$$

whence,

$$v = \frac{ur}{\frac{2u}{\cos \phi} - r}.$$

(30.) To interpret equation (11) geometrically, we should observe that the ratio of MX to RM, that is, $\left(\frac{MX}{RM}\right)$, measures the divergency of the incident pencil LRN. But in the triangle MXL, $MX = ML \,.\, \cos LMX$, and since the angles XMR and LMC

are nearly right angles, LMX is nearly equal to RMC, or $MX = ML.\cos\phi$. $\frac{MX}{RM}$ therefore varies with $\frac{\cos\phi}{u}$, which, consequently measures the vergency of the incident rays. In like manner, $\frac{\cos\phi}{v}$ expresses the vergency of the reflected pencil.

We infer, then, from (11), that the *sum of the vergencies of the incident and reflected pencils is a constant quantity.*

If $\phi = o$, or the rays proceed *from a point in the axis*, $\cos\phi = 1$, and formula (11) becomes

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r},$$

agreeing with equation (1).

If the rays are *parallel*, $\frac{\cos\phi}{u} = o$, and

$$\frac{\cos\phi}{v} = \frac{2}{r}, \text{ or,}$$

$$v = \frac{r}{2}.\cos\phi.$$

(31.) To find the position of the point F', *for the plane perpendicular to RMC*, we have, in the triangle RMC,

$$MC : MR :: \sin MRC : \sin MCR, \text{ or,}$$

calling the angle RCD, θ, whence $MRC = \theta - \phi$,

$$r : u : \sin(\theta - \phi) : \sin\theta,$$

$$\frac{r}{u} = \frac{\sin(\theta - \phi)}{\sin\theta},$$

and, by substituting for $\sin(\theta - \phi)$ its value,

$$\frac{r}{u} = \frac{\sin\theta\cos\phi - \cos\theta.\sin\phi}{\sin\theta}.$$

If MF' be called v', we may deduce from the triangle CMF' by a method similar to that just used,

$$\frac{r}{v'} = \frac{\sin(\theta + \phi)}{\sin\theta}, \text{ or,}$$

$$\frac{r}{v'} = \frac{\sin\theta.\cos\phi + \cos\theta.\sin\phi}{\sin\theta}.$$

Adding together the values found for $\frac{r}{u}$, and $\frac{r}{v'}$, and reducing, we find,

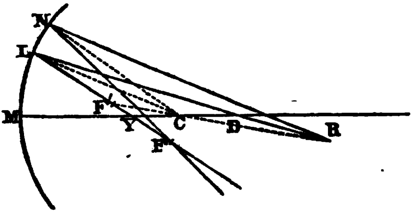
$$\frac{r}{u} + \frac{r}{v'} = 2.\cos\phi, \text{ or,}$$

$$\frac{1}{u} + \frac{1}{v'} = \frac{2.\cos\phi}{r} \ldots\ldots (12).$$

Since $\frac{\cos\phi}{u}$ represents the vergency of the incident rays, and $\frac{\cos\phi}{v'}$ that of the reflected rays, it is evident, from (12), that the sum of these quantities, for the incident and reflected rays, in the secondary plane, is not constant for the same mirror, but depends upon the angle, ϕ, of inclination of the axis of the pencil, to the axis of the mirror.

(32.) **Prop. VII.** *A small oblique pencil crosses the axis before meeting the mirror, required the form of the reflected pencil.*

Fig. B.



In the figure, R is the radiant point, RL the axis of a small pencil, of which one of the extreme rays is RN; MC is the axis of the mirror, which is cut by RL at the point D. F is the focus of the reflected pencil in the plane RMC. F' (found as in the last proposition) is the focus in the plane perpendicular to RMC. In this proposition must be found, besides the distances LF and LF', the point Y, at which LF cuts the axis, and the relation of the angle LYM to the angle LDM, which latter angle is given.

Call, as in article (29.), $RL = u$, $LF = v$, $LF' = v'$, $CL = r$; also let $MY = z$, $MD = b$.

Since CL bisects the angle DLY, we have,

$$YL : DL :: YC : DC,$$

or, taking for YL and DL, the distances YM and DM, which are nearly equal to them,

$$YM : DM :: YC : DC, \text{ that is,}$$

$$z : b :: r - z : b - r, \text{ or,}$$

$$\frac{r-z}{z} = \frac{b-r}{b}, \text{ or,}$$

$$\frac{1}{z} - \frac{1}{r} = \frac{1}{r} - \frac{1}{b}, \text{ and}$$

$$\frac{1}{z} + \frac{1}{b} = \frac{2}{r} \; \ldots\ldots (13).$$

An equation from which z may be determined.

The approximate value of the angle LYM may be obtained by supposing that the tangents of LYM and LDM are to each other as the distances YM and DM, or,

$$\frac{\tan Y}{\tan D} = \frac{YM}{DM} = \frac{z}{b}.$$

The distances LF and LF' will be given, as before, by the equations

$$\frac{\cos\phi}{u} + \frac{\cos\phi}{v} = \frac{2}{r} \; \ldots\ldots (11), \text{ and}$$

$$\frac{1}{u} + \frac{1}{v'} = \frac{2 \cdot \cos\phi}{r} \; \ldots\ldots (12).$$

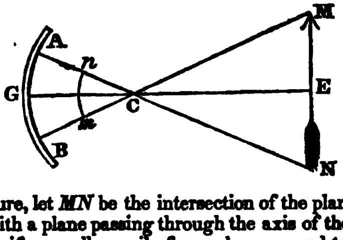
CHAP. II.

FORMATION OF IMAGES BY REFLEXION.

33.) In discussing the subject of the formation of images by reflexion, the surface of the mirror will be assumed to be spherical, and the most useful case will be solved; namely, that of a plane perpendicular to the axis of the mirror. The objects, of which images are to be formed by mirrors, when not plane, are generally of irregular forms, and the images can be found, only, by points.

Prop. VIII. *To determine the image of a plane, perpendicular to the axis of a mirror.*

Fig. C



In the figure, let MN be the intersection of the plane constituting the object, with a plane passing through the axis of the mirror. It is evident that if a small pencil of rays be supposed to proceed from each point of MN, and the several foci of the pencils be determined, the assemblage of the points thus found will give the image. From M and N, through C, the centre of the mirror, draw MB and NA, the axes of two pencils of rays from M and N, and upon which, therefore, the foci of the pencils will be found. Let the focus of the pencil, of which MB is the axis, be at m.

By our former notation, (art. 21.), $MC = u'$, $Cm = v'$, and $CB = r$. Call CE, the distance of the object from the centre of the mirror, d; and the angle MCE, θ: then, since

$$CE = CM \,.\, \cos MCE,$$

$$d = u' \,.\, \cos \theta, \text{ or,}$$

$$u' = \frac{d}{\cos \theta}.$$

The general formula, for the relation of the distance of the radiant point, and of the focus, from the centre, was, art. (23.),

$$\frac{1}{v'} = \frac{1}{f} + \frac{1}{u'}.$$

and, substituting the value of u', just found,

$$\frac{1}{v'} = \frac{1}{f} + \frac{\cos \theta}{d} \text{ .....} (14).$$

The polar equation of a conic section, the focus being the pole, is (Young's Analyt. Geom., Chap. V.)

$$r = A \frac{1 - e^2}{1 + e \,.\, \cos \omega}, \text{ or,}$$

$$\frac{1}{r} = \frac{1}{A(1 - e^2)} + \frac{e \,.\, \cos \omega}{A(1 - e^2)}$$

This equation will be identical with (14), if we suppose $\frac{1}{r} = \frac{1}{v'}$, $\frac{1}{A(1 - e^2)} = \frac{1}{f}$, $\frac{e}{A(1 - e^2)} = \frac{1}{d}$, and $\omega = \theta$.

The image of the line MN is, therefore, a portion of a *conic section, of which C, the centre of the mirror, is one of the foci.*

(34.) Since $f = A(1 - e^2) = A\left(1 - \frac{e^2}{A^2}\right) = \frac{B^2}{A}$, (Analyt. Geom.) the semi-parameter of the conic section, and f is constant, it follows that the radius of curvature at the vertex of the conic section, which is equal to the semi-parameter, is independent of the distance of the object from the centre of the mirror.

(35.) We proceed to examine the variation in the figure of the conic section just found, when the distance of the object from the mirror varies.

First, let the object be infinitely distant. Then, $\frac{1}{d} = o$, and $\frac{e}{A(1-e^2)} = o$, that is, $e = o$, and the image is a portion of a *circle*, corresponding to the equation $\frac{1}{v'} = \frac{2}{r}$, or, $v' = \frac{r}{2}$. The radius of the circle is half that of the mirror.

As the object is brought nearer to the mirror, d diminishes, or $\frac{1}{d}$ increases; but, $\frac{1}{d} = \frac{e}{A(1-e^2)}$, and, since $A(1-e^2) = f$, or is constant, e must vary as $\frac{1}{d}$, and, therefore, increases. As long as $d > f$, or $\frac{1}{d} < \frac{1}{f}$, $\frac{e}{A(1-e^2)} < \frac{1}{A(1-e^2)}$, or $e < 1$, and the image is a portion of an *ellipse*.

When $d = f$, $e = 1$, and the curve is a *parabola*. For $d < f$, $e > 1$, and the curve becomes a branch of a *hyperbola*.

If $d = o$, or the object passes through the centre of the mirror, $\frac{1}{d} = \infty$, and e is infinite, or the image is a *straight line*, coinciding with the object.

When the object passes the centre, towards the mirror, d becomes negative, and the equation (14) changes, if we reckon θ from GC, to

$$\frac{1}{v'} = \frac{1}{f} - \frac{\cos\theta}{d} \; \ldots\ldots (15).$$

This equation gives a hyperbola while $d < f$, a parabola when $d = f$, an ellipse when $d > f$.

Each of these cases presents curious circumstances. For example, in the case, $d < f$, if a point be taken in the object, so that $u' = f$, that is, $\frac{d}{\cos\theta} = f$, the equation for the focal distance of the pencil proceeding from that point, is

$$\frac{1}{v'} = o, \; v' = \infty;$$

the image is infinitely remote from the mirror. If we suppose the object to be sufficiently extended to cut the mirror, the point common to the object and mirror is its own image, and for that point $u' = r$, and $v' = r$; between the points for which $u' = r$ and $u' = f$, the distance of the image has varied from r to infinity, and, therefore, that portion of the object which is between these limits, has a virtual image, the part of each branch of which, between $v' = r$ and the vertex, is wanting.

The part of the object between $u' = d$ and $u' = f$, has its image beyond the centre; it is the branch of a hyperbola conjugate to the first.

(36.) If the section of the object be an arc concentric with the mirror, u' is constant, and

$$\frac{1}{v'} = \frac{1}{f} + \frac{1}{u'}$$

is constant, or the image is also a circular arc concentric with the mirror. In this case, the relative magnitudes of the object and image are as their distances from the centre of the mirror.

(37.) We have considered a section of the object, of the image, and of the mirror, made by a plane passing through the axis of the mirror; if these sections be supposed to revolve about the common axis, the section of the object will generate a plane, and that of the mirror, and of the image, surfaces of revolution corresponding to the sections.

(38.) The case of a convex mirror is embraced by equation (14), if r be made negative.

For the plane mirror, $r = \infty$, and $\frac{2}{r} = o$, whence,

$$\frac{1}{v'} = \frac{\cos\theta}{d} \;\ldots\ldots (16),$$

and the *image is similar to the object.*

REFRACTION.

CHAP. III.

REFRACTION BY PRISMS AND LENSES.

(39.) The most simple case of the refraction of light, is that in which it takes place at a plane surface. The perpendicular being drawn, the refracted ray is connected with the incident, by the law (p. 29, text,) that the sine of the angle of refraction bears a constant ratio, for a given medium, to the sine of the angle of incidence. To represent this law analytically, suppose a ray passing from a rarer to a denser medium, call the angle of incidence ϕ, the angle of refraction ϕ', and let the sign of incidence be to that of refraction, as m is to 1, when m will represent the index of refraction of the denser medium, that of the rarer medium being unity; we have

$$\sin\phi : \sin\phi' :: m : 1, \text{ or,}$$

$$\sin\phi = m \,.\, \sin\phi' \;\ldots\ldots (17).$$

When the ray passes from a denser to a rarer medium, ϕ' represents the angle of incidence, and ϕ that of refraction.

If the angles ϕ and ϕ' be very small, they may be taken instead of their sines, in which case,

$$\phi = m \cdot \phi'.$$

(40.) The difference between the angles of incidence and refraction is termed the *deviation* of the ray, for a single surface. When the angles are very small, we have, for the deviation,

$$\phi - \phi' = m\phi' - \phi' = (m - 1)\,\phi',$$

$$\phi - \phi' = \frac{m-1}{m} \cdot \phi \;\ldots\ldots\; (18).$$

The deviation, therefore, when the angle of incidence is small, bears a constant ratio to that angle.

(41.) The case of the *total reflexion* of a ray moving in a denser medium, and arriving at the separating surface of the denser and of a rarer medium, (pp. 34, 35, text), is comprehended in equation (17). The ray passing from a denser to a rarer medium, if ϕ' be taken to represent the angle of incidence, and ϕ that of refraction, m will remain greater than unity, the angles being, as before, connected by the equation

$$m \cdot \sin\phi' = \sin\phi \;\ldots\ldots\; (17).$$

In this equation, since $\sin\phi$ can never exceed unity, $m \sin\phi'$ cannot exceed unity, whence $\sin\phi'$ cannot exceed $\frac{1}{m}$, (in which case, $m \cdot \sin\phi' = 1$,) or, the equation cannot be satisfied if $\sin\phi' > \frac{1}{m}$. The ray then ceases to be refracted; it is wholly reflected.

The angle, at which, light, passing through a denser medium, and meeting the separating surface of the denser and of a rarer medium, ceases to be refracted, is found from the equation $\sin\phi' = \frac{1}{m}$. If the denser medium be glass, and the rarer, air, $\sin\phi' = \frac{1}{m} = \frac{2}{3}$, whence, $\phi' = 41° 48'$.

(42.) The phenomena of reflexion might be derived, analytically, from those of refraction, by considering that the angle of reflexion is measured with the same perpendicular as that of refraction, or, is the supplement of that measured by ϕ', (see text, *fig.* 22., p. 35), and that the angles of incidence and reflexion are equal, but on opposite sides of the perpendicular; so that, in the case of reflexion, $\sin\phi = -\sin\phi'$, or, $m = -1$.

(43.) PROP. IX. *To determine the course of a single ray, or of a small pencil of rays, refracted by a prism.* (*Fig.* 20., p. 32, text.)

Let the angle, *HRM*, of incidence, upon the first surface, be called ϕ; *H'RN*, the corresponding angle of refraction, ϕ'; *H'R'N*, the angle of incidence on the second surface, ψ'; *n'R'b'*, the angle of emergence from the prism, ψ; the angle, *A*, of the prism, *a*. The angles, *RAR'*, *ARR'*, and *AR'R*, together, are equal to two right angles: but $ARR' = 90^\circ - \phi'$, and $AR'R = 90^\circ - \psi'$ whence,

$$a + (90 - \phi') + (90 - \psi') = 180, \text{ or,}$$

$$a = \phi' + \psi' \ldots\ldots (19).$$

The angle of total deviation, *DEH*, is equal to the sum of the partial deviations, *ERR'* and *ER'R*, that is, calling the deviation δ,

$$\delta = \phi - \phi' + \psi - \psi', \text{ or,}$$

$$\delta = \phi + \psi - (\phi' + \psi'), \text{ or,}$$

$$\delta = \phi + \psi - a \ldots\ldots (20).$$

This value of the deviation contains the given angles ϕ and a, and the angle ψ, which may be found by means of the relations of ϕ, ϕ', ψ', and ψ, as given by the following equations:

$$\sin\phi = m \,.\, \sin\phi' \ldots\ldots (17),$$

$$\psi' = a - \phi' \ldots\ldots\ldots (19),$$

$$\sin\psi = m \,.\, \sin\psi' \ldots\ldots (17').$$

(44.) If we suppose the angles very small, we have, from (17),

$$\phi = m\phi', \text{ whence,}$$

$$\phi - \phi' = (m - 1)\,\phi'.$$

Also, from (17'),

$$\psi = m\psi',$$

$$\psi - \psi' = (m - 1)\,\psi', \text{ and}$$

$$\delta = \phi - \phi' + \psi - \psi' = (m - 1)\,(\phi' + \psi');$$

but (19) gives

$$\phi' + \psi' = a, \text{ whence,}$$

$$\delta = (m - 1) \,.\, a \ldots\ldots (21),$$

a value depending only upon the refractive power of the material of the prism, and upon its refracting angle.

(45.) There are two other cases in which the deviation, as given by equations (20), (17), (17'), and (19), assumes a somewhat simple form. These we shall consider, in order.

(46.) **Prop. X.** *To find the deviation of a ray, or of a small pencil of rays, incident perpendicularly on one of the surfaces of a prism.*

When the incidence upon the first surface is perpendicular, $\phi = o$, and $\phi' = o$, and equation (20) becomes,

$$\delta = \psi - a,$$

and (19),

$$\psi' = a, \text{ whence,}$$

$$\sin \psi' = m \,.\, \sin \psi' = m \,.\, \sin a;$$

but from the value just found for δ, we have,

$$\psi = a + \delta, \text{ whence,}$$

$$\sin (a + \delta) = m \,.\, \sin a \,\ldots\ldots (22):$$

from which equation, δ becomes known when a and m are given; or,

$$m = \frac{\sin (a + \delta)}{\sin a},$$

may be found by determining δ and a. This is one method of *determining the refractive power of a substance.* Another method is furnished by the next proposition.

(47.) **Prop. XI.** *To determine the deviation of a ray, or of a small pencil of rays, when the angles of incidence and emergence are equal.* (*Fig.* 20., p. 32, text.)

By the condition of the question $\phi = \psi$, and since, from (17),

$$\sin \phi' = \frac{\sin \phi}{m}, \text{ and } \sin \psi' = \frac{\sin \psi}{m},$$

$$\sin \phi' = \sin \psi', \text{ or, } \phi' = \psi'.$$

Equation (19), gives,

$$2\phi' = a, \text{ or, } \phi' = \frac{a}{2},$$

and from (20), by making $\phi = \psi$,

$$\delta = 2\phi - a, \text{ and } \phi = \frac{a + \delta}{2}.$$

Substituting these values for ϕ and ϕ' in (17), it becomes

$$\sin \frac{1}{2} (a + \delta) = m \,.\, \sin \frac{1}{2} a \,\ldots\ldots (23);$$

an equation from which δ may be determined when a and m are given.

By transmitting light through a prism so that $\phi = \psi$, we have a method of *measuring the refractive power* of the substance of which it is composed, for,

$$\sin \tfrac{1}{2}(a + \delta) = m . \sin \tfrac{1}{2} a, \text{ whence,}$$

$$m = \frac{\sin \frac{1}{2}(a + \delta)}{\sin \frac{1}{2} a} \ldots\ldots (24).$$

The equality of the angles of incidence and emergence may be ascertained, by measurement, with the instrument which has been devised for this purpose, or by the use of the proposition which follows.

(48.) **Prop. XII.** *The angles of incidence and emergence, of a ray passing through a prism, are equal, when the deviation is a minimum.*

The proposition requires the deviation to be a minimum. We therefore find its value, differentiate it, and put the differential coefficient equal to zero. The value of the deviation, from equation (20), is,

$$\delta = \phi + \psi' - a,$$

the differential of which, a being constant, is,

$$d\delta = d\phi + d\psi, \text{ whence,}$$

$$\frac{d\delta}{d\phi} = 1 + \frac{d\psi}{d\phi}.$$

Equation (19), is

$$a = \phi' + \psi',$$

by the differentiation of which, we obtain

$$\frac{d\psi}{d\phi'} = -1.$$

From equation (17), by a similar process, we have,

$$d . \sin \phi = m . d \sin \phi', \text{ or,}$$

$$\cos \phi . d\phi = m . \cos \phi' . d\phi', \text{ or,}$$

$$d\phi = m . \frac{\cos \phi'}{\cos \phi} . d\phi'.$$

In like manner, we obtain, by differentiating (17'),

$$d\psi' = m . \frac{\cos \psi'}{\cos \psi} . d\psi'.$$

Dividing the second of these equations by the first,

$$\frac{d\psi}{d\phi} = \frac{\cos \phi . \cos \psi'}{\cos \phi' . \cos \psi} . \frac{d\psi'}{d\phi'}, \text{ or, since}$$

$$\frac{d\psi'}{d\phi'} = -1,$$

$$\frac{d\psi}{d\phi} = -\frac{\cos \phi . \cos \psi'}{\cos \phi' . \cos \psi}$$

Substituting this value for $\frac{d\psi}{d\phi}$ in the value of $\frac{d\delta}{d\phi}$ found above, it becomes,

$$\frac{d\delta}{d\phi} = 1 - \frac{\cos\phi \cdot \cos\psi'}{\cos\phi' \cdot \cos\psi}.$$

This value found for the differential coefficient of δ, is to be made equal to zero, whence,

$$\frac{\cos\phi \cdot \cos\psi'}{\cos\phi' \cdot \cos\psi} = 1 \ldots\ldots (25);$$

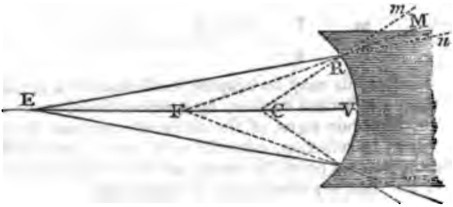
an equation which is satisfied if $\phi = \psi$, or *the angles of incidence and emergence are equal.* The method of using this proposition is described in art. 35, pp. 33, 34, of the text.

Refraction by Lenses.

(49.) In discussing the subject of refraction by lenses, we shall consider the refracting surfaces as spherical. The pencil of incident rays will be, first, assumed as indefinitely small, and the thickness of the lens neglected; next, the correction for thickness will be introduced, and, lastly, under the title of *Aberration of Lenses,* the case of a pencil of any magnitude will be considered. Refraction through spheres and parallel plane surfaces, will be deduced from a consideration of the general case.

(50.) **Prop. XIII.** *To determine the course of an indefinitely small pencil of rays, passing from a rarer to a denser medium, through a spherical surface.*

Fig. D.



Let E be the radiant point of a small pencil, of which EV is the axis, and ER the extreme ray; the ray ER, passing into the denser medium, will be refracted towards the perpendicular, CR, making the angle mRM less than ERC, and in such a proportion that $\sin ERC : \sin mRM :: m : 1$, m representing the index of refraction of the denser medium, that of the rarer medium being unity.

Call $EV = u$, $FV = u'$, $CV = r$. In the triangle, ERC, we have,

$$\frac{EC}{ER} = \frac{\sin ERC}{\sin ECR},$$

and in FRC,

$$\frac{FC}{FR} = \frac{\sin FRC}{\sin FCR}.$$

Dividing the first of these equations by the second,

$$\frac{EC}{ER} \cdot \frac{FR}{FC} = \frac{\sin ERC}{\sin FRC}, \text{ and since}$$

$$\sin ERC = m \,.\, \sin mRM = m \,.\, \sin FRC,$$

$$\frac{EC}{ER} \cdot \frac{FR}{FC} = m.$$

The pencil of rays being very small, the point R is very near to V, and for ER and FR we may take their approximate values EV and FV, whence, $EC = EV - CV = u - r$, and $FC = FV - CV = u' - r$. Substituting these values of EC and FC, and the values of ER and FR for those lines in the equation just found, it becomes

$$\frac{u-r}{u} \cdot \frac{u'}{u'-r} = m, \text{ or,}$$

$$\frac{u-r}{u} = m \,.\, \frac{u'-r}{u'}; \text{ and, dividing by } r,$$

$$\frac{1}{r} - \frac{1}{u} = \frac{m}{r} - \frac{m}{u'}, \text{ or, by transposition,}$$

$$\frac{m}{u'} - \frac{1}{u} = \frac{m-1}{r} \;\ldots\ldots\; (26).$$

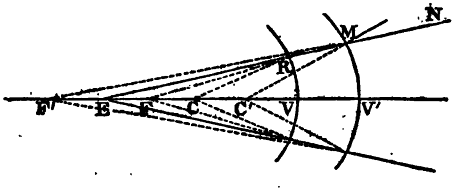
(51.) This formula may be adapted to all cases of a small pencil incident upon a spherical surface, by a conventional mode of considering the algebraic signs, of the different quantities involved in it. Let us, as in reflexion, consider u, u', and r, essentially *positive* when the radiant point, the focus, and centre, are, respectively, in *front* of the lens, *negative* in the contrary case.

The *positive* sign of u' will, then, correspond to an *imaginary*, or *virtual*, focus; and the negative sign, to a real focus. This it is important to the student to recollect, since the reverse has been the case in reflexion.

It would hardly be useful to discuss the variations of formula (26), by changes in the quantities concerned in it, since we must consider, in refraction by lenses, the action of two surfaces.

(52.) **Prop. XIV.** *To determine the course of a small pencil of rays falling upon a lens; the radiant point of the pencil being in the axis of the lens.*

Fig. E.



Let ER, as before, be the extreme ray of the pencil, EV the axis of the pencil and of the lens, RM the ray refracted at the first surface, CR the radius of that surface, M the point in which RM meets the second surface, $C'M$ the radius of the second surface drawn to the point M. Producing MR until it meets the axis, F is the virtual focus of rays refracted by the first surface. Since the refraction at M is from a denser to a rarer medium, the ray is refracted from the perpendicular, taking the direction MN, which, prolonged until it intersects the axis, gives F' for the virtual focus of the pencil refracted by the lens.

Keeping the notation of the last proposition $EV = u$, $FV = u'$, $CV = r$, and the equation for refraction at the first surface is, (26),

$$\frac{m}{u'} - \frac{1}{u} = \frac{m-1}{r} \; \ldots\ldots (26).$$

The equation for the second surface may be inferred from (26), or may be obtained directly, by proceeding in the triangles FMC' and $F'MC'$, as was done, in the last proposition, in ERC and FRC. Thus,

$$\frac{FC'}{FM} = \frac{\sin FMC'}{\sin FC'M}, \text{ and } \frac{F'C'}{F'M} = \frac{\sin F'MC'}{\sin F'C'M},$$

and, by division,

$$\frac{FC'}{FM} \cdot \frac{F'M}{F'C'} = \frac{\sin FMC'}{\sin F'MC'} = \frac{1}{m}.$$

Call VV', the thickness of the lens, t; $F'V'$, v; $C'V'$, r'; then $FV' = u' + t$, $FC' = u' + t - r'$, and $F'C' = v - r'$. These values being substituted, in the equation just found, instead of FM, $F'M$, &c., we have,

$$\frac{u' + t - r'}{u' + t} \cdot \frac{v}{v - r'} = \frac{1}{m}, \text{ whence,}$$

$$\frac{v-r'}{v} = m \cdot \frac{u'+t-r'}{u'+t}, \text{ or,}$$

$$1-\frac{r'}{v} = m\left(1-\frac{r'}{u'+t}\right);$$

dividing by r', and collecting the terms,

$$\frac{1}{v} - \frac{m}{u'+t} = \frac{1-m}{r'};$$

m being greater than unity, the sign of the second member is really negative, and as it will be convenient to shew this, we change the form of that member, and the equation becomes,

$$\frac{1}{v} - \frac{m}{u'+t} = -\frac{m-1}{r'} \ldots\ldots (27).$$

This is the general equation between u', v, r', t, and m, which, combined with (26), will determine v in terms of u, r, r', t, and m, all of which are given quantities.

(53.) If the *thickness* of the lens is so small that it *may be neglected*, equation (27) becomes,

$$\frac{1}{v} - \frac{m}{u'} = -\frac{m-1}{r'}, \text{ but, from (26),}$$

$$\frac{m}{u'} = \frac{1}{u} + \frac{m-1}{r}, \text{ whence,}$$

$$\frac{1}{v} - \frac{1}{u} - \frac{m-1}{r} = -\frac{m-1}{r'}, \text{ or,}$$

$$\frac{1}{v} - \frac{1}{u} = (m-1)\left(\frac{1}{r} - \frac{1}{r'}\right) \ldots\ldots (28).$$

Since $\frac{1}{u}$ represents the divergency, or convergency, of the incident pencil, and $\frac{1}{v}$ that of the refracted pencil, we deduce, from (28), that the *difference of the vergencies of the refracted and incident pencils is a constant quantity for the same lens.*

This formula applies to the different cases of the incident pencil and lens, as in the single surface (art. 51), if we consider the distances of the radiant point, focus, and centre, *positive* when in *front* of the lens, and negative in the contrary case. The same remark applies to the general equations (26) and (27).

(54.) In most of the cases which occur in practice, the thickness of the lens may be neglected, and, therefore, equation (28) is applicable to them; in all cases this equation determines an approximate value of the focal distance, to which a correction for the thickness of the lens may be, conveniently, applied.

(55.) To obtain this *correction for thickness*, expand $\frac{m}{u'+t}$ in equation (27), into a series, by division,

$$\frac{m}{u'+t} = \frac{m}{u'} - \frac{mt}{u'^2} + \frac{mt^2}{u'^3} \mp \&c.$$
$$= \frac{m}{u'} - \frac{m}{u'}\left(\frac{t}{u'} - \frac{t^2}{u'^2} \pm \&c.\right)$$

If the thickness of the lens is not very great compared with the distance of the point F, the powers of $\frac{t}{u'}$, higher than the first, may be neglected, and we have,

$$\frac{m}{u'+t} = \frac{m}{u'} - \frac{mt}{u'^2}.$$

Substituting this value for $\frac{m}{u'+t}$ in (27), it becomes,

$$\frac{1}{v} - \frac{m}{u'} + \frac{mt}{u'^2} = -\frac{m-1}{r'},$$

or substituting for $\frac{m}{u}$ its value from (26),

$$\frac{1}{v} - \frac{1}{u} - \frac{m-1}{r} + \frac{mt}{u'^2} = -\frac{m-1}{r'}; \text{ or,}$$

by transposing and collecting the terms,

$$\frac{1}{v} = \frac{1}{u} + (m-1)\left(\frac{1}{r} - \frac{1}{r'}\right) - \frac{mt}{u'^2} \; \ldots\ldots (29),$$

in which we perceive the approximate value of $\frac{1}{v}$ given by equation (28), and a *correction* $-\frac{mt}{u'^2}$ for the *thickness* of the lens.

(56.) **Prop. XV.** *To determine the form of a small pencil refracted by a medium, bounded by parallel planes.*

For plane surfaces, r and r' are infinite, whence $\frac{1}{r} = 0$, and $\frac{1}{r'} = 0$.

Equation (28) becomes,

$$\frac{1}{v} - \frac{1}{u} = 0, \text{ or, } \frac{1}{v} = \frac{1}{u} \; \ldots\ldots (30).$$

The vergency of the refracted pencil is the same with that of the incident pencil, when the plate is indefinitely thin.

(57.) Applying the correction from equation (29), we obtain,

$$\frac{1}{v} = \frac{1}{u} - \frac{mt}{u'^2},$$

but, from (29), by making $\frac{1}{r} = 0$,

$$\frac{m}{u'} = \frac{1}{u}, \text{ whence, } u' = mu;$$

Substituting this value of u' in that last given, for $\frac{1}{v}$,

$$\frac{1}{v} = \frac{1}{u} - \frac{t}{mu^2}, \text{ or,}$$

$$\frac{1}{v} = \frac{1}{u}\left(1 - \frac{t}{mu}\right) \ldots\ldots (31).$$

Equation (31) contains the theory of refracting plates of considerable thickness.

(58.) If the incident rays are *parallel*, $\frac{1}{u} = 0$, and $\frac{1}{v} = 0$, or, $v = \infty$, or parallel rays are unchanged by the refraction (fig. 23, p. 36, text.)

(59.) The value of $\frac{1}{v}$ for *diverging* rays, given by (31), is positive, zero, or negative, according as we have

$$1 > \frac{t}{mu},\ 1 = \frac{t}{mu},\ 1 < \frac{t}{mu}, \text{ that is,}$$

as $t < mu$, $t = mu$, $t > mu$, in which m is greater than unity. For ordinary cases of relation between u, and t, $(t < mu)$ $\frac{1}{v}$ is positive, and therefore the focus imaginary, or the rays still diverge after refraction. As u is measured from the first surface, and v from the second, the effect of refraction in bringing the object nearer to, or removing it farther from the plate, is not expressed by the relation of v and u, but by that of $v - t$, and u. To ascertain the effect of refraction in this point of view, we take the value of v in (31), or,

$$v = \frac{mu^2}{mu - t};$$

this, by dividing and neglecting the powers of t above the first, gives

$$v = u + \frac{t}{m}, \text{ whence}$$

$$v - t = u + t\left(\frac{1}{m} - 1\right) \ldots\ldots (32).$$

In which since $m > 1$, $\frac{1}{m} < 1$, and $\frac{1}{m} - 1$ is subtractive, or, $v - t < u$; the point of divergence, therefore, is brought nearer the first surface by the distance, $t\left(1 - \frac{1}{m}\right)$.

In a glass plate, $m = \frac{3}{2}$, and $1 - \frac{1}{m} = \frac{1}{3}$. The point of divergence is, therefore, brought nearer the first surface by one third the thickness of the plate.

For water, $m = \frac{4}{3}$, and $1 - \frac{1}{m} = \frac{1}{4}$.

(60.) If the incident rays *converge*, u is negative, whence from (31),

$$\frac{1}{v} = -\frac{1}{u}\left(1 + \frac{t}{mu}\right) \ldots\ldots (33),$$

in which $\frac{1}{v}$ is always negative, and, therefore, the rays still converge.

Proceeding to find the value of $v - t$, as before, we have

$$v - t = -\frac{mu^2}{mu + t} - t = -u + \frac{t}{m} - t,$$

$$v - t = -u - t\left(1 - \frac{1}{m}\right),$$

$$v - t = -\left(u + t\left(1 - \frac{1}{m}\right)\right) \ldots\ldots (34,)$$

from which it appears that the focus is farther from the first surface than the imaginary radiant point, by the distance $t\left(1 - \frac{1}{m}\right)$. The reverse of the result for diverging rays.

(61.) Prop. XVI. *To determine the form of a small pencil of rays refracted by a double convex lens.*

We will first consider the case in which the thickness of the lens may be neglected. To this, equation (28) will be adapted by making r, the radius of the first surface, negative, since its centre is turned from incident light. This gives

$$\frac{1}{v} - \frac{1}{u} = -(m-1)\left(\frac{1}{r} + \frac{1}{r'}\right) \ldots\ldots (35),$$

since $\frac{1}{v}$ represents the vergency of the refracted pencil, and $\frac{1}{u}$ that of the incident pencil, and $\frac{1}{v} - \frac{1}{u}$ is negative, m, r, r', being constant, for the same lens, we infer that the ***divergency destroyed, or convergency produced, by this lens, is a constant quantity.***

(62.) For *parallel* rays, $\frac{1}{u} = o$, and

$$\frac{1}{v} = -(m-1)\left(\frac{1}{r} + \frac{1}{r'}\right) \ldots\ldots (36).$$

In the refracting media of which lenses are made, $m > 1$, and this value of $\frac{1}{v}$ is negative: hence the focus lies behind the lens, and is real. For the distance of the principal focus we have by taking the value of v from (36),

$$v = -\frac{1}{m-1} \cdot \frac{rr'}{r+r'} \ldots\ldots (37).$$

If the lens is of glass, $m = \frac{3}{2}$, and $\frac{1}{m-1} = 2$, whence,

$$v = -\frac{2rr'}{r+r'},$$

corresponding to the rule given in the text (page 42).

If the glass is equally convex, $r = r'$, and

$$v = -\frac{2r^2}{2r} = -r.$$

The principal focal distance is equal to the radius of the surfaces of the lens.

(63.) The principal focal distance may serve as a convenient term of comparison for the focal distances of diverging and converging rays. Denoting it by f, we have the value of f given by (37), and of $\frac{1}{f}$ by (36), substituting in (35) $\frac{1}{f}$ for its equal; and transposing, we have,

$$\frac{1}{v} = \frac{1}{u} - \frac{1}{f} \ldots\ldots (38).$$

(64.) *Diverging rays.* Equation (38) applies to this case.

From that equation it appears that for the same lens, the vergency of the refracted rays $\left(\frac{1}{v}\right)$ is less than the divergency of the incident rays $\left(\frac{1}{u}\right)$ by a constant quantity $\left(\frac{1}{f},\right)$ depending upon the index of refraction of the material of the lens, and upon the curvature of its surfaces.

If $u > f$ (*fig.* 29., p. 43), $\frac{1}{u} < \frac{1}{f}$, and $\frac{1}{v}$ is negative, or the rays are brought to a focus. The reciprocal of the focal distance is, from (38),

$$\frac{1}{v} = -\left(\frac{1}{f} - \frac{1}{u}\right).$$

Since $\frac{1}{f} - \frac{1}{u} < \frac{1}{f}$, $\frac{1}{v} < \frac{1}{f}$, or $v > f$, and the focus is farther from the lens than the principal focus. As the radiant point approaches the lens, $\frac{1}{u}$ increases, and, of course, $\frac{1}{v}$, or $\frac{1}{f} - \frac{1}{u}$, diminishes, or v increases: that is, as the radiant point approaches the lens, the focus recedes, and vice versa.

When $u = 2f$, $\frac{1}{v} = -\left(\frac{1}{f} - \frac{1}{2f}\right) = -\frac{1}{2f}$, and $v = -2f$. The focus is as far from the lens as the radiant point.

If the rays proceed from a point as far from the lens as the principal focus (as from O', *fig.* 29.), $u = f$, and $\frac{1}{v} = 0$; the refracted rays are parallel.

The radiant point being still supposed to approach the lens, we have $u < f$, or $\frac{1}{u} > \frac{1}{f}$; $\frac{1}{u} - \frac{1}{f}$, or $\frac{1}{v}$, is then positive, and the rays are no longer brought to a focus.

(65.) Equation (35) will give the value of the focal distance for *diverging* rays: to determine this, transpose $\frac{1}{u}$ and bring the terms on the right-hand side of the equation to a common denominator, whence,

$$\frac{1}{v} = \frac{rr' - u\,(m - 1)\,(r + r')}{urr'}, \text{ and}$$

$$v = \frac{urr'}{rr' - u(m-1)(r+r')},$$

or changing the sign to correspond to a real focus, to which we have found the rays to be brought as long as $u > f$,

$$v = -\frac{urr'}{u(m-1)(r+r') - rr'}.$$

For a *glass* lens, $m = \frac{3}{2}$, and $m - 1 = \frac{1}{2}$; whence,

$$v = -\frac{2urr'}{u(r+r') - 2rr'} \quad \ldots\ldots (40).$$

This value of v gives the rule found in art. 45, of the text. The arithmetical operation there directed is changed for the subtraction of $u(r+r')$ from $2rr'$, when $2rr' > u(r+r')$, or $u < \frac{2rr'}{r+r'}$, or $u < f$; the algebraic expression shows by its change of sign, in that case, that the focus is imaginary.

If $r = r'$, or the lens is equally convex, (40) becomes

$$v = -\frac{2ur^2}{2ur - 2r^2}, \text{ or,}$$

$$v = -\frac{ur}{u-r};$$

agreeing with the rule just referred to.

(66.) For *converging* rays falling upon a double convex lens, we make $\frac{1}{u}$, in equations (35) and (38), negative, whence,

$$\frac{1}{v} = -\left[\frac{1}{u} + (m-1)\left(\frac{1}{r} + \frac{1}{r'}\right)\right] \ldots\ldots (41), \text{ and}$$

$$\frac{1}{v} = -\left(\frac{1}{u} + \frac{1}{f}\right) \ldots\ldots (42).$$

The sign of $\frac{1}{v}$ being always negative, whatever be the relation of u and f, the focus is always real. Since $\frac{1}{u} + \frac{1}{f} > \frac{1}{u}$, the convergency of the rays is increased by refraction.

Taking the value of v from (41), and making, in it, $m = \frac{3}{2}$, as was done in the case of diverging rays in the last article, we find for a *glass* lens,

$$v = -\frac{2urr'}{u(r + r') + 2rr'} \;\ldots\ldots (43).$$

For a *glass* lens *equally convex*, we have,

$$v = -\frac{ur}{u + r} \;\ldots\ldots (44).$$

These values for the focal distance give the rules on p. 44, of the text.

(67). In what precedes, we have neglected the thickness of the lens, and next proceed to show how a correction, for the effect of the thickness, may be introduced,

PROP. XVII. *To show the method of applying, to the approximate focal length found for a double convex lens, a correction for the effect of the thickness.*

As an example, let us take the case of *parallel* rays falling upon the lens. Equation (29) is applied to this case by making r negative, whence,

$$\frac{1}{v} = \frac{1}{u} - (m-1)\left(\frac{1}{r} + \frac{1}{r'}\right) - \frac{mt}{u'^2} \;\ldots\ldots (45).$$

And for parallel rays, for which $\frac{1}{u} = o$,

$$\frac{1}{v} = -(m-1)\left(\frac{1}{r} + \frac{1}{r'}\right) - \frac{mt}{u'^2} \;\ldots\ldots (46).$$

Equation (26) adapted to the case of a convex surface, gives,

$$\frac{m}{u'} = \frac{1}{u} - \frac{m-1}{r}$$

and for parallel rays,

$$\frac{m}{u'} = -\frac{m-1}{r}, \text{ whence,}$$

$$\frac{m}{u'^2} = \frac{(m-1)^2}{mr^2}$$

Substituting this value of $\frac{m}{u'^2}$ in (46), we have,

$$\frac{1}{v} = -(m-1)\left(\frac{1}{r} + \frac{1}{r'}\right) - \frac{(m-1)^2 t}{mr^2}, \text{ or,}$$

$$\frac{1}{v} = -\frac{1}{f} - \frac{(m-1)^2 t}{mr^2} \;\ldots\ldots (47).$$

To determine from this equation the correction to be applied to the focal length, v, we reduce the terms of the second member to a common denominator, whence,

$$\frac{1}{v} = -\frac{mr^2 + (m-1)^2 ft}{mr^2 f} \text{ and}$$

$$v = -\frac{mr^2 f}{mr^2 + (m-1)^2 ft},$$

$$v - (-f) = v + f = -\frac{mr^2 f}{mr^2 + (m-1)^2 ft} + f = \frac{(m-1)^2 f^2 t}{mr^2 + (m-1)^2 ft}.$$

Dividing, and neglecting the terms containing powers of t higher than the first,

$$v + f = \frac{(m-1)^2 f^2 t}{mr^2} \ldots\ldots (48),$$

the correction which is to be applied to the focal distance obtained by equation (37).

When the lens is *equiconvex* and of *glass*, we find (art. 62) that $f = -r$, to which a correction,

$$v + f = \frac{(\frac{1}{2})^2 \,.\, r^2 t}{\frac{3}{2}\, r^2} = \frac{1}{6}\, t,$$

is to be applied. The sign of the correction is contrary to that of the focal distance, and the effect is therefore subtractive. The corrected focal length is

$$v = -r + \tfrac{1}{6}\, t.$$

(68.) The method which has just been shown gives, at last, only an approximate value of the focal distance, which, however, is sufficiently accurate for all cases in which the thickness does not bear a considerable relation to the focal distance. In the case of a sphere used as a lens, the thickness is too considerable to use the method of correction already exhibited.

(69.) **Prop. XVIII.** *To find the focal length of a sphere for parallel rays.*

The supposition that the rays are parallel simplifies the question, without deducting much from its utility. Since $\frac{1}{u} = 0$, equations (26) and (27) become, by making r negative,

$$\frac{m}{u'} = -\frac{m-1}{r} \ldots\ldots (49), \text{ and}$$

$$\frac{1}{v} = \frac{m}{u' + t} - \frac{m-1}{r'} \ldots\ldots (50).$$

For the sphere $t = 2r$, $r = r'$; and (50) gives

$$\frac{1}{v} = \frac{m}{u' + 2r} - \frac{m-1}{r}, \text{ but from (49),}$$

$$u' = -\frac{mr}{m-1}, \text{ whence}$$

$$u' + 2r = 2r - \frac{mr}{m-1} = \frac{2mr - 2r - mr}{m-1}, \text{ or,}$$

$$u' + 2r = \frac{mr - 2r}{m-1} = \frac{r(m-2)}{m-1}.$$

Substituting this value of $u' + 2r$ in (50),

$$\frac{1}{v} = \frac{m(m-1)}{r(m-2)} - \frac{m-1}{r}, \text{ or}$$

$$\frac{1}{v} = \frac{m(m-1) - (m-1)(m-2)}{r(m-2)} = \frac{(m-1)(m-m+2)}{r(m-2)},$$

and

$$\frac{1}{v} = \frac{2(m-1)}{r(m-2)}, \text{ whence}$$

$$v = \frac{r(m-2)}{2(m-1)}.$$

The value just found is the distance of the focus from the second surface; call f the distance from the centre, then

$$f = v - r = \frac{r(m-2)}{2(m-1)} - r,$$

or, bringing to a common denominator and reducing,

$$f = -\frac{mr}{2(m-1)} \,.\,.\,.\,.\,.\, (51).$$

The rule on page 40 of the text, is given by the value of f just found.

If the refracting sphere be of *tabasheer*, $m = 1.11145$, and $f = -5r$, of course FQ (fig. 26, text,) $= -4r$. If the sphere be of water, $m = 1.3358$ and $f = -2r$ nearly, or FQ (fig. 26) $= -r$. For a sphere of glass, $m = 1.5$, $f = -1\frac{1}{2}r$, and $FQ = -\frac{1}{2}r$. For a sphere of zircon, $m = 2$, $f = -r$, and $FQ = o$.

(70.) Returning to the discussion of the formula for the refracted pencil when the lens is indefinitely thin, we take up the case next in order.

Prop. XIX. *To determine the form of a small pencil after refraction by a plano-convex lens.*

As in other discussions, the refractive power of the substance of the lens is assumed to exceed that of the medium traversed by the incident pencil, or $m > 1$.

The question obviously includes two cases; in the one, the plane side is turned towards incident light, in the other the curved side is thus directed.

(71.) First: when the *plane side* is turned to *incident* rays, $\frac{1}{r} = o$, whence from (28),

$$\frac{1}{v} - \frac{1}{u} = -\frac{m-1}{r'} \quad \ldots\ldots (52).$$

From this we infer, that the *divergency destroyed*, or *convergency produced*, *by this lens*, *is a constant quantity*, as in the double convex lens (art. 61), but the effect is less than in that lens by $\frac{m-1}{r}$, the power of the first surface; it is not necessary, therefore, to carry out the discussion of the properties of this lens. There will be no correction for thickness, for parallel rays, no refraction being produced by the first surface. This is shown by the analysis, the term $\frac{mt}{u'^2}$ (in 29) vanishing, since from (26), $\frac{m}{u'} = o$.

The *principal* focal distance given by making $\frac{1}{u} = o$ in (52), and inverting, is

$$f = -\frac{r'}{m-1}.$$

For a *glass* lens,

$$f = -2r'$$

(72.) Second: when the *convex side* is turned to *incident* light, $\frac{1}{r'} = o$, and r is negative; from (28),

$$\frac{1}{v} - \frac{1}{u} = -\frac{m-1}{r} \quad \ldots\ldots (53).$$

The effect is, as in the first case, to *destroy divergency* in the incident pencil, or to *produce, or increase, convergency;* and if we suppose $r = r'$, that is, the same lens to be used in both cases, the effect produced is the same.

For the *principal* focal distance,

$$f = -\frac{r}{m-1},$$

and for a *glass* lens,

$$f = -2r.$$

(73.) The thickness of the lens produces in this case an effect on the principal focal length, since the rays refracted by the first surface fall obliquely upon the second.

To introduce the correction into the value of the principal focal distance, we recur to equations (26) and (27); making, in these, r negative, $\frac{1}{r'} = 0$, and $\frac{1}{u} = 0$, we obtain,

$$\frac{m}{u'} = -\frac{m-1}{r} \;\ldots\ldots\; (54),$$

$$\frac{1}{v} = \frac{m}{u'+t} \;\ldots\ldots\; (55).$$

The value of u' from (54), is

$$u' = -\frac{mr}{m-1}, \text{ and}$$

$$u' + t = -\frac{mr}{m-1} + t = -m\left(\frac{r}{m-1} - \frac{t}{m}\right), \text{ but from (55),}$$

$$v = \frac{u'+t}{m},$$

and substituting for $u' + t$ the value just found,

$$v = -\frac{r}{m-1} + \frac{t}{m} \;\ldots\ldots\; (56).$$

The correction, therefore, shortens the approximate focal length of the lens by $\frac{1}{m}$th part of its thickness; if the lens be of glass,

$$v = -2r + \tfrac{2}{3}t, \text{ or,}$$
$$v = -(2r - \tfrac{2}{3}t).$$

(74.) **Prop. XX.** *To determine the form of a small pencil, after refraction by a double concave lens.*

In this form the radius of the first surface is positive, that of the second negative. When the thickness of the lens may be neglected, we have from (28),

$$\frac{1}{v} - \frac{1}{u} = (m-1)\left(\frac{1}{r} + \frac{1}{r'}\right) \;\ldots\ldots\; (57),$$

and where an approximate value of the thickness may be used, from (29,)

$$\frac{1}{v} - \frac{1}{u} = (m-1)\left(\frac{1}{r} + \frac{1}{r'}\right) - \frac{mt}{u'^2} \;\ldots\ldots\; (58),$$

in which u' is determined from equation (26), or

$$\frac{m}{u'} = \frac{1}{u} + \frac{m-1}{r} \;\ldots\ldots\; (59).$$

(75.) When the incident rays are *parallel*, (fig. 31, p. 44, text,) (57) gives

$$\frac{1}{v} = (m-1)\left(\frac{1}{r} + \frac{1}{r'}\right)$$

or calling f the principal focal distance, and determining it from the equation just given,

$$f = \frac{1}{(m-1)} \cdot \frac{rr'}{(r+r')} \dots\dots (60).$$

This value being positive, the focus is imaginary, and at a distance expressed by the product of the radii divided by the index of refraction, less one, into the sum of the radii. The rule corresponds to that for a double convex lens; in fact, equations (60) and (37), differ only in their sign.

(76.) *Diverging* rays. In this case $\frac{1}{u}$ is positive, and, therefore, as long as $m > 1$, the value of v, from equation (57), will always be positive and the focus imaginary; since

$$\frac{1}{v} = \frac{1}{u} + (m-1)\left(\frac{1}{r} + \frac{1}{r'}\right) \dots\dots (57),$$

it appears that $\frac{1}{v} > \frac{1}{u}$, or the divergency of the rays is increased by the refraction, (fig. 32, text.)

In a *glass* lens,

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{2}\left(\frac{1}{r} + \frac{1}{r'}\right), \text{ or}$$

$$v = \frac{2urr'}{2rr' + u(r+r')},$$

whence the rule on page 45 of the text.

(77.) When the rays *converge*, $\frac{1}{u}$ is negative, and (57) becomes

$$\frac{1}{v} = -\frac{1}{u} + (m-1)\left(\frac{1}{r} + \frac{1}{r'}\right) \dots\dots (61).$$

The pencil still converges, is rendered parallel, or diverges, according to the relation between $\frac{1}{u}$ and $(m-1)\left(\frac{1}{r} + \frac{1}{r'}\right)$, or its equal $\frac{1}{f}$. If $\frac{1}{u} < \frac{1}{f}$ or $u > f$, $\frac{1}{v}$ is positive, and the rays still converge; if $\frac{1}{u} = \frac{1}{f}$, or $u = f$, $\frac{1}{v} = o$, and the refracted rays are parallel; if $\frac{1}{u} > \frac{1}{f}$, or $u < f$, $\frac{1}{v}$ is negative, and the rays are brought to a focus. This real focus is as far behind the lens, as the virtual focus of parallel rays is in front of it, if $u = \frac{f}{2}$, or $\frac{1}{u} = \frac{2}{f}$; for then $-\frac{1}{u} + \frac{1}{f} = -\frac{1}{f}$, and $v = -f$: the distance exceeds that just named if $u > \frac{f}{2}$ when we shall have $\frac{1}{u} < \frac{2}{f}$ and $-\frac{1}{u} + \frac{1}{f} < -\frac{1}{f}$, or $\frac{1}{v} < -\frac{1}{f}$ and $v > -f$: the reverse will of course be true if $u < \frac{f}{2}$.

If, as was first supposed, $u > f$ or $\frac{1}{u} < \frac{1}{f}$, though the rays still converge after refraction, they converge less than before it, for $-\frac{1}{u} + \frac{1}{f} < \frac{1}{f}$.

We shall not introduce the correction for thickness, as it would be determined by the same method with that for the double convex lens. Practically the thickness of double concave lenses is of little importance, since it is least at the central parts.

(78.) **Prop. XXI.** *To determine the form of a small pencil of rays, after refraction by a plano-concave lens.*

First. When the *concave* side is turned to incident rays, $\frac{1}{r'} = o$, and r is positive; equation (28) gives

$$\frac{1}{v} - \frac{1}{u} = \frac{(m-1)}{r} \; \ldots\ldots (62).$$

The divergency produced by this lens is, therefore, less than that produced by a double concave lens, by $\frac{(m-1)}{r'}$, the effect of the second surface.

Second. When the *plane* side is towards incident light. Then $\frac{1}{r} = o$, and r' is negative, whence,

$$\frac{1}{v} - \frac{1}{u} = \frac{(m-1)}{r'} \; \ldots\ldots (63),$$

agreeing with the expression found above, if the lens is the same in each case, or $r = r'$.

(79.) The next form to be considered is the meniscus.

Prop. XXII. *To determine the form of a small pencil of light after refraction by a meniscus.*

When the *convex* side of the meniscus is turned towards incident light, the signs of both r and r' are negative. The general formula (28) gives

$$\frac{1}{v} - \frac{1}{u} = -(m-1)\left(\frac{1}{r} - \frac{1}{r'}\right) \ldots\ldots (64),$$

in which, by the nature of this lens, $r' > r$, or $\frac{1}{r'} < \frac{1}{r}$

From this relation of $\frac{1}{r'}$ and $\frac{1}{r}$ it follows, that $\frac{1}{r} - \frac{1}{r'}$ is a positive quantity, and therefore the sign of the right hand side of this equation is negative. The equation corresponds to that for the double convex lens (35), but the divergency destroyed by the meniscus is $(m-1)\left(\frac{1}{r} - \frac{1}{r'}\right)$, while that by the double convex lens was $(m-1)\left(\frac{1}{r} + \frac{1}{r'}\right)$. *The power of the meniscus is the difference between the powers of its two surfaces.*

(80.) When the *concavity* of the meniscus is turned to incident light, r and r' are positive, and $r > r'$, or $\frac{1}{r} < \frac{1}{r'}$.

Equation (28) applies directly to this case, and

$$\frac{1}{v} - \frac{1}{u} = -(m-1)\left(\frac{1}{r'} - \frac{1}{r}\right) \ldots\ldots (65).$$

Since $\frac{1}{r} < \frac{1}{r'}$, $\frac{1}{r'} - \frac{1}{r}$ is a positive quantity, hence $\frac{1}{v} - \frac{1}{u}$ is negative. Equations (65) and (64) are identical, the surface which first received the incident rays in the case of (64), being now the second surface.

(81.) For the focus of *parallel* rays, we have, from (64),

$$f = -\frac{1}{m-1} \cdot \frac{rr'}{r'-r} \ldots\ldots (66).$$

(82.) The formulæ just found for the meniscus apply to the *concavo-convex* lens, recollecting that when the convexity is turned to incident light $r > r'$, and the reverse, $r' > r$, when the concavity is thus turned.

For the first of these cases we have (64),

$$\frac{1}{v} - \frac{1}{u} = -(m-1)\left(\frac{1}{r} - \frac{1}{r'}\right) \ldots\ldots (64),$$

which, since $\frac{1}{r} < \frac{1}{r'}$, will be more expressive if written

$$\frac{1}{v} - \frac{1}{u} = (m-1)\left(\frac{1}{r'} - \frac{1}{r}\right) \ldots\ldots (67).$$

The second member of this equation is positive, and by referring to the case of the double concave lens (art. 74), we shall find that the convergency destroyed by the concavo-convex lens is the *difference of the effects of its two surfaces*, while in the double concave lens it was the sum of the same effects.

It is obvious that turning the concave side of this lens to incident light does not alter the effect of the lens, as was shown in the case of the meniscus.

The virtual focal length of the concavo-convex lens for *parallel* rays, when the convex side of the lens is turned to the incident pencil, is

$$f = \frac{1}{m-1} \cdot \frac{rr'}{r-r'} \ldots\ldots (68).$$

(83). For two *spherical surfaces* of the *same curvature*, we have $r = r'$, and (28) gives

$$\frac{1}{v} - \frac{1}{u} = 0.$$

The effect is that of a plane glass.

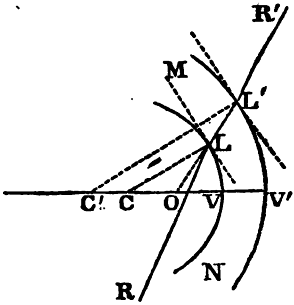
CHAP. IV.

FORMATION OF IMAGES BY REFRACTION.

(84.) The subject of the formation of images by lenses becomes simple, by introducing the consideration of the ray which passes through the two surfaces of the lens, at points where the tangents are parallel.

Prop. XXIII. *All the rays which suffer no deviation by refraction, by a lens, pass through a single point.*

Fig. F.



In the figure, let RL be a ray, refracted by the first surface of the lens MN into LL', and finally emerging in the direction $L'R'$, parallel to RL. Produce LL' until it intersects the axis of the lens in O. Since, by hypothesis, the tangents at the points L and L' are parallel, the radii CL and $C'L'$ are also parallel, and the triangles COL and $C'OL'$ are similar. Whence,

$$C'O : CO :: C'L' : CL, \text{ or,}$$

$$C'O = CO \cdot \frac{C'L'}{CL}, \text{ and}$$

$$C'C = C'O - CO = CO \cdot \left(\frac{C'L'}{CL} - 1\right).$$

Calling $CL = r$, $C'L' = r'$, the thickness of the lens $= t$, and $CO = u'$, we have

$$C'C = C'V - CV = C'V' - VV' - CV = r' - t - r, \text{ and}$$

$$r' - r - t = u' \cdot \left(\frac{r'}{r} - 1\right), \text{ or,}$$

$$u' = (r' - r - t) \cdot \left(\frac{r}{r' - r}\right), \text{ and}$$

$$u' = r - t \cdot \frac{r}{r' - r} \; \ldots\ldots (69).$$

This value of CO is made up of quantities, constant for the same lens; from which we infer, that *all the rays which experience no deviation in passing through a lens, would, if produced after the first refraction, meet in a single point in the axis of the lens.* This point is called the *centre* of the lens.

(85.) *The distance of the centre of a lens from the vertex of the first surface* is found, readily, from equation (69); for, since $VO = CV - CO = r - u'$, we have, by taking the value of u' from (69) and calling $r - u'$, x.

$$x = r - u' = t \cdot \frac{r}{r' - r} \; \ldots\ldots (70).$$

The distance from the centre of a lens to the vertex of its first surface, is equal to the thickness of the lens, multiplied by the radius of that surface, and divided by the *difference* of the radii of the two surfaces.

In the double convex lens, r is negative and r' positive, whence,

$$x = -\frac{rt}{r' + r}.$$

The sign of (x) VO shows that, in this case, it lies on the right-hand side of the vertex. Since $\frac{r}{r' + r}$ is a fraction, $x < t$, the *centre* is therefore between the two surfaces.

In the equiconvex lens $r' = r$, and

$$x = -\frac{t}{2}.$$

The centre is midway between the vertices. It is from this circumstance that the point, which we have defined, derives its name. The same remarks apply to the double concave lens, since for that lens r is positive and r' negative, whence,

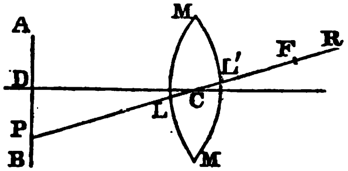
$$x = -\frac{rt}{r' + r},$$

the same expression which we have above.

(86.) To use the position of the centre of the lens, in determining the image formed by an object, we observe, that one ray of the pencil, which proceeds from every point of the object to the lens, passes through this centre. This ray is called the principal ray or axis of the pencil, and when the lens is thin may be regarded as suffering no refraction. It does not fall perpendicularly, nor nearly so, upon the surface which it meets, and, therefore, in strictness, the refraction of an oblique pencil should be investigated and applied to this case. Approximate results may, however, be obtained, by taking the focal distance already determined for a direct pencil; this distance being found, for the pencil proceeding from each point of the object, we have a series of points, the assemblage of which constitutes the image. An application of this method is given in the following proposition.

(87.) Prop. XXIV. *The object, of which the image by a convex lens is required, is a plane perpendicular to the axis of the lens.*

Fig. G.



Let *AB* represent a section of the object, *MM* that of a double convex lens, *PC* a line drawn from any point in the object through the centre, *C*, of the lens; this line may be regarded as the axis of a pencil of rays proceeding from *P*, and may, farther, be considered to suffer no refraction. Call a the distance *DC*; u, *PC*; and θ the angle *DCP*: we have from the triangle *DCP*,

$$a = u \cdot \cos\theta, \text{ whence}$$

$$\frac{1}{u} = \frac{\cos\theta}{a},$$

but from equation (38), article 63,

$$\frac{1}{v} = \frac{1}{u} - \frac{1}{f} \; \ldots\ldots (38),$$

or substituting for $\frac{1}{u}$ its value just found,

$$\frac{1}{v} = \frac{\cos\theta}{a} - \frac{1}{f} \; \ldots\ldots (71).$$

The polar equation of a *conic section* referred to the focus.

From this equation, inferences might be drawn similar to those, found in the chapter on the formation of images by mirrors.

(88.) When the *object subtends a small angle*, we may consider its section as a circular arc; the image will be, also, a circular arc, since if u is constant (38), v will be so; and the arcs will, evidently, be similar. If the distance of the object and image, respectively, from the centre of the lens, be called a and v, their magnitudes d and d', we shall have

$$\frac{d'}{d} = \frac{v}{a} \ldots\ldots (72);$$

θ being assumed very small, equation (71) gives, making $\cos\theta = 1$,

$$\frac{1}{v} = \frac{f-a}{af}, \text{ or}$$

$$v = \frac{af}{f-a}:$$

this value of v substituted in (72), gives

$$\frac{d'}{d} = \frac{f}{f-a} \ldots\ldots (73).$$

As long as $a > f$, $f - a$ is negative, and the image is real. To show the results of this case more clearly, put equation (73) under the form

$$\frac{d'}{d} = -\frac{f}{a-f}.$$

If $a > 2f$, $a - f > f$ and $\frac{d'}{d}$ is a fraction, or the image is less than the object.

When $a = 2f$, $\frac{d'}{d} = 1$, the image is equal to the object.

For $a < 2f$, $a - f < f$, and the image is larger than the object.

The object still approaching the lens when $a = f$, $\frac{d'}{d} = \infty$, and no image is formed.

Next, if $a < f$, equation (73) gives for

$$\frac{d'}{d} = \frac{f}{f-a},$$

a positive value, and the image is now a virtual one, on the same side of the lens with the object. It is greater than the object until $f - a = f$, or $a = o$, that is, until the object touches the lens.

Since the rays cross at the centre of the lens, it is evident that when the object and image are on the same side of the lens, or the image is virtual, the latter is erect; when on different sides, it is inverted. The sign of $\frac{d'}{d}$, therefore, determines whether the image will be erect or inverted with respect to the object, the positive sign corresponding to an erect image, the negative to an inverted one.

(89.) For the *double concave lens* from (57), article 76,

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f}.$$

Whence we derive, by following the same process as for the convex lens,

$$\frac{1}{v} = \frac{\cos\theta}{a} + \frac{1}{f}, \text{ and}$$

$$\frac{d'}{d} = \frac{f}{f+a},$$

an expression which is always positive, and a fraction: hence the image formed by a concave lens is on the same side of the lens with the object, erect, and less than the object.

We have spoken only of sections of the object, image, and lens; the remarks made in the chapter on the formation of images by mirrors, (Chap. II., art. 37,) apply equally to this case.

(90.) Having found an expression for the ratio of the linear magnitudes of an object and its image formed by a double convex lens, if we would view this image at the distance of distinct vision, (pp. 48 and 49, text,) the apparent magnitude will be increased, in the ratio of the distance of the object from the eye, to the limit of distinct vision. Let δ' express the former distance, δ the latter, the magnifying power of a convex lens used as above described, will be expressed by

$$\mu = \frac{\delta'}{\delta} \cdot \frac{d'}{d}, \text{ or } \mu = \frac{\delta'}{\delta} \cdot \frac{f}{f-a} \ldots\ldots (74).$$

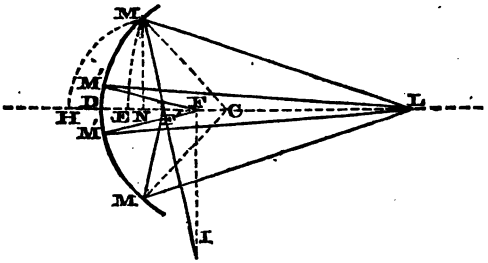
where f is the focal length, and a the distance of the object from the lens.

CHAP. V.

SPHERICAL ABERRATION OF MIRRORS AND LENSES.

(91.) In the text, pages 57 and 58, the fact is stated that the rays which fall upon a spherical mirror, at a distance from the axis, are not converged to the same point, with those nearer to the axis. This is illustrated in the annexed figure, in which LM, LM are the extreme rays of a pencil diverging from L, and F' is the point on the axis at which the reflected rays MF', MF' meet; LM', LM' are two rays meeting the mirror near to its vertex D, the focus of the reflected rays $M'F$ and $M'F$ being at F.

Fig. H.



FF' is the aberration in length, or ***longitudinal aberration***, of the reflected pencil, and if from F a perpendicular to the axis be drawn meeting the reflected ray MF' in I, FI will be half the aberration in breadth, or ***lateral aberration*** of the same pencil.

(92.) Prop. XXV. ***To determine the aberration of a pencil of rays reflected by a spherical mirror.***

First: To find the amount of the ***longitudinal*** aberration.

With the centre L and radius LM describe an arc cutting the axis of the mirror in E. According to the usual notation $LD = u$, $CD = r$; to distinguish between DF' and DF, call $DF' = v'$ and $DF = v$, and let $MN = y$, the semi-breadth of the portion of the mirror occupied by the pencil.

The relation of the segments CF' and LC to the sides $F'M$ and LM in the triangle LMF', gives (as in Prop. I.),

$$\frac{F'C}{F'M} = \frac{LC}{LM}$$

But $LM = LE = LD - ED$, and $ED = ND - NE$, or, since ND is the versed sine of the arc MD to the radius CD,

$$ND = \frac{y^2}{2r};$$

for the same reason $NE = \frac{MN^2}{2LM}$, or, using for LM its approximate value LD,

$$NE = \frac{y^2}{2u}, \text{ whence,}$$

$$ED = \frac{y^2}{2}\left(\frac{1}{r} - \frac{1}{u}\right), \text{ and}$$

$$LE = u - \frac{y^2}{2}\left(\frac{1}{r} - \frac{1}{u}\right),$$

$$LE = u\left[1 - \frac{y^2}{2u}\left(\frac{1}{r} - \frac{1}{u}\right)\right].$$

Taking the reciprocal of this value of LE, and neglecting the terms containing the powers of the sine, y, divided by the diameter $2u$, after $\frac{y^2}{2u}$ we have,

$$\frac{1}{LM} = \frac{1}{LE} = \frac{1}{u}\left[1 + \frac{y^2}{2u}\left(\frac{1}{r} - \frac{1}{u}\right)\right].$$

By a similar mode of proceeding, using the versed sines NH and ND, instead of ND and NE, we should obtain

$$\frac{1}{F'M} = \frac{1}{F'H} = \frac{1}{v'}\left[1 - \frac{y^2}{2v'}\left(\frac{1}{v'} - \frac{1}{r}\right)\right].$$

But $F'C = r - v'$, and $LC = u - r$; and substituting the values of $F'C$, $F'M$, LC and LM in the ratio found in the beginning of this article,

$$\frac{r - v'}{v'}\left[1 - \frac{y^2}{2v'}\left(\frac{1}{v'} - \frac{1}{r}\right)\right] =$$

$$= \frac{u - r}{u}\left[1 + \frac{y^2}{2u}\left(\frac{1}{r} - \frac{1}{u}\right)\right].$$

Dividing by r and performing the multiplications by the quantities outside of the vinculum, in each member of the equation just found,

$$\frac{1}{v'} - \frac{1}{r} - \frac{y^2}{2v'}\left(\frac{1}{v'} - \frac{1}{r}\right)^2 =$$

$$= \frac{1}{r} - \frac{1}{u} + \frac{y^2}{2u}\left(\frac{1}{r} - \frac{1}{u}\right)^2 \ldots\ldots (75).$$

We have thus a general relation between v' and u in terms of the radius and semi-aperture of the mirror.

(93.) If in (75) we use for $\frac{1}{v'} - \frac{1}{r}$, in the multiplier of $\frac{y^2}{2v'}$, its approximate value, derived from equation (1), art. (7), namely

$$\frac{1}{v} - \frac{1}{r} = \frac{1}{r} - \frac{1}{u}, \text{ we obtain}$$

$$\frac{1}{v'} - \frac{1}{r} - \frac{y^2}{2v'}\left(\frac{1}{r} - \frac{1}{u}\right)^2 =$$

$$= \frac{1}{r} - \frac{1}{u} + \frac{y^2}{2u}\left(\frac{1}{r} - \frac{1}{u}\right)^2, \text{ or,}$$

$$\frac{1}{v'} - \frac{1}{r} = \frac{1}{r} - \frac{1}{u} + \frac{y^2}{2}\left(\frac{1}{u} + \frac{1}{v'}\right)\left(\frac{1}{r} - \frac{1}{u}\right)^2,$$

farther, by using for $\left(\frac{1}{u} + \frac{1}{v'}\right)$ the value given by (1),

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r},$$

and substituting this in the equation last found,

$$\frac{1}{v'} - \frac{1}{r} = \frac{1}{r} - \frac{1}{u} + \frac{y^2}{r}\left(\frac{1}{r} - \frac{1}{u}\right)^2, \text{ or,}$$

$$\frac{1}{v'} = \frac{2}{r} - \frac{1}{u} + \frac{y^2}{r}\left(\frac{1}{r} - \frac{1}{u}\right)^2 \ldots\ldots (76).$$

In this equation, which gives the value of $\frac{1}{v'}$, corresponding to a point of incidence distant from the vertex, we find the reciprocal of the approximate focal length, obtained when the rays were supposed to meet the mirror near the vertex, namely, $\frac{2}{r} - \frac{1}{u}$, and a correction for aberration. This correction contains $\frac{y^2}{r}$, a quantity proportional to the versed sine of the semi-angle of the pencil, and therefore depending upon this angle, or the semi-aperture of the mirror; and also r, and u, the radius of the mirror and distance of the radiant point. If these latter quantities are constant, the aberration is a function of the semi-aperture of the mirror. The correction for aberration is additive, showing that the reciprocal of the focal length, for rays distant from the vertex, is greater than the reciprocal focal length of those near the vertex, or that the point F' is nearer to the mirror than F.

(94.) For *parallel* rays $\frac{1}{u} = o$, and from equation (76),

$$\frac{1}{v'} = \frac{2}{r} + \frac{y^2}{r^3}, \text{ whence,}$$

$$v' = \frac{r^3}{2r^2 + y^2};$$

performing the division, and neglecting the powers of y higher than the second,

$$v' = \frac{r}{2} - \frac{y^2}{4r} \ldots\ldots (77).$$

The *correction for aberration is*, therefore, $-\frac{y^2}{4r}$, or $-\frac{y^2}{8f}$, or is *subtractive, and equal to the square of the semi-aperture of the mirror divided by eight times the principal focal length.*

(95.) Second: To find the *lateral* aberration of the extreme ray.

The value of FI, which measures the lateral aberration of the extreme ray, may be obtained as follows. In the similar triangles $F'FI$ and $F'MN$,

$$\frac{FI}{FF'} = \frac{MN}{F'N}, \text{ and } FI = FF' \cdot \frac{MN}{F'N}.$$

To approximate to the ratio $\frac{MN}{F'N}$, $F'D$ may be taken instead of $F'N$, and the value of the aberration is

$$FI = FF' \cdot \frac{MN}{F'D} \ldots\ldots (78),$$

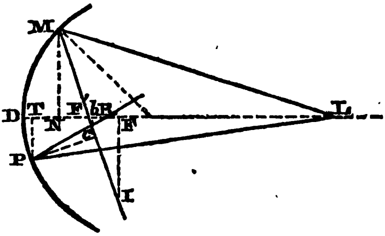
in which all the terms are known when FF' has been determined.

(96.) We propose in this article to determine the position and magnitude of the physical focus of a mirror, or of the circle which includes all the rays of a reflected pencil, when they are spread over the least space.

Prop. XXVI. *To determine the position and magnitude of the circle of least aberration, in a pencil of rays reflected by a concave mirror.*

In the figure let LM be the extreme ray of a pencil, incident upon the mirror, MF' the corresponding reflected ray, F the focus of rays very near the vertex. Farther, let LP be any incident ray, in the lower portion of the pencil; PR the corresponding reflected ray intersecting MF' produced in c: draw cb perpendicular to the axis, from the point c. If we suppose the arc DP very small, the reflected ray PR will coincide very nearly with the axis, and the distance cb will be indefinitely small; as the arc DP increases, the reflected rays being removed farther from the axis, *cb*, at first, increases; it afterwards diminishes as the point of intersection of *PR* with the axis approaches to *F′*, and when $DP = DM$, *PR* coincides with *PF′*, and *cb* vanishes. Between the case, then, in which *DP* is very small and $DP = DM$, there is a position of the incident ray *LP*, for which the reflected ray *PR* gives a maximum value for *cb* When *cb* is a maximum, all the rays of the reflected pencil pass through the circle of which that line is the radius, which is, thus, the physical focus of the mirror. The question resolves itself into determining the values of *F′b* and *cb* when the latter is a maximum.

Fig. I.



Call MN, y; PT, y'; DF, v; DF', v'; FF', the longitudinal aberration, a; $F'b = x$; $bc = z$. Since (art. 93) the aberration of a ray is proportional to the square of the distance of its point of incidence, from the axis of the mirror,

$$FR : FF' :: PT^2 : MN^2 :: y'^2 : y^2 \text{, or}$$

$$FF' - FR : FF' :: y^2 - y'^2 : y^2 \text{, whence}$$

$$F'R = a \, . \, \frac{y^2 - y'^2}{y^2}.$$

But from the similar triangles $F'bc$ and $F'NM$,

$$bc : bF' :: MN : F'N \text{, or}$$

$$bc = bF' \, . \, \frac{MN}{F'N}.$$

And from the similar triangles Rbc and RTP

$$Rb : bc :: RT : TP \text{, or}$$

$$Rb = bc \, . \, \frac{RT}{TP};$$

substituting for bc in this expression the value found above,

$$Rb = bF' \cdot \frac{MN}{F'N} \cdot \frac{RT}{TP},$$

and by the notation,

$$Rb = x \cdot \frac{y}{F'N} \cdot \frac{RT}{y'}.$$

If we approximate, by considering RT and $F'N$ to be equal,

$$Rb = x \cdot \frac{y}{y'}, \text{ and}$$

$$F'R = Rb + bF' = x \cdot \frac{y}{y'} + x = x \cdot \frac{y + y'}{y'}.$$

Equating this value of $F'R$ with the one before found,

$$x \cdot \frac{y + y'}{y'} = a \cdot \frac{y^2 - y'^2}{y^2}, \text{ whence}$$

$$x = a \cdot \frac{y'}{y^2} \cdot \frac{y^2 - y'^2}{y + y'}, \text{ or}$$

$$x = a \cdot \frac{y'}{y^2} (y - y') \ldots\ldots (79).$$

As we have supposed the ray LM to remain fixed, and LP to take different positions, and have found,

$$z = bc = bF' \cdot \frac{MN}{F'N},$$

bc (or z) will be a maximum when bF' (or x) is a maximum; but from (79), x is a maximum, since a and y are constant, when $y'(y - y')$ is a maximum, or when

$$y'(y - y') = y'^2, \text{ or}$$

$$y' = \frac{y}{2}.$$

In that case, from (79),

$$x = \frac{ay^2}{4y^2} = \frac{a}{4} \ldots\ldots (80), \text{ and,}$$

$$z = x \cdot \frac{MN}{F'N} = \frac{a}{4} \cdot \frac{MN}{F'N}.$$

If a perpendicular, FI, be drawn from the focus F, of rays incident near the vertex, to the axis, meeting the extreme ray MF' in I, by article (95),

$$\frac{MN}{F'N} = \frac{FI}{FF'},$$

whence the value of z, or $\frac{a}{4} \cdot \frac{MN}{F'N}$, becomes

$$z = \frac{FI}{4} \ldots\ldots (81).$$

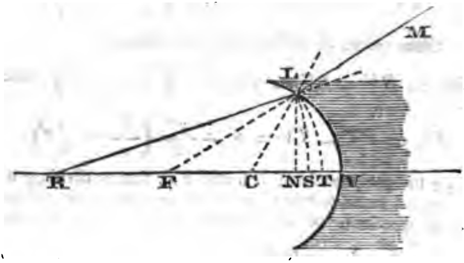
From these values, (80) and (81), of x and z, it appears, that *the distance of the circle of least aberration, from the focus of rays near the vertex, is three fourths of the longitudinal aberration of the extreme ray, and that the radius of the same circle is one fourth of the lateral aberration of the extreme ray.*

Spherical Aberration of Lenses.

(97.) In this investigation we begin by determining the aberration produced by a single surface. We shall assume the light to pass from a rarer into a denser medium, as when it enters a lens through its first surface.

Prop. XXVII. *To determine the aberration produced by a single refracting surface.*

Fig. K.



Let the ray RL fall upon the spherical surface LV at any point L, and be refracted into the direction LM. Continue LM until it intersects the axis of the surface, at F. Draw the radius CL. Call m the ratio of the sine of incidence to that of refraction, in the passage of the ray from the rarer to the denser medium, the sine of refraction being unity; then, by proceeding as in article (80), Chap. III., we find

$$\frac{RC}{RL} = m \cdot \frac{FC}{FL}.$$

From the centres R and F with the radii RL and FL, respectively, describe the arcs LS and LT cutting the axis in S and T; SV will be the difference between RV and RL, and TV that between FV and FL. If the perpendicular LN be let fall, from L, upon the axis RV, $SV = NV - NS$, and $TV = NV - NT$. As in the notation of Chap. III., let $RV = u$, $FV = u'$, $CV = r$; and call LN, y. NS is the versed sine of the arc LS, NT of LT, and NV of LV; and if for the chord of each of those arcs we substitute, as an approximate value, the sine, we have

$$NS = \frac{y^2}{2RL},$$

$$NT = \frac{y^2}{2FL},$$

$$NV = \frac{y^2}{2CV},$$

or substituting for RL and FL, the approximate values RV and FV,

$$NS = \frac{y^2}{2u}, \quad NT = \frac{y^2}{2u'}, \quad NV = \frac{y^2}{2r}, \text{ whence,}$$

$$SV = NV - NS = \frac{y^2}{2}\left(\frac{1}{r} - \frac{1}{u}\right), \text{ and}$$

$$TV = NV - NT = \frac{y^2}{2}\left(\frac{1}{r} - \frac{1}{u'}\right).$$

From these values of SV and TV we obtain,

$$RL = RV - SV = u - \frac{y^2}{2}\left(\frac{1}{r} - \frac{1}{u}\right), \text{ and}$$

$$FL = FV - TV = u' - \frac{y^2}{2}\left(\frac{1}{r} - \frac{1}{u'}\right).$$

Taking the reciprocals of RL and FL, that is dividing unity by the values just found for those lines, and neglecting the terms which involve the quotients of the powers of y^2 by those of u, after the first term, $\left(\frac{y^2}{2u^2}\right)$, we have

$$\frac{1}{RL} = \frac{1}{u}\left[1 + \frac{1}{u}\left(\frac{1}{r} - \frac{1}{u}\right)\frac{y^2}{2}\right],$$

$$\frac{1}{FL} = \frac{1}{u'}\left[1 + \frac{1}{u'}\left(\frac{1}{r} - \frac{1}{u'}\right)\frac{y^2}{2}\right].$$

From the figure we have $RC = RV - CV = u - r$, and $FC = FV - CV = u' - r$, and the equation for the relation of RC, RL, FC and FL, becomes

$$\frac{u-r}{u}\left[1 + \frac{1}{u}\left(\frac{1}{r} - \frac{1}{u}\right)\frac{y^2}{2}\right] =$$

$$= m\,\frac{u'-r}{u'}\left[1 + \frac{1}{u'}\left(\frac{1}{r} - \frac{1}{u'}\right)\frac{y^2}{2}\right].$$

Performing the divisions by u and u', indicated by the terms of the equation, and dividing both sides of the equation by r,

$$\left(\frac{1}{r} - \frac{1}{u}\right)\left[1 + \frac{1}{u}\left(\frac{1}{r} - \frac{1}{u}\right)\frac{y^2}{2}\right] =$$

$$= m\left(\frac{1}{r}-\frac{1}{u'}\right)\left[1+\frac{1}{u'}\left(\frac{1}{r}-\frac{1}{u'}\right)\frac{y^2}{2}\right],$$

performing the multiplications required by the expressions,

$$\frac{1}{r}-\frac{1}{u}+\frac{1}{u}\left(\frac{1}{r}-\frac{1}{u}\right)^2\frac{y^2}{2}=$$

$$= m\left(\frac{1}{r}-\frac{1}{u'}\right)+\frac{m}{u'}\left(\frac{1}{r}-\frac{1}{u'}\right)^2\frac{y^2}{2}\text{, whence,}$$

$$\frac{m}{u'}=\frac{1}{u}+(m-1)\frac{1}{r}+\frac{y^2}{2}$$

$$\left[\frac{m}{u'}\left(\frac{1}{r}-\frac{1}{u'}\right)^2-\frac{1}{u}\left(\frac{1}{r}-\frac{1}{u}\right)^2\right]\ldots\ldots(82).$$

In this value of $\frac{m}{u'}$, the first two terms correspond to the value found, (26) art. 50, on the supposition that the pencil is small, and the third contains the correction for the aberration, produced by the single spherical surface.

(98.) The expression just found, may be simplified by substituting in the terms of the second member for u', its approximate value from equation (26), art. 50. From that equation

$$\frac{m}{u'}=\frac{1}{u}+(m-1)\frac{1}{r}\text{, whence,}$$

$$\frac{1}{u'}=\frac{1}{m}\left(\frac{1}{u}+(m-1)\frac{1}{r}\right)\text{, and}$$

$$\frac{1}{r}-\frac{1}{u'}=\frac{1}{r}-\frac{1}{m}\left(\frac{1}{u}+(m-1)\frac{1}{r}\right)\text{, or,}$$

$$\frac{1}{r}-\frac{1}{u'}=\frac{1}{m}\left(\frac{1}{r}-\frac{1}{u}\right)\text{, and}$$

$$\left(\frac{1}{r}-\frac{1}{u'}\right)^2=\frac{1}{m^2}\left(\frac{1}{r}-\frac{1}{u}\right)^2.$$

In order to reduce, with greater convenience, the coefficient of $\frac{y^2}{2}$ in (82) to its simplest form, call that coefficient k, then substituting in it the approximate values of $\frac{m}{u'}$ and $\left(\frac{1}{r}-\frac{1}{u'}\right)^2$, just obtained, we have,

$$k=\left(\frac{1}{u}+(m-1)\frac{1}{r}\right)\cdot\frac{1}{m^2}\left(\frac{1}{r}-\frac{1}{u}\right)^2$$

$$-\frac{1}{u}\left(\frac{1}{r}-\frac{1}{u}\right)^2\text{, or,}$$

$$k = \left(\frac{1}{r} - \frac{1}{u}\right)^2 \cdot \left[\frac{1}{m^2}\left(\frac{1}{u} + (m-1)\frac{1}{r}\right) - \frac{1}{u}\right],$$

$$k = \left(\frac{1}{r} - \frac{1}{u}\right)^2 \cdot \left[\frac{1-m^2}{u} + \frac{m-1}{r}\right] \frac{1}{m^2},$$

$$k = \left(\frac{1}{r} - \frac{1}{u}\right)^2 \cdot \left(\frac{1}{r} - \frac{m+1}{u}\right) \cdot \frac{m-1}{m^2}$$

whence (82) becomes

$$\frac{m}{u'} = \frac{1}{u} + \frac{m-1}{r} + \frac{m-1}{m^2}$$

$$\cdot \left(\frac{1}{r} - \frac{m+1}{u}\right) \cdot \left(\frac{1}{r} - \frac{1}{u}\right)^2 \cdot \frac{y^2}{2} \ldots\ldots (83).$$

(99.) When the surface is convex, r is negative, and (83) takes the form,

$$\frac{m}{u'} = \frac{1}{u} - \frac{m-1}{r} - \frac{m-1}{m^2}$$

$$\cdot \left(\frac{1}{r} + \frac{m+1}{u}\right) \cdot \left(\frac{1}{r} + \frac{1}{u}\right)^2 \cdot \frac{y^2}{2} \ldots\ldots (84).$$

And for converging rays, u being negative,

$$\frac{m}{u'} = -\frac{1}{u} - \frac{m-1}{r} - \frac{m-1}{m^2}$$

$$\left(\frac{1}{r} - \frac{m+1}{u}\right) \cdot \left(\frac{1}{r} - \frac{1}{u}\right)^2 \cdot \frac{y^2}{2} \ldots\ldots (85).$$

The term in (85) which contains the correction for aberration, will vanish if either of the factors composing it should be equal to zero.

First, let

$$\frac{1}{r} - \frac{m+1}{u} = o, \text{ then } \frac{m+1}{u} = \frac{1}{r}, \text{ or,}$$

$$1 : r :: m + 1 : u.$$

There is, therefore, no aberration for converging rays, falling upon a convex spherical surface, when the distance of the radiant point is a fourth proportional to 1, r, and $m + 1$. From whence the result on page 56, of the text, is easily deduced.

Next, let

$$\frac{1}{r} - \frac{1}{u} = o, \text{ and } u = r,$$

or the incident rays converge to the centre of the spherical surface.

(100.) In art. 42, it was remarked that making $m = -1$ in the formulæ for refraction, the cases would represent the corresponding ones in reflexion. Making $m = -1$ in (83) we have,

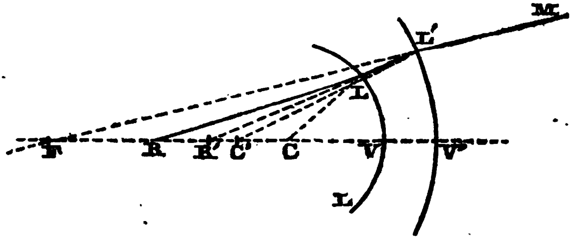
$$-\frac{1}{u'} = \frac{1}{u} - \frac{2}{r} - \frac{2}{r}\left(\frac{1}{r} - \frac{1}{u}\right)^2 \cdot \frac{y^2}{2}\text{, or,}$$

$$\frac{1}{u'} = \frac{2}{r} - \frac{1}{u} + \frac{y^2}{r}\left(\frac{1}{r} - \frac{1}{u}\right)^2,$$

a result which agrees with equation (76), art. 93.

(101.) PROP. XXVIII. *To determine the aberration in a pencil of rays, after refraction by a spherical lens.*

Fig. L.



R being the radiant point of a pencil of rays falling upon the lens *LVV'L'*, let *RL* be the extreme ray of the pencil, and *R'* the virtual focus of the extreme rays, after refraction by the first surface of the lens. If now we suppose a pencil to proceed from *R'*, considered as in the denser medium, the extreme ray of this pencil, *R'L'*, will be refracted into the direction, *L'M*, which, if continued backward to *F*, will give the virtual focus of the extreme rays.

As before, represent *RV* by u, *R'V* by u', and *CL* by r; farther, let $R'V' = v'$, $FV' = v$, $CL' = r'$, and $VV' = t$.

By the preceding proposition (equation 83,) we have

$$\frac{m}{u'} = \frac{1}{u} + \frac{m-1}{r} + \frac{m-1}{m^2}$$

$$\cdot\left(\frac{1}{r} - \frac{m+1}{u}\right)\cdot\left(\frac{1}{r} - \frac{1}{u}\right)^2 \cdot \frac{y^2}{2} \;\ldots\ldots (83).$$

The case of the second surface will correspond to that of the first, if we consider *F* the radiant point, and *R'* the virtual focus; v must be written in (83), for u, v' for u', and r' for r; we then obtain

$$\frac{m}{v'} = \frac{1}{v} + \frac{m-1}{r'} + \frac{m-1}{m^2}$$

$$\left(\frac{1}{r'} - \frac{m+1}{v}\right)\cdot\left(\frac{1}{r'} - \frac{1}{v}\right)^2 \cdot \frac{y^2}{2}\text{, or,}$$

$$\frac{1}{v} = \frac{m}{v'} - \frac{m-1}{r'} - \frac{m-1}{m^2}$$

$$\cdot\left(\frac{1}{r'} - \frac{m+1}{v}\right)\cdot\left(\frac{1}{r'} - \frac{1}{v}\right)^2\cdot\frac{y^2}{2} \ldots\ldots (86);$$

but $v' = u' + t$, whence $\frac{m}{v'} = \frac{m}{u'+t}$, performing the division and neglecting the powers of t above the first,

$$\frac{m}{v'} = \frac{m}{u'} - \frac{mt}{u'^2}.$$

We may farther approximate to this value of $\frac{m}{v'}$, by substituting for $\frac{1}{u'^2}$, in the second member of the equation, its approximate value, from (26), art. 50, namely,

$$\frac{1}{u'^2} = \frac{1}{m^2}\left(\frac{1}{u} + \frac{m-1}{r}\right)^2; \text{ whence,}$$

$$\frac{m}{v'} = \frac{m}{u'} - \frac{t}{m}\left(\frac{1}{u} + \frac{m-1}{r}\right)^2:$$

in which the value of $\frac{m}{u'}$, from (83), being written,

$$\frac{m}{v'} = \frac{1}{u} + \frac{m-1}{r} - \frac{t}{m}\left(\frac{1}{u} + \frac{m-1}{r}\right)^2 +$$

$$\frac{m-1}{m^2}\left(\frac{1}{r} - \frac{m+1}{u}\right)\cdot\left(\frac{1}{r} - \frac{1}{u}\right)^2\cdot\frac{y^2}{2}.$$

By substituting for $\frac{m}{v'}$, in equation (86), its value just found, we have

$$\frac{1}{v} = \frac{1}{u} + (m-1)\left(\frac{1}{r} - \frac{1}{r'}\right) - \frac{t}{m}\left(\frac{1}{u} + \frac{m-1}{r}\right)^2 +$$

$$\frac{m-1}{m^2}\left[\left(\frac{1}{r} - \frac{m+1}{u}\right)\left(\frac{1}{r} - \frac{1}{u}\right)^2 - \left(\frac{1}{r'} - \frac{m+1}{v}\right)\right.$$

$$\left.\cdot\left(\frac{1}{r'} - \frac{1}{v}\right)^2\right]\cdot\frac{y^2}{2} \ldots\ldots (87).$$

We see, in this formula, first, the two terms which denote the reciprocal of the focal distance of an indefinitely small pencil; second, the correction for thickness; and, in the last term, the correction for aberration.

(102.) The general formula, (87), becomes less complex, and gives results of considerable practical importance, when applied to the case of parallel rays.

Prop. XXIX. *The incident rays being parallel, to determine the aberration of the pencil after refraction by a spherical lens.*

In this case, $\frac{1}{u} = 0$, and (87) becomes,

$$\frac{1}{v} = (m-1)\left(\frac{1}{r} - \frac{1}{r'}\right) - \frac{t}{m} \cdot \frac{(m-1)^2}{r^2} +$$
$$\frac{m-1}{m^2}\left[\frac{1}{r^3} - \left(\frac{1}{r'} - \frac{m+1}{v}\right) \cdot \left(\frac{1}{r'} - \frac{1}{v}\right)^2\right] \cdot \frac{y^2}{2} \ldots\ldots (88)$$

The correction for thickness, contained in the second term, has already been separately considered, articles 55, 67, &c.; we may therefore leave it out of the question here, making in (88) $t = 0$. Farther, to approximate to the value of v, we may substitute for $\frac{1}{v}$ in the second member of the same equation, the approximate value $\frac{1}{f}$, or $(m-1)\left(\frac{1}{r} - \frac{1}{r}\right)$, obtained by making $\frac{1}{u} = 0$ in (28), art. 53.

We have, then, from (88),

$$\frac{1}{v} = \frac{1}{f} + \frac{m-1}{m^2}\left[\frac{1}{r^3} - \left(\frac{1}{r'} - \frac{m+1}{f}\right) \cdot \left(\frac{1}{r'} - \frac{1}{f}\right)^2\right] \cdot \frac{y^2}{2},$$

or, taking the value of v, dividing by the denominator thus found, and neglecting the powers of f higher than the second,

$$v = f - \frac{m-1}{m^2}\left[\frac{1}{r^3} - \left(\frac{1}{r'} - \frac{m+1}{f}\right) \cdot \left(\frac{1}{r'} - \frac{1}{f}\right)^2\right] \cdot \frac{y^2 f^2}{2};$$

the *aberration in length*, therefore, is represented by

$$a = -\frac{m-1}{m^2}\left[\frac{1}{r^3} - \left(\frac{1}{r'} - \frac{m+1}{f}\right) \cdot \left(\frac{1}{r'} - \frac{1}{f}\right)^2\right] \cdot \frac{y^2 f^2}{2} \ldots\ldots (89).$$

(103.) To apply the formula just obtained, to a *double convex* lens, r and f (art. 62,) must be made negative, whence

$$a = -\frac{m-1}{m^2}\left[-\frac{1}{r^3} - \left(\frac{1}{r'} + \frac{m+1}{f}\right) \cdot \left(\frac{1}{r'} + \frac{1}{f}\right)^2\right] \cdot \frac{y^2 f^2}{2}, \text{ or,}$$

$$a = \frac{m-1}{m^2}\left[\frac{1}{r^3} + \left(\frac{1}{r'} + \frac{m+1}{f}\right)\right.$$

$$\left.\cdot\left(\frac{1}{r'} + \frac{1}{f}\right)^2\right] \cdot \frac{y^2 f^2}{2} \; \ldots\ldots (90).$$

This value of the aberration having the positive sign, while the approximate focal length has the negative sign, its effect on the focal length, for rays not near the vertex, is subtractive; showing that the focus of such rays is nearer the lens, than the focus of rays incident near the vertex.

(104.) For an *equi-convex, glass* lens, $r = r'$, $m = \frac{3}{2}$, and $f = r$, disregarding the sign, since f has already been made negative in (90); and from (90),

$$a = \frac{2}{9}\left[\frac{1}{r^3} + \left(\frac{1}{r} + \frac{5}{2r}\right) \cdot \frac{4}{r^2}\right] \cdot \frac{y^2 r^2}{2}, \text{ or,}$$

$$a = \frac{1}{9}\left[1 + \left(1 + \frac{5}{2}\right) \cdot 4\right] \cdot \frac{y^2}{r},$$

$$a = \frac{5}{3} \cdot \frac{y^2}{r}.$$

If we suppose the beam of light to occupy the whole aperture of the lens, y becomes the semi-breadth, and $y^2 = \frac{t}{2} \cdot 2r$ nearly, or $y^2 = rt$, and $t = \frac{y^2}{r}$; writing t for $\frac{y^2}{r}$ in the value of a, just found,

$$a = 1\tfrac{2}{3}t,$$

the result stated in paragraph 3, page 53, of the text.

(105.) If $m = \frac{3}{2}$, and $r : r' :: 2 : 5$, or $r' = \frac{5}{2}r$, we have from (36),

$$\frac{1}{f} = -(m-1)\left(\frac{1}{r} + \frac{1}{r'}\right) = -\frac{7}{10} \cdot \frac{1}{r}, \text{ and}$$

$$f = -\frac{10}{7}r.$$

Substituting these values of m, r', and f in (90), recollecting that f has been already made negative in that equation, and that now its value is to be placed there without regard to the sign, it gives,

$$a = \frac{2}{9}\left[\frac{1}{r^3} + \left(\frac{2}{5r} + \frac{5}{2} \cdot \frac{7}{10r}\right)\right.$$

$$\cdot \left(\frac{2}{5r} + \frac{7}{10r}\right)^2\Bigg] \cdot \frac{10^3 r^3}{7^3} \cdot \frac{y^2}{2f},$$

$$a = \frac{1}{9}\left[1 + \left(\frac{2}{5} + \frac{35}{20}\right) \cdot \left(\frac{2}{5} + \frac{7}{10}\right)^2\right] \frac{10^3}{7^3} \cdot \frac{y^2}{f},$$

$$a = \frac{7}{6} \cdot \frac{y^2}{f}.$$

This is the case of an unequally convex lens, in which the more convex side is turned to incident light.

(106.) In the *plano-convex* lens, if the plane side be turned towards parallel rays, $\frac{1}{r} = 0$, and $f = 2r'$; if the material be glass, $m = \frac{3}{2}$, and from (90) we obtain

$$a = \frac{2}{9}\left[\left(\frac{1}{r'} + \frac{5}{4r'}\right) \cdot \left(\frac{1}{r'} + \frac{1}{2r'}\right)^2\right] \cdot (2r')^3 \cdot \frac{y^2}{2f}, \text{ or,}$$

$$a = \frac{1}{9}\left[\left(1 + \frac{5}{4}\right) \cdot \frac{9}{4}\right] 8 \cdot \frac{y^2}{f}, \text{ or,}$$

$$a = 4.5\,\frac{y^2}{f} = 4.5\,t.$$

The result given in paragraph 1, page 53, of the text.

In the same lens, with the convex side turned to parallel rays, $\frac{1}{r'} = 0$, and $f = 2r$, whence from (90), r and f having already been made negative,

$$a = \frac{1}{9}\left[1 + \frac{5}{4} \cdot \frac{1}{4}\right] 8 \cdot \frac{y^2}{f}, \text{ or,}$$

$$a = 1.17\,\frac{y^2}{f} = 1.17 \cdot t,$$

The result stated in paragraph 2, page 53, of the text.

(107.) PROP. XXX. *To determine the ratio of the radii of the surfaces of a double convex lens, which shall produce the least aberration, with a given focal length and aperture.*

To solve this question we must determine the ratio of r and r' when a is a minimum, f and y being constant.

Differentiating the value of a given in (90), considering r and r' as variable, and disregarding the constant multipliers, we obtain, after changing all the signs,

$$da = \frac{3dr}{r^4} + \left(\frac{1}{r'} + \frac{m+1}{f}\right)\cdot\left(\frac{1}{r'} + \frac{1}{f}\right)\frac{2dr'}{r'^2}$$
$$+ \left(\frac{1}{r'} + \frac{1}{f}\right)^2\cdot\frac{dr'}{r'^2} \ldots\ldots (91).$$

But from equation (36), art. 62, we have

$$\frac{1}{f} = -(m-1)\left(\frac{1}{r} + \frac{1}{r'}\right), \text{ or,}$$

$$\frac{1}{r} + \frac{1}{r'} = -\frac{1}{m-1}\cdot\frac{1}{f}, \text{ whence, by differentiating,}$$

$$\frac{dr}{r^2} + \frac{dr'}{r'^2} = o, \text{ or,}$$

$$\frac{dr'}{r'^2} = -\frac{dr}{r^2}.$$

Substituting in (91) this value of $\frac{dr'}{r'^2}$, and dividing by dr

$$\frac{da}{dr} = \frac{3}{r^4} - \left(\frac{1}{r'} + \frac{m+1}{f}\right)$$
$$\cdot\left(\frac{1}{r'} + \frac{1}{f}\right)\frac{2}{r^2} - \left(\frac{1}{r'} + \frac{1}{f}\right)^2\frac{1}{r^2},$$

which, by the question, is equal to zero. Multiplying by r^2 we obtain

$$\frac{3}{r^2} - 2\cdot\left(\frac{1}{r'} + \frac{m+1}{f}\right)\cdot\left(\frac{1}{r'} + \frac{1}{f}\right)$$
$$-\left(\frac{1}{r'} + \frac{1}{f}\right)^2 = o \ldots\ldots (92).$$

From equation (36), disregarding the sign of f,

$$\frac{1}{r} = \frac{1}{m-1}\cdot\frac{1}{f} - \frac{1}{r'}.$$

Substituting this value in (92), and arranging the terms

$$\frac{3}{r'^2} - \frac{6}{(m-1)fr'} + \frac{3}{(m-1)^2f^2} - \frac{2}{r'^2} - \frac{2(m+2)}{fr'} -$$
$$\frac{2(m+1)}{f^2} - \frac{1}{r'^2} - \frac{2}{fr'} - \frac{1}{f^2} = o, \text{ or,}$$

$$-\left(\frac{6}{m-1} + 2m + 6\right)\cdot\frac{1}{fr'} +$$
$$\left(\frac{3}{(m-1)^2} - 2m - 3\right)\cdot\frac{1}{f^2} = o,$$

and by transposition and multiplication,

$$\left(\frac{6}{m-1} + 2m + 6\right) \cdot \frac{1}{r'} =$$
$$= \left(\frac{3}{(m-1)} - 2m - 3\right) \cdot \frac{1}{f} \ldots\ldots (93).$$

If the lens is of glass, or $m = \frac{3}{2}$,

$$\frac{21}{r'} = \frac{6}{f}, \text{ or } r' = \frac{21}{6} \cdot f;$$

but from (36)

$$\frac{1}{r} = \frac{2}{f} - \frac{1}{r'} = \frac{2}{f} - \frac{6}{21} \cdot \frac{1}{f} = \frac{12}{7} \cdot \frac{1}{f}, \text{ and}$$
$$r = \frac{7}{12} \cdot f.$$

Comparing together the values obtained for r and r',

$$r : r' :: 1 : 6.$$

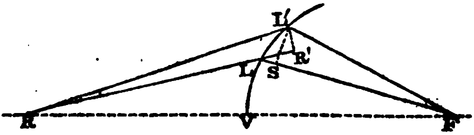
This lens is known to opticians as the *crossed* lens. With the more convex side turned to parallel rays the aberration is $\frac{15}{14} \cdot \frac{y^2}{f}$, which is less than that for the plano-convex lens with the convex side turned to parallel rays.

(108.) It would carry us beyond the limits of this Appendix, to go into the investigation of the aberration of combined lenses.

Before leaving this subject we purpose to show a method by which the surfaces which refract rays accurately to a point, may be determined.

Prop. XXXI. *To determine the curvature of the surface of a medium, so that rays passing into it, from a rarer medium, may be refracted to a point.*

Fig. M.



As we have found a concave surface to give only a virtual focus, we proceed, at once, to examine the case in which the surface of the denser medium is convex. Let R be the radiant point, RL a ray meeting the surface at L and refracted to F: let L' be a point farther from the vertex V than L, RL' being the incident and

$L'F$ the refracted ray for this point. Draw the perpendiculars $L'R'$ and $L'S$ upon the incident and refracted rays RL' and $L'F$, respectively. LR' will be nearly equal to the increment of the incident ray, and LS to the decrement of the refracted ray, in passing from the point L to L'. Call RL, u', and LF, $-v'$. Then if L' be supposed very near to L, $LR = du'$, and $LS = dv'$.

In the triangle $L'LR'$, $\frac{LR'}{LL'} = \cos. RLL' = \sin.$ incidence,

and in $L'LS$, $\frac{LL'}{LS} = \frac{1}{\cos. SLL'} = \frac{1}{\sin. \text{ refraction}}$;

whence,

$$\frac{LR'}{LS} = \frac{\sin. \text{ incidence}}{\sin. \text{ refraction}}, \text{ or,}$$

$$\frac{du'}{dv'} = m, \text{ or,}$$

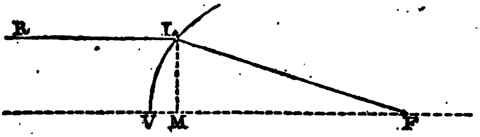
$$du' - m dv' = 0 \; \ldots\ldots (94),$$

the differential equation of the curve which, by a revolution about the axis RF, will produce the surface required. To integrate, let $RV = u$, and $FV = -v$, the complete integral of (94) will be

$$u' - u = m\,(v' - v) \; \ldots\ldots (95).$$

(109.) If the incident rays be *parallel*, $u' - u = VM$, *fig*. N.

Fig. N.



If we put $VM = A - x$, (95) becomes

$$A - x = m\,(v' - v), \text{ whence,}$$

$$v = v' - \frac{A - x}{m}, \text{ or,}$$

$$v = v' - \frac{A}{m} + \frac{x}{m} \; \ldots\ldots (96).$$

The equation for the distance of any point in an ellipse from the farther focus is, (Young's Analyt. Geom. art. 47, p. [illegible]),

$$v = A + ex,$$

in which $e < 1$; with this (96) agrees in form, and will be identical if

$$\frac{1}{m} = e = \frac{c}{A}, \text{ and } v' - \frac{A}{m} = A.$$

Substituting for m in the second of these equations, its value from the first;

$$v' - c = A, \text{ or } v' = A + c.$$

We find, then, that *an ellipsoid of which the semi-transverse axis is to the excentricity as the index of refraction is to unity* $\left(\frac{A}{c} = m\right)$ *will refract parallel rays, accurately, to the farther focus.*

If a lens be formed, of which the first surface is a portion of the ellipsoid just determined, the second surface should be (art. 99.) a portion of a sphere, having the farther focus of the ellipsoid as its centre (*fig.* 38, text).

(110.) Equation (95) may be applied to the case in which the incident pencil passes from a denser to a rarer medium, through a concave surface. Then FL, FL', *fig.* M, would represent the incident rays, and LR, $L'R$ the refracted rays, and the ratio of the sine of incidence to the sine of refraction would be represented by the fraction $\frac{1}{m}$; substituting this for m in (95) we have

$$u' - u = \frac{1}{m}(v' - v) \; \ldots\ldots (97).$$

For the case of parallel rays, (*fig.* 40., p. 55, text,) by proceeding as in the last article, making $u' - u = A - x$,

$$v' - v = m(A - x), \text{ and}$$

$$v = v' - mA + mx;$$

an equation of the same form with that before obtained, and representing the distance of a point in a conic section from the farther focus; in it

$$m = e = \frac{c}{A}, \text{ and } v' - mA = A.$$

Since $m > 1$, $e > 1$, and the equation belongs to a hyperbola, (Young's Analyt. Geom., article 79, p. 104,) the equation of which is

$$v' = A + mA = A + c.$$

If, then, we form a lens with the *first surface plane, and the second that of a hyperboloid of which the excentricity is to the semi-transverse as the index of refraction, of the material of the lens, is to unity, parallel rays, incident perpendicularly upon the first surface of the lens, will be refracted to the farther focus of the hyperboloid which forms the second surface* (*fig.* 40, text).

(111.) The cases in which the aberration of converging rays upon a spherical surface is zero, (art. 99,) are contained in (95); it is unnecessary, however, to discuss it farther

(112.) The forms of mirrors without aberration may also be inferred from the equations just discussed. The *convex* mirror will be given by making $m = -1$ in (94), whence,

$$du' + dv = o, \text{ and integrating}$$

$$u' - v'(-v) = C \; \ldots\ldots (98).$$

By this property we recognize the hyperbola, the distances u' and $-v'$ being those of the point, from the two foci.

For a concave mirror, u' and v' have the same sign, in equation (94), and

$$du' + dv' = o, \text{ or,}$$

$$u' + v' = C \; \ldots\ldots (99).$$

The mirror is an *ellipsoid, the radiant point coinciding with one focus, and the rays being collected at the opposite focus.*

If one focus remove to an infinite distance, the ellipsoid becomes a paraboloid, into the focus of which the rays which have been supposed parallel are collected.

Caustics by Reflexion.

(113.) It is not intended to enter fully into this subject in relation to both reflexion and refraction, but to confine the discussion to examples of the caustics produced by reflexion.

The formula for the oblique pencil, art. 29, &c., gives, in certain cases, an elegant and easy method of determining the form of a section of the caustic surface, produced by reflexion from a spherical mirror.

Prop. XXXII. *To determine the form of the caustic produced by the reflexion of a pencil of rays from a spherical mirror, when the rays are parallel; and also when the radiant point is at a diameter's distance from the vertex of the mirror.*

First. When the radiant point is infinitely distant, or the rays parallel.

LDM representing a section of the mirror, let *RL* be a ray incident upon it and reflected into *LB*; then, the focus of a small pencil meeting the mirror near to *L* will be the point *F* found from the value of v in the equation which concludes art. 30, namely,

$$v = \frac{r}{2} \cdot \cos. \phi.$$

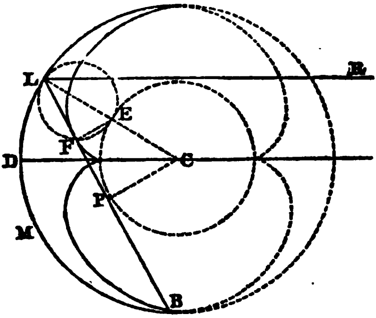
To construct this value of v; let fall from *C*, *CP* perpendicular to the reflected ray *LB*, then

$$LP = LC \cdot \cos. \phi = r \cdot \cos. \phi, \text{ whence,}$$

$$v = \frac{LP}{2}.$$

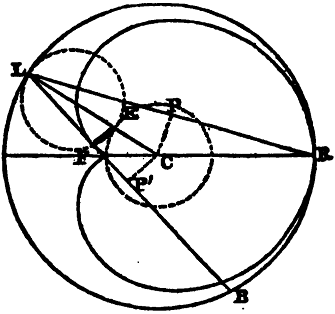
Bisect LC in E, and LP in F, then LFE is a right angle; the point F, of the caustic, is, therefore, in the circumference of a circle of which $LE = \frac{1}{2} LC$ is the diameter. This being true of each point in the curve, the caustic curve is an epicycloid, the diameter of the generating circle of which is equal to the radius of the base; this latter being half the radius of the mirror. This curve and the circle LDM revolving about DC, as an axis, would generate, respectively, the surface of the caustic and that of the mirror.

Fig. O.



SECOND. Let the radiant point be at the extremity of the diameter of the mirror.

Fig. P.



The ray RL is reflected into LB. To find the position of F upon LB, we recur to equation (11), art. 29.

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r \,.\, \cos.\, \phi}.$$

Drawing CP perpendicular to RL,

$$LP = r \,.\, \cos.\, \phi, \text{ whence,}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{LP} = \frac{4}{RL} = \frac{4}{u}$$

$$\frac{1}{v} = \frac{3}{u}, \text{ or,}$$

$$v = \frac{u}{3} = \frac{LB}{3}.$$

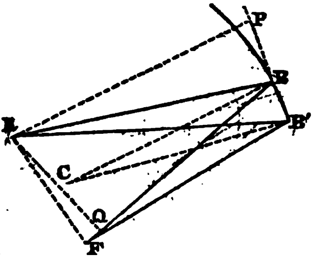
If CE be made $= \frac{CL}{3}$, the perpendicular from E to LB will intersect it in the focus F. The locus of these foci is, therefore, an epicycloid, of which the diameter of the generating circle is to the radius of the base as two to one. This curve is the cardioid

(114.) In considering the subject in a more general point of view, we may determine the equation of the curve of section of the caustic, the position of the radiant point and section of the mirror being given.

Prop. XXXIII. *To determine the equation of the curve which is the section of a caustic formed by a curved mirror. The section being made by a plane passing through the axis of the mirror.*

We refer the curve to polar co-ordinates, the radiant point being the pole.

Fig. Q.



Let B and B' be two points very near each other upon the curve which is a section of the mirror; let C be the centre of the osculating circle to the curve at either of these points, so that the portion of the circle and curve nearly coincide between B and B'. RB, RB' representing two incident rays, BF, $B'F$ are the reflected rays. Call $RB = u$, $BF = v$, the angle $RBC = \phi$, $RB'C = \phi'$, the perpendicular RP, upon the tangent BP, $= p$, the radius $CB = CB' = r$. Join FR and let fall the perpendicular RQ, upon BF. F being a point in the caustic, FR is the radius vector of that point and RQ a perpendicular upon the tangent; call RF, u', and RQ, p'. An equation between u' and p' will be that of the caustic curve.

In the acute angled triangle RFB, since the segment $BQ = RB \,.\, \cos. RBQ$,

$$u'^2 = u^2 + v^2 - 2uv \,.\, \cos. 2\phi \;\ldots\ldots (100);$$

and in the right angled triangle RBQ

$$p' = u \,.\, \sin. 2\phi \;\ldots\ldots (101).$$

To eliminate cos. 2ϕ and sin. 2ϕ, we proceed as follows. Since RP and CB are perpendicular to BP, they are parallel, and the angle $PRB = RBC = \phi$, and

$$\cos. \phi = \frac{p}{u}.$$

But, by trigonometry,

$$\cos. 2\phi = 2 \cos.^2 \phi - 1,$$

and by substituting for cos. ϕ the value just given,

$$\cos. 2\phi = \frac{2p^2}{u^2} - 1.$$

We have also, by trigonometry,

$$\sin. 2\phi = \sqrt{1 - \cos.^2 2\phi}, \text{ or,}$$

$$\sin. 2\phi = \sqrt{\frac{4p^2}{u^2} - \frac{4p^4}{u^4}},$$

$$\sin. 2\phi = \frac{2p}{u}\sqrt{1 - \frac{p^2}{u^2}}.$$

Substituting these values of cos. 2ϕ and sin 2ϕ in (100), and (101), respectively, we obtain

$$u'^2 = u^2 + v^2 - 2uv\left(\frac{2p^2}{u^2} - 1\right), \text{ and}$$

$$p' = 2p\sqrt{1 - \frac{p^2}{u^2}} \;\ldots\ldots (102).$$

The value of u'^2 may be written under the more simple form,

$$u'^2 = (u + v)^2 - \frac{4p^2 v}{u} \;\ldots\ldots (103).$$

The relation between u and v given by equation (11), art. 29, or

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{r\,.\cos.\phi},$$

may be applied to this case by taking r to represent the radius of the osculating circle, which is,

$$r=\frac{udu}{dp}.$$

Substituting this value for r,

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{\frac{udu}{dp}\,.\cos.\phi}=\frac{2dp}{udu\,.\cos.\phi}\text{, or, since}$$

$$\cos.\phi=\frac{p}{u},$$

$$\frac{1}{u}+\frac{1}{v}=\frac{2dp}{pdu}\text{, whence,}$$

$$\frac{1}{v}=\frac{2udp-pdu}{pudu}\text{, and}$$

$$v=\frac{pudu}{2udp-pdu}\ldots\ldots(104).$$

If this value of v be substituted in equation (103), we shall obtain a new equation, which, in conjunction with (102), will give the relation of u' and p' in terms of u, p, du, and dp. The relation of the last four quantities mentioned will be given by the equation of the reflecting curve and by its differential; eliminating these quantities, there will result a single equation between u' and p', the equation of the caustic curve.

(115.) To give an example of this method of proceeding, let the reflecting curve be any portion of a *logarithmic spiral*, of which the equation is,

$$p=mu.$$

The general value of v (104), is first to be applied to this particular case.

Differentiating the equation of the curve,

$$dp=mdu\text{, whence (104) becomes}$$

$$v=\frac{pudu}{2mudu-pdu}=\frac{pu}{2mu-p}=\frac{pu}{2p-p}=u.$$

This value of v substituted in equation (103), gives

$$u'^2=4u^2-\frac{4p^2u}{u}=4u^2-4p^2\text{, or,}$$

$$u'^2=4u^2-4m^2u^2=4u^2(1-m^2)\text{, and}$$

$$u' = 2u\sqrt{1-m^2}.$$

Also, from (102),

$$p' = 2mu\sqrt{1-\frac{m^2u^2}{u^2}} = 2mu\sqrt{1-m^2},$$

or since we have just found

$$2u\sqrt{1-m^2} = u'$$

$$p' = mu'.$$

The section of the caustic surface is, therefore, a *logarithmic spiral differing only in position from the reflecting curve.*

CHAP. VI.

ON THE DOUBLE REFRACTION AND POLARIZATION OF LIGHT.

(116.) Although it does not enter into the design of this Appendix to show the method of deducing, from theoretical considerations, any of the general laws of Optics, I have thought that it may assist the student to give the formulæ to which these considerations lead, or which have been deduced from experiment, in certain particular cases, discussed in the text. The formula, or general law, once remembered, the details of the phenomena flow naturally from it, and the memory is not tasked to recollect individual results.

Double Refraction of Light.

(117.) The formula which represents the law of extraordinary refraction in doubly refracting crystals, becomes, when the incident ray is in a plane passing through the axis of the crystals,

$$m'^2 = m^2 - (m^2 - m'^2) \,.\, \text{sin.}\,^2\phi \;\ldots\ldots (105),$$

in which m' is the index of refraction of the extraordinary ray, m that of the ordinary ray, and ϕ the inclination to the axis. In the spheroids constructed in the text (*figs.* 77. and 79.), to give the index of refraction of the extraordinary ray, if the axis which coincides with that of the rhomb be called b, and that perpendicular to the same axis a, then by the construction $a = \frac{1}{m'}$, and $b = \frac{1}{m}$, whence (105) becomes

$$m'^2 = \frac{1}{b^2} - \left(\frac{1}{b^2} - \frac{1}{a^2}\right) . \text{sin.}\,^2\phi.$$

As long as $m > m'$, or $\frac{1}{b} > \frac{1}{a}$, that is $a > b$,—$\left(\frac{1}{b^2} - \frac{1}{a^2}\right)$

(*fig.* 77), will be negative, whence the term *crystals* with a *negative axis* which applies to this class. When $a < b$ (*fig.* 79), $\frac{1}{b} < \frac{1}{a}$ or $m' > m$, and $-\left(\frac{1}{b^2} - \frac{1}{a^2}\right)$ becomes additive, and the crystals are said to be *crystals* with a *positive axis* of double refraction.

(118.) In the plane of principal section the tangents of the angles of extraordinary and of ordinary refraction are in a constant ratio to each other. In the plane perpendicular to this, the law of the sines applies equally to the extraordinary and to the ordinary ray, but the value of the constant quantity is different for the two rays. These are the only two cases, in which the extraordinarily refracted ray is contained in the plane of incidence.

(119.) When light which has been polarized by double refraction, in the plane of principal section of a crystal Iceland spar (*figs.* 84. and 85., text), passes through a second crystal, the relative brightness of each image, supposing that no light is lost by reflexion or absorption, may be expressed by the following formula; in which *Oo*, *Ee*, *Oe*, and *Eo* represent the images formed as described on page 140 of the text, a is the angle which the plane of principal section of the second rhomb makes with the same plane in the first, and A is the brightness of the incident ray.

$$Oo = \frac{1}{2} A \,.\, \cos.^2 a = Ee \; \ldots\ldots \; (106).$$

$$Oe = \frac{1}{2} A \,.\, \sin.^2 a = Eo \; \ldots\ldots \; (107).$$

The sum of the brightness of the four images,

$$Oo + Ee + Oe + Eo = A\,(\cos.^2 a + \sin.^2 a) = A.$$

From the foregoing formulæ (106. and 107.) we may trace the changes of brightness in the several images, as described in pages 140, 141, of the text (*fig.* 86.)

When the principal sections are parallel, $a = 0$, $\cos. a = 1$, and $\sin. a = 0$, therefore

$$Oo = Ee = \frac{1}{2}A \qquad Oe = Eo = 0.$$

By turning the lower crystal, a assumes a finite value and the images *Oe*, *Eo* appear. As a increases, sin. a increases and cos. a diminishes; *Oe* and *Eo*, therefore, increase in brightness, and *Oo*, *Ee* decrease. When $a = 45°$, $\cos. a = \sin. a$, and the four images are equally bright. The angle a increasing farther, *Oo* and *Ee* become more and more faint, and disappear when $a = 90°$; at this angle $Oe = Eo = \frac{1}{2} A$. The rotation of the lower crystal being continued beyond 90°, cos. a takes the negative sign and increases negatively, while sin. a again diminishes; when $a =$ 180°, cos. $a = -1$, sin. $a = 0$, and Oe, Eo again disappear. At this angle the two images Oo, Ee coalesce, the two extraordinary refractions taking place in opposite directions.

Polarization of Light by Reflexion.

(120.) When light has been *polarized by reflexion* from a surface, upon which it falls at the *maximum polarizing angle*, the following empyrical formula, determined by Malus, will represent the intensity of the light reflected from another surface, upon which the pencil is incident at the polarizing angle: (*fig.* 87, page 143, text.)

$$I = A. \cos.^2 a \; \ldots\ldots \; (108),$$

in which I is the intensity of the reflected light, A that of the incident light, and a the angle between the plane of incidence and that of the second reflexion, or the azimuth of the plane of the second reflexion. When $a = 0$, or 180°, I is a maximum, and when $a =$ 90°, or 270°, $I = 0$, and no light is reflected.

As a consequence of this law, a beam of common light, as far as brightness is concerned, may be represented by two beams of polarized light, having their planes of polarization at right angles to each other: for, the angle between the planes of polarization and of reflexion of the one being called a, that of the other will be $90° - a$, and from (108) we shall have, for the brightness of the two reflected pencils,

$$I = A. \cos.^2 a$$

$$I' = A. \cos.^2 (90 - a) = A. \sin.^2 a;$$

whence,

$$I + I' = A (\cos.^2 a + \sin.^2 a) = A,$$

the sum of the intensities, of the two supposed pencils, remaining the same whatever be the angle a, which is characteristic of common light.

Equation (108) applies to the case of light polarized by refraction, and incident upon a reflecting surface at the angle of complete polarization, a being the angle between the plane of polarization of the incident ray and the plane of reflexion.

(121.) The law, deduced by Sir David Brewster, as expressing the relation between the phenomena of refraction and *polarization by reflexion*, when light falls upon the first surface of a body, is

$$\tan. P = m \; \ldots\ldots \; (109).$$

P being the polarizing angle, and m the index of refraction of the material used.

From this formula, if we suppose the light to be incident at the polarizing angle, and call R the angle of refraction at this incidence,

$$\tan. P = \frac{\sin. P}{\sin. R} \text{ ; but } \tan. P = \frac{\sin. P}{\cos. P} \text{ , whence}$$

$$\sin. R = \cos. P, \ldots\ldots (110),$$

or the maximum *polarizing angle* is the complement of the corresponding angle of *refraction*, and the reflected ray is perpendicular to the refracted ray.

(122.) If the light which has passed through the first surface fall upon a second, parallel to the first, the angle of incidence upon the first surface being P, that on the second is R, and $R = 90 - P$ (110); whence,

$$\tan. R = \cot. P \text{ : but } \cot. P = \frac{1}{\tan. P} = \frac{1}{m} \text{ , and therefore,}$$

$$\tan. R = \frac{1}{m}$$

or the tangent of the incidence upon the second surface is the index of the refraction from the denser to the rarer medium. R is, therefore, the angle of polarization for the second surface, and the light reflected from that surface, as well as that from the first, will be polarized.

Law of Partial Polarization of Light by Reflexion.

(123.) Sir David Brewster has verified by an extensive series of experiments a law, which is due to Fresnel, by which the effect of any number of reflexions, on the inclination of the planes of polarization of a beam of light, may be determined. The effect of a single reflexion at an angle differing from the polarizing angle, is given by the equation

$$\tan. \phi = \tan. x \frac{\cos. (i + i')}{\cos. (i - i')} \ldots\ldots (111),$$

in which formula, i is the angle of incidence, i' the corresponding angle of refraction, x the primitive inclination of the plane of polarization of the polarized ray to the plane of reflexion, and ϕ the inclination of the same planes after reflexion. The angle $i - i'$ is evidently the deviation produced by refraction, and $i + i'$ is the supplement of the angle between the refracted and reflected rays. When $x = 45°$, the case considered in the text, p. 150, $\tan. x = 1$, and

$$\tan. \phi = \frac{\cos. (i + i')}{\cos. (i - i')} \ldots\ldots (112).$$

(124.) The effect of ***successive reflexions*** of a pencil of common light, or in which $x = 45°$, may be deduced from the first equation for the value of tan. ϕ (111); for if θ represent the inclination of the plane of polarization to that of reflexion after n reflexions, $\tan.\theta = \tan.^n \phi$, x and ϕ preserving the same relations to each other after any number of reflexions; whence,

$$\tan.\theta = \frac{\cos.(i + i')^n}{\cos.(i - i')^n} \ldots\ldots (113).$$

Since $\tan.\theta = \tan.^n \phi$, and by the supposition tan. ϕ is not zero, it appears that although partially polarized light may have its planes brought indefinitely near to parallelism, by increasing the number of reflexions, yet tan. θ, and therefore θ, cannot become ***absolutely*** equal to zero by any number of reflexions.

The formula for the quantity of the apparently polarized light could not, advantageously, be introduced in this place.*

Polarization of Light by ordinary Refraction.

(125.) From an examination of the effect produced by a single surface upon the two planes of polarization in the beam of common light, Sir David Brewster inferred, that it depended upon the angle of deviation of the ray, and was represented by the formula,

$$\cot.\phi = \cos.(i - i') \ldots\ldots (114).$$

in which ϕ is the inclination of the planes of polarization to the plane of the refraction, and i and i' the angles of incidence and refraction of the ray. When $i - i' = 0°$ or $i = 90°$, $\cos.(i - i') = 1$, and $\cot.\phi = 1$, or $\phi = 45°$, and no change is produced in the inclination. When $i - i' = 90°$, $\cos.i - i' = 0$, and $\cot.\phi = 0$, or $\phi = 90°$.

When the light is not common light, or light in which the planes of polarization are inclined 45° to the plane of refraction, if x be taken to represent the inclination of the planes of polarization of the beam to the plane of refraction,

$$\cot.\phi = \cot.x.\cos.(i - i') \ldots\ldots (115).$$

If the light fall upon a second surface, parallel to the first, x for that surface is the value of ϕ found for the first, and if θ be called the inclination after n refractions,

$$\cot.\theta = \cot.^n \phi = \cot.^n x.\cos.^n (i - i') \ldots\ldots (116).$$

When $\cot.x = 1$, that is, in the case of common light,

$$\cot.\theta = \cos.^n (i - i') \ldots\ldots (117).$$

(126.) By combining this formula with that for the partial polarization by reflexion, we can readily obtain the effect produced upon light, which should reach the eye, after two refractions, at the first surface of a plate, and an intermediate reflexion at the second surface.

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* Sir D. Brewster, in *Phil. Trans.* (Lon.) 1830.

Let ϕ represent the inclination of the plane of polarization to that of refraction, after refraction by the first surface of the plate; ϕ' the inclination produced by the reflexion at the second surface; and ϕ'' that produced by the second refraction at the first surface, as the ray emerges. Calling, as before, i the angle of incidence on the first surface, i' that of refraction, and x the inclination of the planes of polarization of the incident light to the plane of incidence; then from (115),

$$\cot.\ \phi = \cot.\ x \,.\, \cos.\ (i - i'),\ \text{or}\ \tan.\ \phi = \frac{\tan.\ x}{\cos.\ (i - i')}.$$

From formula (111), for partial polarization by reflexion,

$$\tan.\ \phi' = \tan.\ \phi\ \frac{\cos.\ (i + i'}{\cos.\ (i - i')} . = \tan.\ x\ \frac{\cos.\ (i + i')}{\cos.\ (i - i')^2}$$

Equation (115), applied to the second surface of the plate, gives

$$\cot.\ \phi'' = \cot.\ \phi' \,.\, \cos.\ (i - i');$$

whence, by substituting for cot. ϕ' the reciprocal of the value just found for tan. ϕ',

$$\cot.\ \phi'' = \frac{1}{\tan.\ x} \,.\, \frac{\cos.\ (i - i')^3}{\cos.\ (i + i')},\ \text{or},$$

$$\cot.\ \phi'' = \cot.\ x \,.\, \frac{\cos.\ (i - i')^3}{\cos.\ (i + i')} \quad \ldots . (118).$$

For common light, in which $x = 45°$,

$$\cot.\ \phi = \cos.\ (i - i'),$$

$$\tan.\ \phi' = \frac{\cos.\ (i + i')}{\cos.\ (i - i')^2},$$

$$\cot.\ \phi'' = \frac{\cos.\ (i - i')^3}{\cos.\ (i + i')} \quad \ldots\ldots (119).$$

If, in this latter case,

$$\cos.\ (i - i')^3 = \cos.\ (i + i'),$$

$$\cot.\ \phi'' = 1,\ \text{or}\ \phi'' = 45°,$$

and the light polarized by the first refraction and the intermediate reflexion, will be restored by the refraction at emerging, to the state of common light. The above equation will be satisfied in glass of which $\frac{\sin.\ i}{\sin.\ i'} = m = 1.525$, at 78° 7'.

If $\cos.\ (i - i')^3 > \cos.\ (i + i')$, which would occur by diminishing i, $\cot.\ \phi'' > 1$, and $\phi'' < 45°$.

If $\cos.\ (i - i')^2 = \cos.\ (i + i')$ $\tan.\ \phi' = 1$, $\phi' = 45°$, or the light polarized by the first refraction is restored to common light by the reflexion. When refracted at the second surface, since,

$$\frac{\cos.\ (i-i')^3}{\cos.\ (i+i')} = \cos.\ (i-i') \cdot \frac{\cos.\ (i-i')^2}{\cos.\ (i+i')} = \cos.\ (i-i'),$$

$$\cot.\ \phi'' = \cos.\ (i-i'),$$

or the light is repolarized at the second refraction, and the effect of the plate is that of a single surface.*

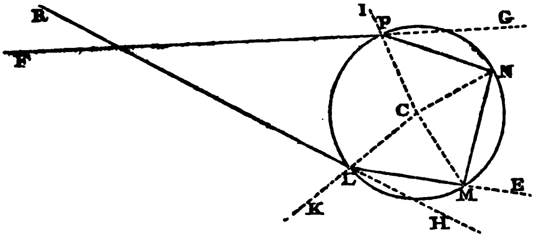
CHAP. VII.

OF THE RAINBOW.

(127.) To explain the theory of the rainbow, we begin by the following proposition.

PROP. XXXIV. *A ray of light enters a refracting sphere, is reflected any number of times, and emerges; to determine the deviation when it is a maximum, or minimum.*

Fig. R.



Let *RL* be a ray of light, meeting the refracting sphere *LMNP* at *L*, and refracted into *LM*; *LM* meeting the second surface of the sphere at *M*, is in part reflected into *MN*, which farther suffers reflexion at *NP*, taking the direction *NP*; that part of *NP* which is not reflected, passes out of the sphere, being refracted into the direction *PF*. By the law of reflexion the angles *CML*, *CMN*, &c., are all equal to *CLM* the angle of refraction at the first surface; the angle of emergence *IPF* is, therefore, equal to the angle of incidence *RLK*. Call the angle of incidence ϕ, that of refraction ϕ'; the angle of deviation of the refracted ray *LM*, or the angle $HLM = \phi - \phi'$; the angle of deviation at emergence, or the angle $NPG = \phi - \phi'$; and the sum of the deviations is $2\ (\phi - \phi'.)$ The deviation produced by the first reflexion, or $EMN = 180 - LMN = 180 - 2\phi'$, and at each succeeding reflexion a new deviation of equal amount is produced; the total deviation, therefore, after n reflexions is $180n - 2n\phi'$. The sum of the effects produced by both refraction and reflection is, calling δ the total deviation,

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* Memoir by Sir D. Brewster, in *Phil. Trans.* (Lon.) 1830

$$\delta = 180n - 2n\phi' + 2\,(\phi - \phi'_{,}) \text{ or}$$

$$\delta = 180n + 2\phi - 2\,(n + 1)\,\phi' \;.\;.\;.\;.\;.\; (120).$$

In which equation ϕ and ϕ' are connected by the equation, (17, art. 39,)

$$\sin.\,\phi = m.\,\sin.\,\phi' \;.\;.\;.\;.\;.\; (17).$$

When δ is a maximum or minimum, $d\delta = o$, and by differentiating (120,) considering ϕ and ϕ' as variable,

$$2d\phi - 2\,(n + 1)\,d\phi' = o, \text{ or}$$

$$d\phi = (n + 1)\,d\phi'.$$

By differentiating (17),

$$d.\,\sin.\,\phi = m.\,d\,\sin.\,\phi', \text{ or}$$

$$\cos.\,\phi\,.\,d\phi = m.\,\cos.\,\phi'\,.\,d\phi',$$

in which substituting the value just found for $d\phi$,

$$(n + 1)\,\cos.\,\phi\,.\,d\phi' = m\,\cos.\,\phi'\,d\phi', \text{ and}$$

$$(n + 1)\,\cos.\,\phi = m\,.\,\cos.\,\phi'.$$

To combine this with (17), square both equations and add, we have

$$\sin.\,^2\phi + (n + 1)^2.\,\cos.^2\,\phi = m^2.\,(\sin.^2\,\phi' + \cos.^2\,\phi');$$

but, $\sin.^2\,\phi = 1 - \cos.^2\,\phi$, and $\sin.^2\,\phi' + \cos.^2\,\phi' = 1$, whence

$$\cos.^2\,\phi\,\Big((n + 1)^2 - 1\Big) + 1 = m^2$$

$$\cos.\,^2\phi = \frac{m^2 - 1}{(n+1)^2 - 1} = \frac{m^2 - 1}{n^2 + 2n} = \frac{m^2 - 1}{n\,(n + 2)} \text{ and}$$

$$\cos.\,\phi = \sqrt{\frac{m^2 - 1}{n\,(n + 2)}} \;.\;.\;.\;.\;.\; (121).$$

(128.) The *primary rainbow* is formed by two refractions and one reflexion of the sun's light, by drops of rain, as shown in *fig.* 134, page 224 of the text.

Producing the incident and emergent rays *RF* and *Oq*, until they meet at q; (120) gives by making $n = 1$,

$$\delta = 180 + 2\phi - 4\phi'.$$

The angle q (RqO) is the supplement of the deviation δ, whence

$$q = 4\phi' - 2\phi \;.\;.\;.\;.\;.\; (122),$$

and since q increases as δ diminishes, q is a maximum when δ is a minimum. Near the maximum value, q will change less for a given change of incidence than at other values; and near this maximum, therefore, the incident pencil will emerge most copiously, and affect the eye most strongly. When δ is a minimum, we have from the preceding article, by making $n = 1$ in equation (121),

$$\cos.\phi = \sqrt{\frac{m^2-1}{3}} \ldots\ldots (123);$$

in which, if for m, the index of refraction of water for the differently colored rays be substituted, the angle of incidence will be found at which each color is most copiously transmitted to the eye. The angle q will be given at the same time by equation (122), and by the relation,

$$\sin.\phi = m\,.\,\sin.\phi' \ldots\ldots (17).$$

To determine the limits of the value of q for the differently colored rays, we take the value of m for the least and most refrangible of those rays: for the *red* $m = \frac{108}{81}$, and for the *violet* $m = \frac{109}{81}$. These values substituted for m, in equation (123), we obtain from (123), for the *red* rays,

$\cos.\phi = .5092$, $\phi = 59°\ 21'$, and $\sin.\phi = .8603$, whence from (17) $\sin.\phi' = .6452$ and $\phi' = 40°\ 11'$; therefore from (122), $q = 160°\ 44' - 118°\ 42' = 42°\ 2'$.

For the *violet* rays the same equations give,

$\cos.\phi = .5199$, $\phi = 58°\ 41\tfrac{1}{2}'$, and $\sin.\phi = .8543$; whence $\sin.\phi' = .6352$, and $\phi' = 39°\ 25'$, and $q' = 157°\ 40' - 117°\ 23' = 40°\ 17'$.

The breadth of the bow is measured by the angle $qOq' = OnR - q' = q - q' = 42°\ 2' - 40°\ 17' = 1°\ 45'$. This supposes the rays to flow from a point. The angle q being greater than q', the line Oq is above Oq', and the red is the highest color in the bow.

(129.) The *secondary rainbow*, shown in the same figure of the text, is formed by two reflexions and two refractions: it corresponds to the case of $m = 2$ in the formula for the deviation. From this formula (120)

$$\delta = 360 + 2\phi - 6\phi';$$

but the angle GqO between the incident and emergent rays is the excess of the angle of deviation above two right angles, whence

$$q = 180 + 2\phi - 6\phi' \ldots\ldots (124).$$

By the same reasoning which was used in the preceding article, it may be shown, that the different colors will be transmitted most copiously, at incidences given by equation (121), in which $n = 2$, and m is the index of refraction corresponding to the colored rays of which the incidence is sought. From (121)

$$\cos.\phi = \sqrt{\frac{m^2-1}{8}} \ldots\ldots (125).$$

The relation of ϕ and ϕ' is given by

$$\sin.\ \phi = m\ .\ \sin.\ \phi' \ldots\ldots (17).$$

The limits of the value of q will be found by placing for m in (125), and (17), the index of refraction for the least and most refrangible rays, or $m = \frac{108}{81}$ and $m = \frac{109}{81}$.

By proceeding as in the last article, we have for the *red* rays: $\cos.\ \phi = .3118$, $\phi = 71° 49\frac{1}{2}'$, $\sin.\ \phi = .9501$, $\sin.\ \phi' = .7126$, $\phi' = 45° 27'$, and $q = 50° 57'$.

For the *violet* rays: $\cos.\ \phi = .3184$, $\phi = 71° 27\frac{1}{2}$, $\sin.\ \phi' = .7046$, $\phi' = 14° 48'$, whence $q' = 54° 7'$.

The angle, q', for the violet rays, being greater than the corresponding angle for the red, the violet is higher than the red, in the bow; the colors are therefore inverted in relation to those of the primary. The angle $q'Oq = q' - q = 3° 10'$.

The angular distance between the bows, $qOq = 50° 57' - 42' 2' = 8° 55'$.

(130.) The breadth of the bows, and of the space between them, having been measured on the supposition that the rays flow from a point, correction must be made for the apparent diameter of the solar disc, which is about 32′. On this account the breadth of each bow is increased by 32′, so that the primary is 2° 17′ in breadth, and the secondary 3° 42′. The breadth of the space between the two bows is, thus, diminished by 32′, and is 8° 23′. The angle, q, for the highest red of the primary bow will be (42° 2′ + 16′) 42° 18′; whence, if the sun is more than 42° 18′ above the horizon, the primary bow is not seen; the corresponding limit for the secondary bow is 54° 23′.

(131.) A portion of the light which enters any drop of rain, is lost at each reflexion: for, by art. 41, in order that total reflexion shall take place at the separating surface of the denser and of the rarer medium, the relation $m \sin.\ \phi' = 1$, or > 1, must subsist; but from the investigation it appears, that $\sin.\ \phi$ is always less than unity, and that the condition necessary to total reflexion is never satisfied. The colors of the secondary bow are therefore fainter than those of the primary.

The method of investigating the theory of the bows formed by three or more reflexions combined with two refractions, must be obvious from what has been said in relation to the primary and secondary bows.

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